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Static and dynamic Young's Modulus for Lower Cretaceous chalk. A low frequency scenario.

Introduction

Deformation properties of a rock are governing factors for production of oil from a reservoir. These properties can be determined either from geotechnical compression testing (also called static tests) or from acoustic data (dynamic tests). The main differences between the two tests lie in the frequency of the measurements and the strain amplitude used in the tests. When an acoustic wave propagates through a porous medium the deformation of the grains is elastic. In the geotechnical test the strain is larger and a non-elastic deformation of the sample can occur.

Several studies have shown that static and dynamic properties are different e. g. Wang (2000). In these studies the acoustic modulus was compared to the static modulus obtained from the first loading curve. The ratio between the dynamic and static modulus was found to be between one and 20 (Wang 2000). The low ratios are for stiff rocks and the higher ratios occur for softer sediments. For a solid material like steel the ratio is 1 (Weast 1986). The difference between static and dynamic moduli is often explained by the difference in frequency and strain amplitude between the static and the dynamic test (Yale et al. 1995).

In this study we show that difference in frequency probably is of little significance in chalk with low permeability, and we will explain the difference by a non-elastic deformation during static loading. This non-elastic deformation is described with a non-elastic modulus determined from the loading curve alone. We introduce a new way of comparing the static and the dynamic modulus. When the non-elastic modulus is taken into account, the static and dynamic modulus are not compared directly but are put into a model relating the static modulus to the elastic and non-elastic modulus.

The mechanical loading can be described as a sum of an elastic and a non-elastic deformation (e.g. Hansen 2001), so that the total strain increment becomes a sum of the elastic strain increment and the non-elastic strain increment.

$$d\varepsilon = d\varepsilon_{elastic} + d\varepsilon_{non-elastic} \qquad (1)$$

This equation can be rewritten to an equation with moduli instead of strains if it is assumed that both strain increments are proportional to stress and controlled by stress alone.

$$\frac{1}{E} = \frac{1}{E_{elastic}} + \frac{1}{E_{non-elastic}} \qquad (2)$$

Based on the principle of simultaneous elastic and non-elastic deformation we present a method to determine the non-elastic modulus from mechanical loading tests. This modulus quantifies the influence from the non-elastic deformation on the static modulus. The elastic modulus equals the acoustic modulus which is determined from sonic logs. The non-elastic and elastic moduli are then used to calculate the modulus for the static loading with Equation (2).

The idea is first to show that the difference between static and dynamic moduli is caused by a non-elastic deformation during the static loading and that this non-elastic deformation can be quantified by a non-elastic modulus. When this non-elastic modulus is combined with the elastic modulus from acoustic data it is possible to calculate a Young's modulus in agreement with the measured Young's modulus from the loading curve. Secondly we show that the unloading modulus from the unloading curve equals the acoustic modulus. If the acoustic modulus equals the unloading modulus where the sample expands elastically then the elastic modulus is not dependent on frequency in the frequency interval from static measurements and up to sonic logging frequency for low permeable chalk. By using the BISQ model by Dvorkin et al. (1994) we thirdly show that the success of the predictions may be a consequence of the small pore size of the Lower Cretaceous Chalk.

The data used in this study is logging data from two wells in the Valdemar field in the North Sea and results from geotechnical testing of core samples taken from the two wells Nord Jens 1 and Valdemar 2p. The geotechnical data are published in a project report (Christensen 1999) and the logging data were kindly provided by Mærsk Oil and Gas AS.

Method

The elastic Young's modulus is determined from logging data and the non-elastic modulus is determined from the non-elastic deformation during loading (Figure 1).

The permanent strain after each of the 3 loading cycles is plotted versus the maximum stress in each of the loading cycles (Figure 2). The strain versus stress (Figure 2) is approximated with a straight line. The inverse slope of this line is the non-elastic modulus.

From the loading curve the Young's modulus during the third loading is determined as the tangent to the loading curve (Figure 1). The third loading curve is chosen in order to minimize influence from bedding. The Young's modulus during unloading is determined as the slope of the unloading curve.

Equation (2) is used to calculate the modulus for the deformation where elastic and non-elastic deformation occurs at the same time. This is done in order to compare the calculated modulus and the Young's modulus measured from the loading curve.

Results and Discussion

When the acoustic modulus and non-elastic modulus are used to calculate the total modulus Equation (2) we find that the calculated modulus is close to the modulus measured from the loading curve (Figure 3).

