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Models for Oil Recovery from Naturally Fractured Reservoirs, Considering Multiple Block Size, Capillarity And Gravity Effects.

Theory

Oil displacement in naturally fractured reservoirs (NFR) under waterflooding is considered to be the result of capillarity and gravity effects only, which implies that external pressure gradients due to water injection are negligible, thus:

$$q_{o_{tot}} = q_{o_{grav}} + q_{o_{imb}} . \qquad (1)$$

On the other hand, for a matrix block of size h_{ma}[1]:

$$q_{o\ imb} = \frac{k_o A}{\mu_o B_o} \frac{p_c}{h_{ma}} \qquad (2)$$

$$q_{o\ grav} = \frac{A k_o}{\mu_o B_o} g \Delta\rho \ , \qquad (3)$$

where pressure gradients due to capillarity and gravity effects are p_c / h_{ma} and $g\Delta\rho$, respectively.

It can be observed that gravity pressure gradients and $q_{o\ grav}$ are independent of h_{ma}, while $q_{o\ imb}$ decreases as h_{ma} increases; so for large blocks $q_{o\ grav} > q_{o\ imb}$, which is a common statement, but not because the increment of $q_{o\ grav}$ with h_{ma}; this is the result of the decrement of $q_{o\ imb}$ as h_{ma} increases.

Eqs. 2 and 3 are in Darcy units; if the flow rate is in bl / day, area in square meters and the rest of the variables in the International Metric System, then the corresponding equations are:

$$q_{o\ imb} = 5.364 \times 10^{-4} \frac{A k_o}{\mu_o B_o} \frac{p_c}{h_{ma}} \qquad (4)$$

$$q_{o\ grav} = 5.364 \times 10^{-4} \frac{A k_o}{\mu_o B_o} g \Delta\rho . \qquad (5)$$

The total flow rate q_{ot} , for a NFR having NB block sizes, is:

$$q_{ot} = \sum_{j=1}^{NB} n_j q_{oj\ tot} \ , \qquad (6)$$

where n_j is the number of blocks of a given size.

If f_j is defined as the volume fraction of the blocks of size h_{maj}:

$$f_j = \frac{A_j n_j h_{maj}}{v_R} = \frac{V_{Rj}}{V_R}, \qquad (7)$$

then Eq. 6 can be written as:

$$q_{ot} = v_R \sum_{j=1}^{NB} \frac{f_j}{A_J h_{ma\,j}} q_{oj\,tot}. \qquad (8)$$

Now, taking into account Eqs. 4 and 5 into Eq. 8:

$$q_{ot} = 5.364 \times 10^{-4} V_R \left[p_c \sum_{i=1}^{NB} \frac{k_{oj} f_j}{\mu_{oj} B_{oj} h_{maj}^2} + g\Delta\rho \sum_{j=1}^{NB} \frac{k_{oj} f_j}{\mu_{oj} B_{oj} h_{maj}} \right] \qquad (9)$$

where

$$q_{ot\,imb} = 5.364 \times 10^{-4} V_R p_c \sum_{j=1}^{NB} \frac{k_{oj} f_j}{\mu_{oj} B_{oj} h_{maj}^2} \qquad (10)$$

$$q_{ot\,grav} = 5.364 \times 10^{-4} V_R g\Delta\rho \sum_{j=1}^{NB} \frac{k_{oj} f_j}{\mu_{oj} B_{oj} h_{maj}}. \qquad (11)$$

are the imbibition and gravity oil flow rates, for the whole NFR, having NB block sizes.

If a continuous block size distribution[2] $f_D(h_D)$ is considered, the corresponding equations for the total imbibition and gravity oil flow rates are:

$$q_{ot\;\;imb} = 5.364\ X10^{-4} V_R p_c \int_{h_{D\,Mín}}^{h_{D\,Máx}} \frac{k_o\ f_D}{\mu_o B_o h_{máx}^2 h_D^2} dh_D \qquad (12)$$

$$q_{ot\;\;grav} = 5.364 \times 10^{-4} V_R g\Delta\rho \int_{h_{D\,mín}}^{h_{D\,máx}} \frac{k_o\ f_D}{\mu_o B_o h_{máx} h_D} dh_D. \qquad (13)$$

