Accurate 3D Bathymetry Representation in Low-Dispersion Finite Differences*

Miguel Ferrer[1], Josep De La Puente[1], and Santiago Fernández Prieto[2]

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Abstract

When performing seismic modeling using a finite-difference approach, the domain is usually modeled using a regular Cartesian grid. While this method is often computationally faster and easier to implement than other modeling solutions, it presents some disadvantages when compared to methods that use unstructured meshes, such as finite-element or finite-volume. One drawback is that Cartesian grids struggle when they must conform to a non-flat interface, like a free surface with topographic features or an ocean floor with bathymetry. In the case of the bathymetry, the strong impedance between water and sediments generates complex wavefields, which, if modeled with the staircase approach on a Cartesian grid, generate high-amplitude spurious diffractions. This usually leads to dispersion errors in the simulated interface/refracted waves. We present a method to overcome this issue, based on the deformation of a fully staggered grid to accommodate grid points to interfaces between a top water layer and a bottom sediment layer. This method extends the solution for free surface and topographic features presented in de la Puente et al. (2014), by adapting the grid to arbitrary bathymetry. The resulting scheme solves spatial derivatives with high order, thus resulting in a low dispersion scheme. In addition, we show that this solution can be easily generalized to simulate any number of interfaces between sediment layers. Finally, we present the results of a comparison between this method and a simple staircase-like approximation of the water-sediment interface to quantify the differences in dispersion error between the two solutions.

Selected References

de la Puente, J., M. Ferrer, M. Hanzich, J.E. Castillo, and J.M. Cela, 2014, Mimetic seismic wave modeling including topography on deformed staggered grids: Geophysics, v. 79/3, p. T125–T141.

Fornberg, B., 1988, The pseudospectral method; accurate representation of interfaces in elastic wave calculations: Geophysics, v. 53/5, p. 625–637.

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Accurate 3d bathymetry representation in low-dispersion finite differences.

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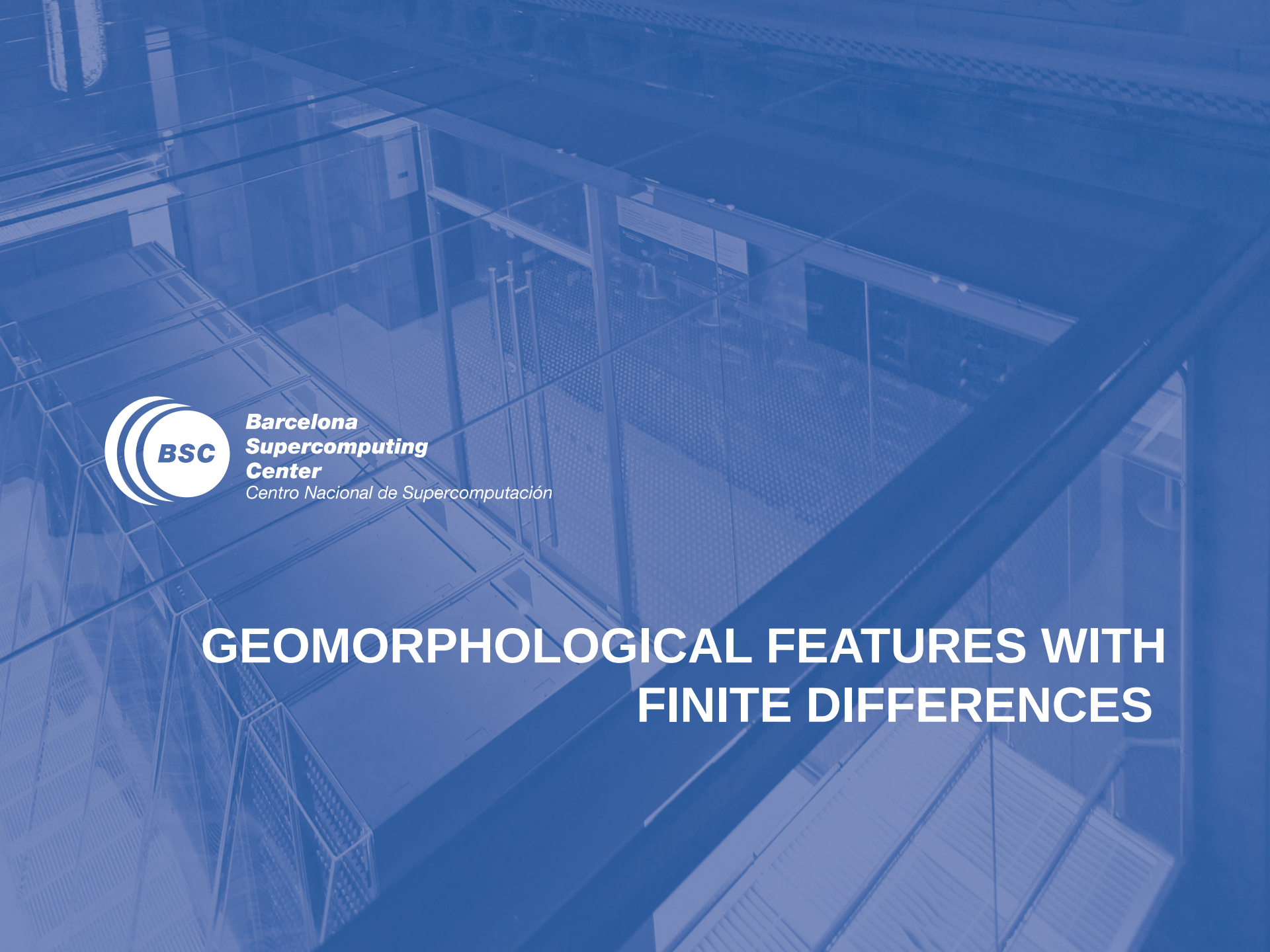
SEG ICE 2016