The acoustic modulus equals the unloading modulus in the beginning of the unloading (Figure 4). During unloading the sample is not forced to deform and we think that the expansion of the sample is pure elastic. Plona & Cook (1995) also found that the unloading modulus and the acoustic modulus equal each other for dry sandstones. The modulus from unloading must be the elastic modulus if the sample only expands elastically. If it equals the elastic modulus from acoustic measurements then it means that the elastic modulus is not dependent on frequency in the interval from static measurements and up to sonic logging frequency.

It is well known from the literature that acoustic velocity can be dependent on frequency and that the acoustic velocity tends to increase with increasing frequency. Dvorkin & Nur (1993) introduced a model to determine the influence of frequency on the acoustic velocity. In this model the characteristic squirt flow length, R, is introduced. This characteristic squirt flow length has the order of the pore diameter. In this study we work with low permeable chalk. From Kozeny's equation the average pore diameter was calculated to be $2.4 \cdot 10^{-7}$ m for lowest porosities and $1.3 \cdot 10^{-6}$m for highest porosities. Based on the model of Dvorkin & Nur (1993) the acoustic velocity as function of frequency was calculated for a low porous and a highly porous sample, with permeabilities of respectively 0.07mD and 4.5 mD (Figure 5).

The velocity data calculated by the equations of Dvorkin et al. (1994) show frequency dependence for pore sizes higher than the actual pore size (figure 5). The Lower Cretaceous chalk does not show any velocity dispersion in the interval of frequency up to 1MHz when the value of R is less then 10^{-5}m for the low porous chalk and 10^{-4}m for highly porous chalk. Above these values of R the velocity starts to increase when the frequency approaches 1MHz.

We thus conclude that Lower Cretaceous chalk will be in the low frequency range for frequencies up to 1MHz.

That the velocity is independent on frequency for low permeable chalk can explain why the elastic modulus in the static tests and elastic modulus from acoustic measurements equal each other. It may also explain why water saturated P-wave velocity as noted by Borre (1998) may be calculated from dry by using Gasmanns equation.

Conclusions

The studied data indicate that the discrepancy between static and dynamic moduli can be explained by the non-elastic strain component invariably present in geotechnical tests and that is absent in dynamic acoustic measurements.

The Young's modulus from the unloading compares well with the acoustic modulus from sonic logs. The elastic modulus is thus not dependent on frequency in the interval of frequency from static tests and up to sonic log frequency.

Acknowledgement

Log data were kindly provided by Mærsk Oil and Gas AS.

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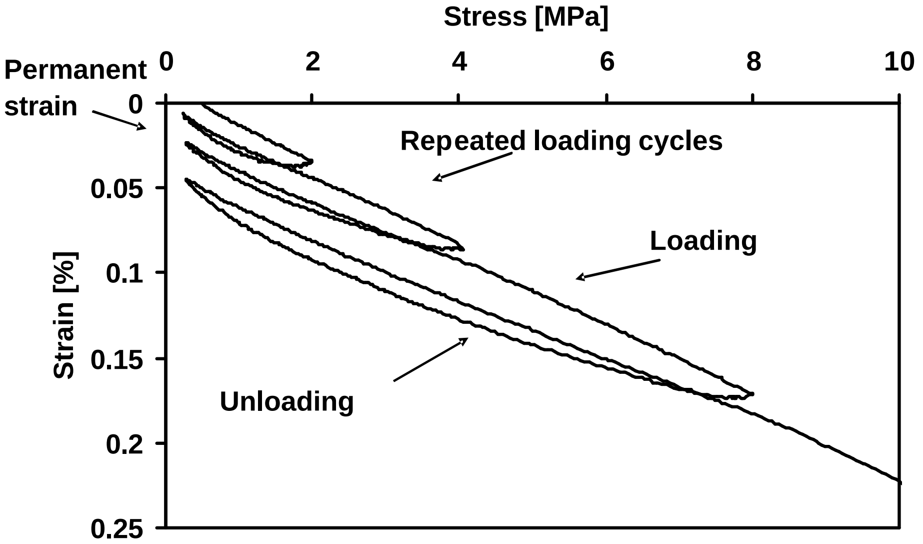

Figure 1: Stress-strain relationship during a uniaxial unconfined compression test. The repeated loading up to 2, 4 and 8 MPa causes increasing permanent deformation. Loading curve from Christensen (1999).