For the particular case of the linear, continuous, block size distribution:

$$f_D(h_D) = mh_D + b, \qquad (14)$$

the above flow rates are:

$$q_{ot\,imb} = 5.364 \times 10^{-4} V_R p_c \int_{h_{Dmín}}^{h_{Dmáx}} \frac{k_o}{\mu_o B_o h_{máx}^2} \left(\frac{m}{h_D} + \frac{b}{h_D^2} \right) dh_D \qquad (15)$$

$$q_{ot\,grav} = 5.364 \times 10^{-4} V_R g\Delta\rho \int_{h_{Dmín}}^{h_{Dmáx}} \frac{k_o}{\mu_o B_o h_{máx}} \left(m + \frac{b}{h_D} \right) dh_D. \qquad (16)$$

In these equations:

$$h_D = \frac{h_{ma}}{h_{máx}}. \qquad (17)$$

The theory just developed can be used to obtain the oil flow rates for a certain time , for instance at the beginning of the process. Based on this theory, the oil recovery behavior as a function of time can be studied as follows:

For a water wet matrix block, the imbibition oil flow rate decline is given by[3]:

$$q_{o\,imb} = \frac{Nrt}{\tau} e^{-t/\tau} , \qquad (18)$$

where Nrt is the recoverable oil by imbibition and τ is the time to recover the amount of oil (1-1/e)Nrt.

From a practical point of view, a better model to represent the imbibition oil flow rate is:

$$q_{o\,imb} = \alpha \frac{Nrt}{\tau} e^{-\beta\, t/\tau} , \qquad (19)$$

where α is a dimensionless constant to adjust the initial oil flow rate for a set of field conditions and β is another dimensionless constant, in this case to adjust the rate at which $q_{o\,imb}$ declines with time. In practice, α and β can be obtained from historical data, laboratory tests or applying the available models (Eqs. 10, 12 or 15).

Cumulative oil production by this mechanism, for each time interval, is calculated by:

$$\Delta Np = \bar{q}_{o\,imb}\ \Delta t , \qquad (20)$$

taking Δt as small as necessary and $\bar{q}_{o\ imb}$ as a constant in this time interval.

The oil flow rate by gravity effects $q_{o\ grav}$, from a matrix block of size h_{ma}, is given by Eq. 5; the variation of $q_{o\ grav}$ with time is given by the way that k_o and μ_o, B_o change as time goes on, through the changes of oil saturation and pressure.

Illustrating examples and discussion of results are presented for each one of the involved models, including discrete and continuous block size distributions.

Nomenclature:

A	=	Area exposed to imbibition and gravity effects, Eqs. 2 and 3.
b	=	Intersection with the verticals axis, Eq. 14.
$\mathbf{B_o}$	=	Oil volume factor.
$\boldsymbol{f_j}$	=	Volume fraction of the blocks of size h_{maj}, Eq. 7.
$\boldsymbol{f_D(h_D)}$	=	Continuous block size distribution, Eqs. 12-14.
$\mathbf{h_D}$	=	Dimensionless matrix block thickness, Eq. 17.
$\mathbf{h_{ma}}$	=	Matrix block thickness.
$\mathbf{h_{máx}}$	=	Maximum matrix block thickness.
$\mathbf{h_{mín}}$	=	Minimum matrix block thickness.
$\mathbf{k_o}$	=	Effective oil permeability.
m	=	Slope in the linear block size distribution , Eq. 14.

n	=	Number of blocks of a given size.
NB	=	Number of block sizes in the discrete distribution, Eq. 6.
NFR	=	Naturally Fractured Reservoirs
Nrt	=	Recoverable oil by capillarity effects, Eq. 18
$\mathbf{p_c}$	=	Capillary pressure, Eqs. 2 and 4.
$\mathbf{q_{o\ grav}}$	=	Oil flow rate due to gravity effects, Eqs. 3 and 5.
$\mathbf{q_{o\ imb}}$	=	Oil flow rate due to capillarity effects, Eqs. 2 and 4.
$\mathbf{q_{ot}}$	=	Total oil flow rate in the NFR, Eq. 6
$\mathbf{q_{o\ tot}}$	=	Oil flow rate due to capillarity and gravity effects, Eq. 1.
$\mathbf{V_R}$	=	Total reservoir rock volume, Eq. 7
$\mathbf{V_{Rj}}$	=	Reservoir rock volume of a given size, Eq. 7.
a	=	Dimensionless constant to adjust the initial oil flow rate, Eq. 19
b	=	Dimensionless constant to adjust the rate at which the imbibition oil flow rate declines, Eq. 19.
D**t**	=	Time interval, Eq. 20
Dr	=	Difference in water and oil densities, Eqs.3 and 5.

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