Outline

- Geomorphological features with finite differences
- From regular to deformed grid
- Bathymetry examples
- Conclusions and future work



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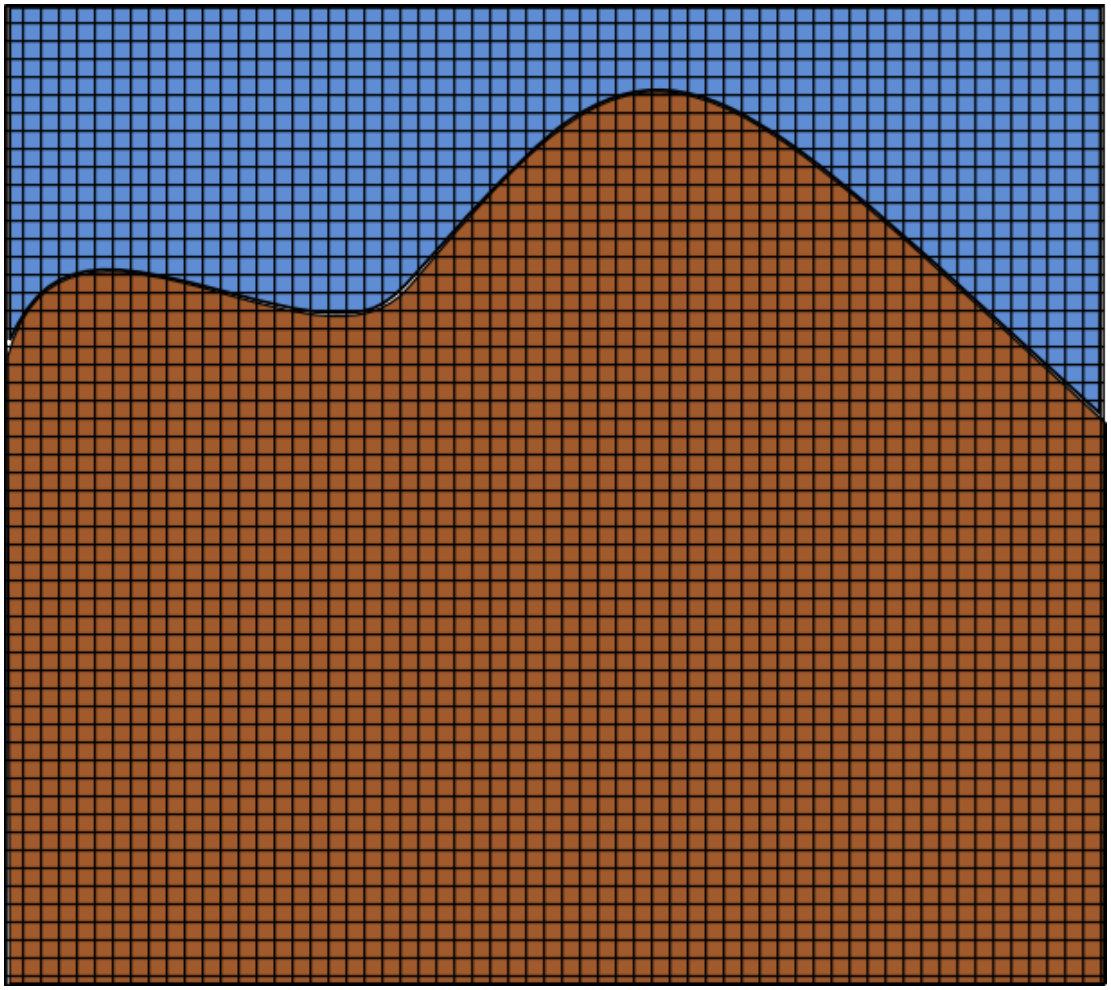
GEOMORPHOLOGICAL FEATURES WITH FINITE DIFFERENCES