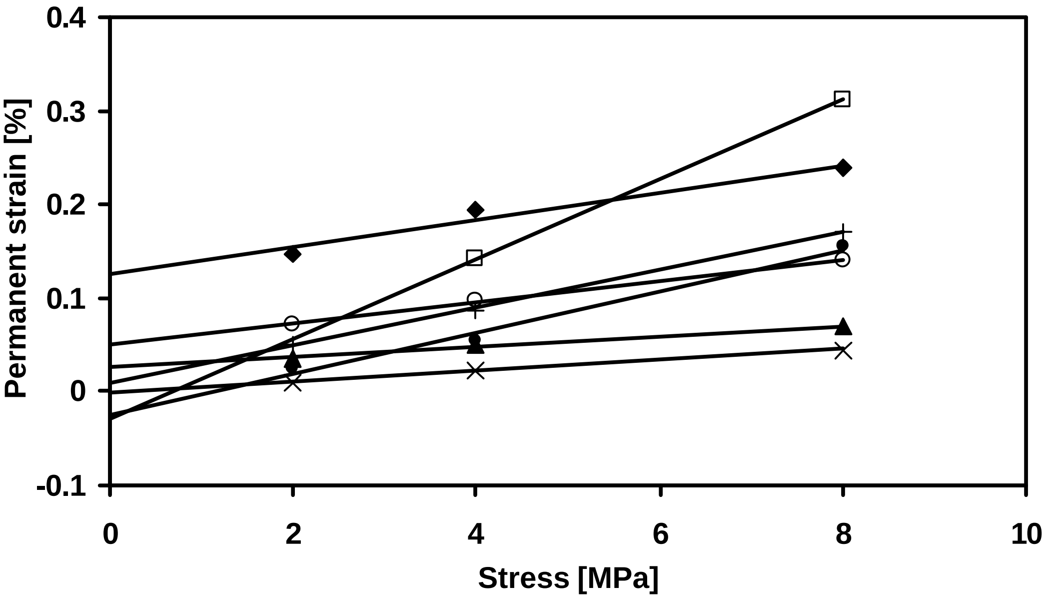

Figure 2: Non-elastic strain vs. maximum stress during repeated loading of seven samples. Strain is measured by LVDT. The plot of strain vs. stress is approximated by a straight line and the slope inversed defines the non-elastic modulus. The intercept on the y-axis is a reflection of bedding in the testing apparatus.

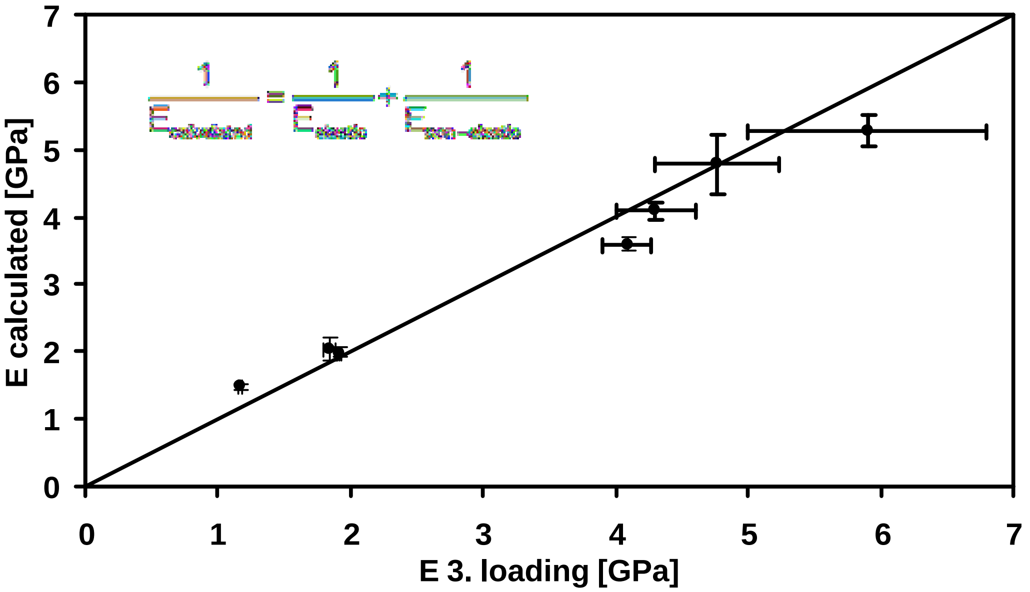

Figure 3: Calculated Young’s modulus vs. Young’s from the unconfined uniaxial compression experiments.

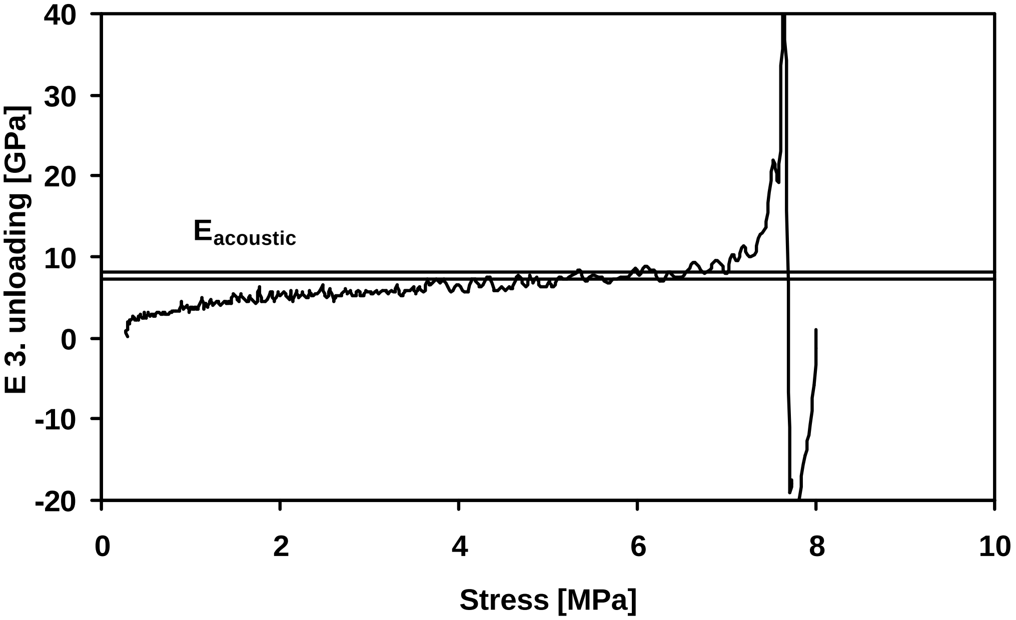

Figure 4: Young's modulus from unloading versus stress compares well with the upper and lower acoustic modulus estimates.

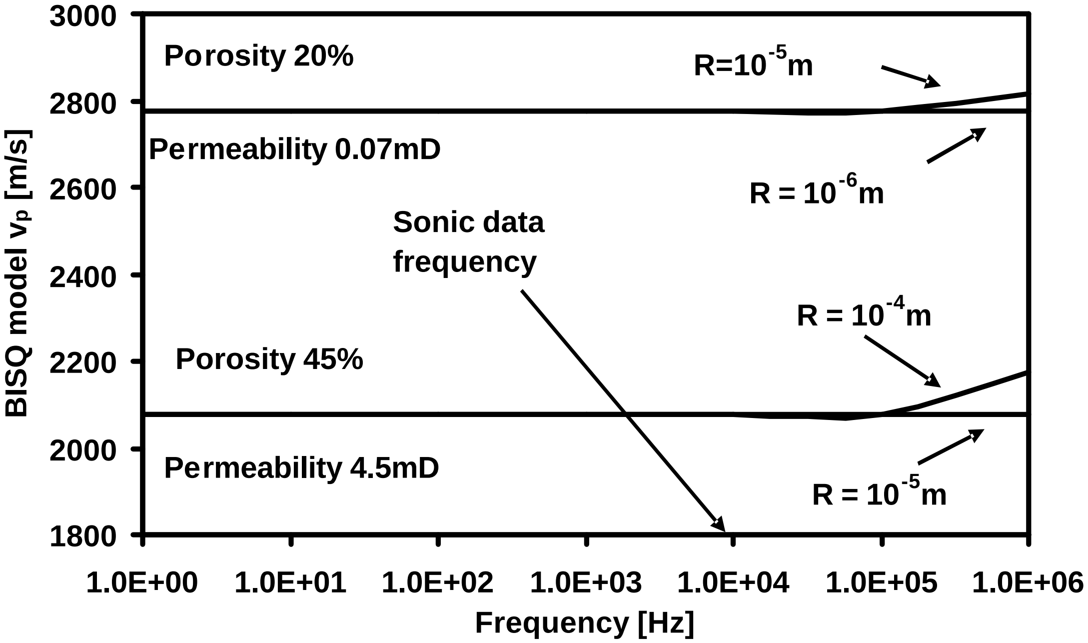

Figure 5: Velocity vs. frequency for two different samples indicates that sonic logging frequency is in the low frequency interval for the lower cretaceous chalk. Curves for the assumptions of pore size is indicated for each sample. For lower cretaceous chalk a pore size of 10^{-6}m is estimated from electron micrographs.