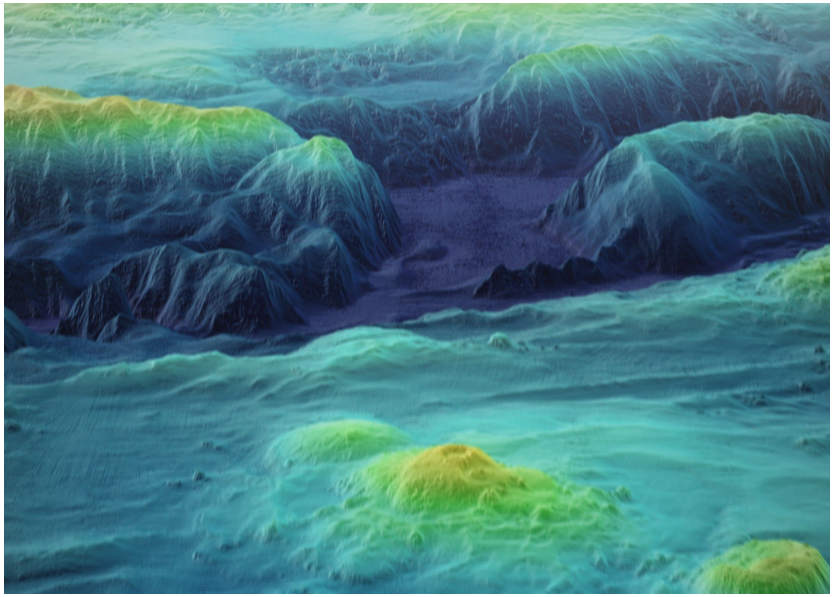
Finite differences

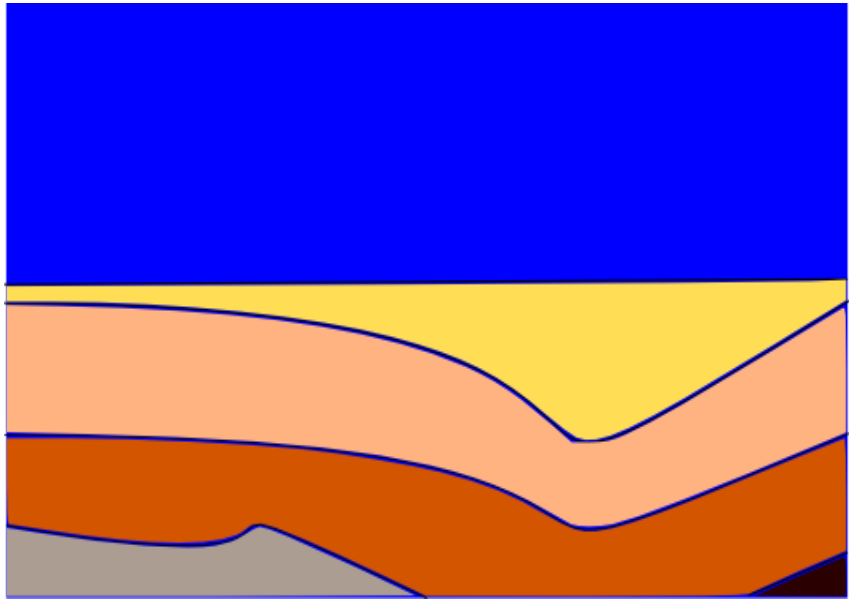
- Offer significative advantages:
  - Easy to implement.
  - Fast, efficient calculations.
  - Generally lower memory footprint than other methods.
- And challenging disadvantages:
  - Low flexibility, especially in non-regular shaped domains.
  - Difficulty in dealing with some boundary conditions.
  - Lower accuracy compared to other methods.
- But these drawbacks can be mitigated.

Non-conforming geometries

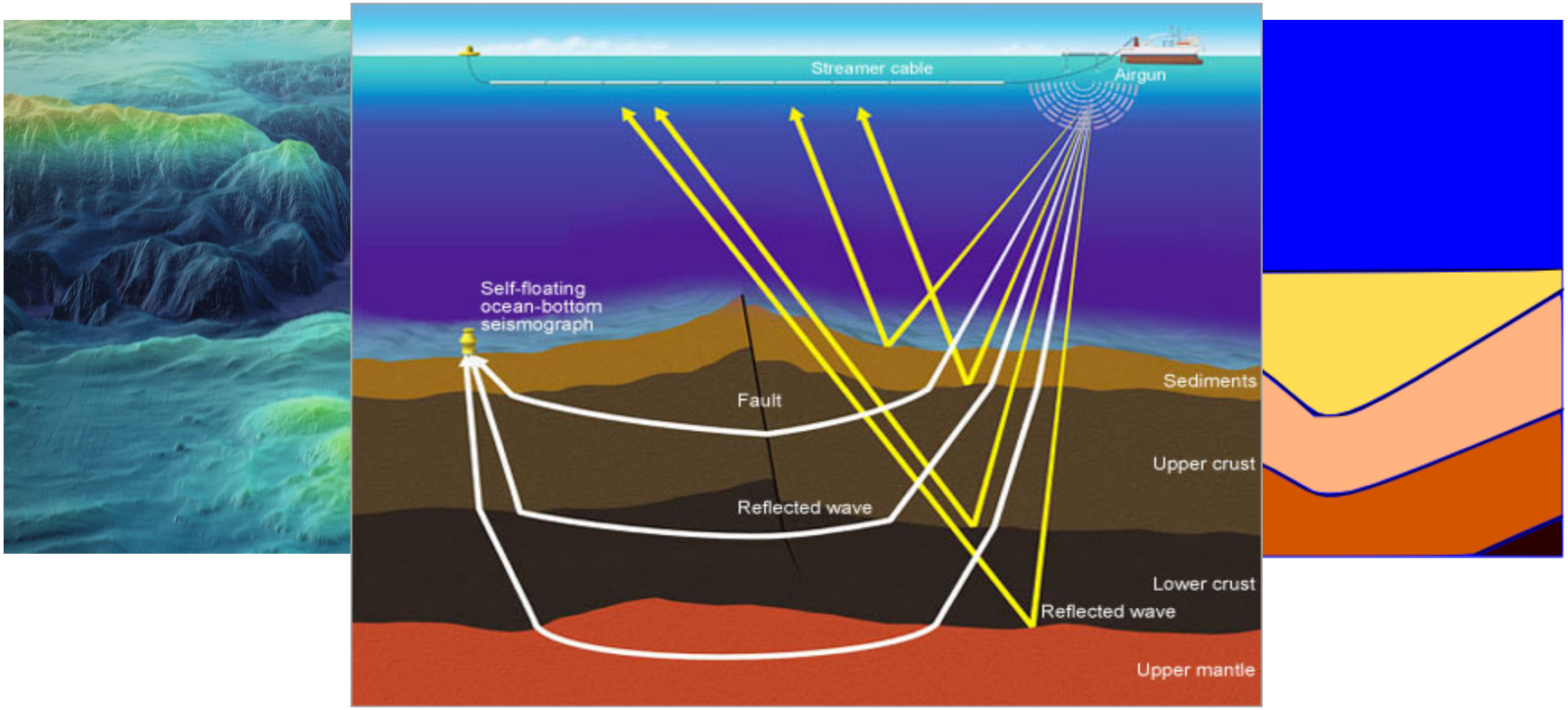


Non-conforming geometries



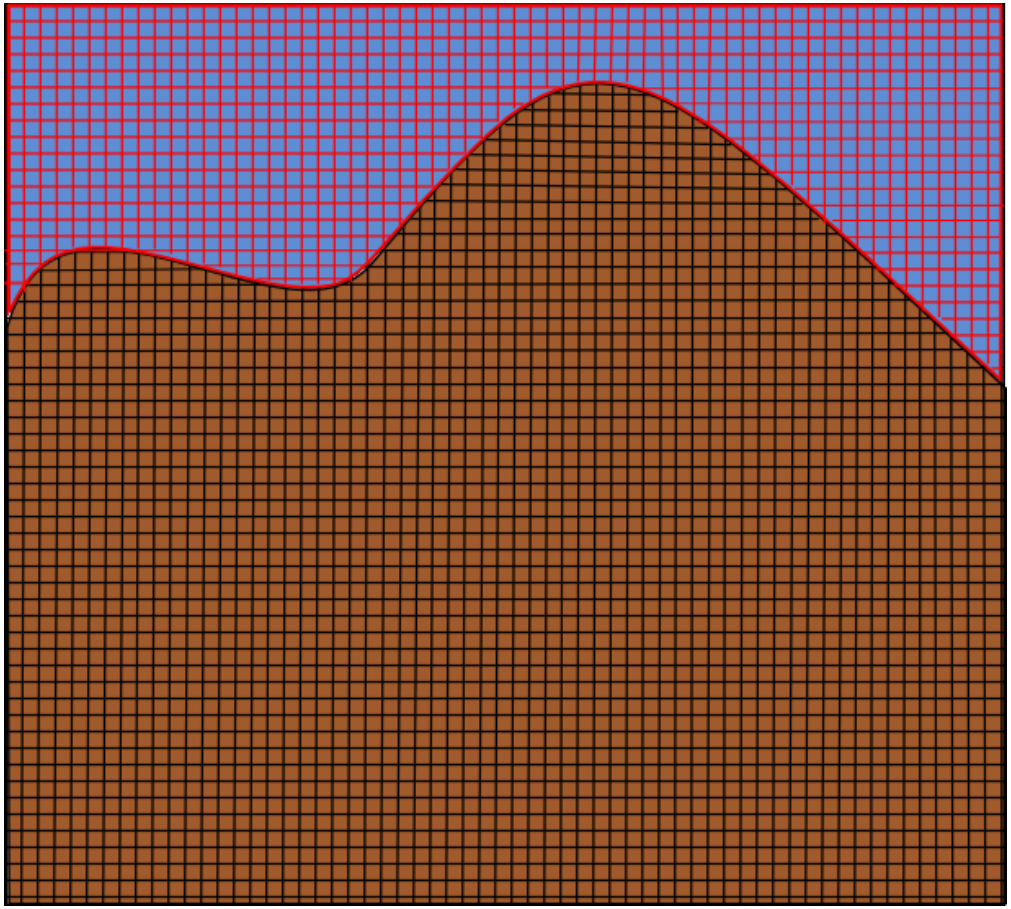


Non-conforming geometries



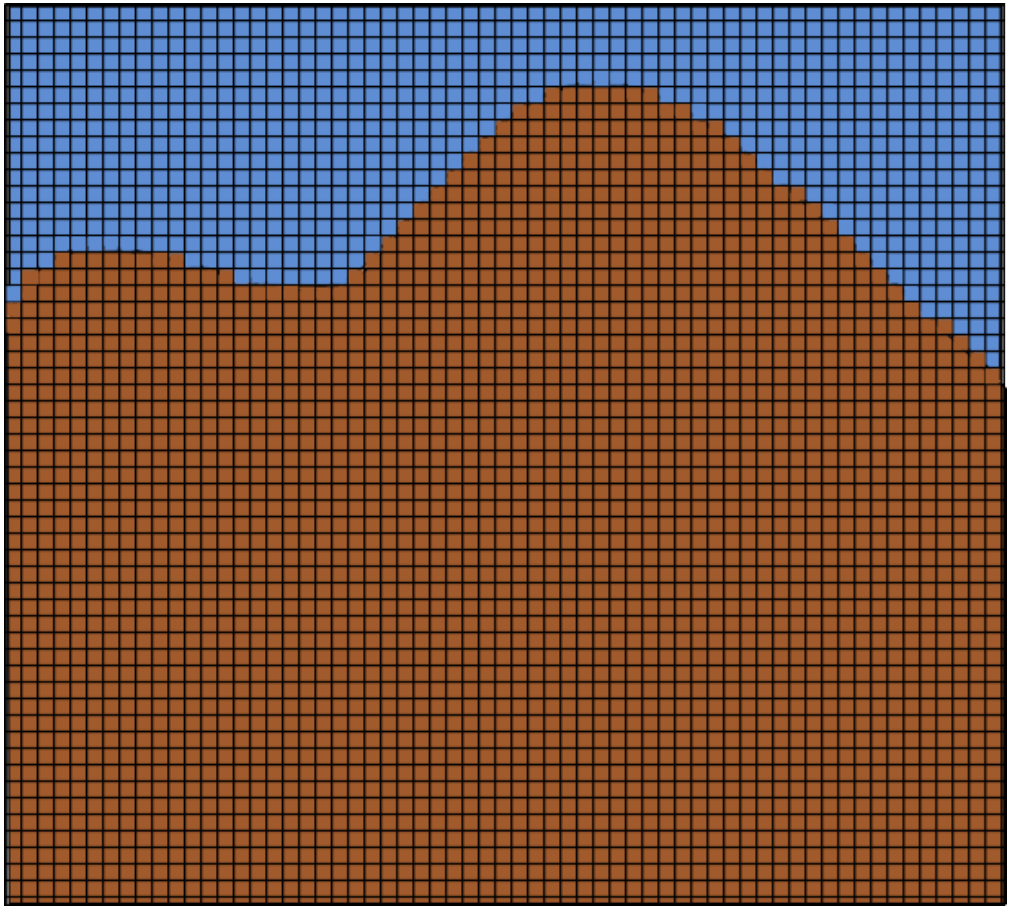
Ideally

- Matching grids at media boundaries
- Non-regular grid, smaller cells present at interfaces.
- Idea similar to FEM.



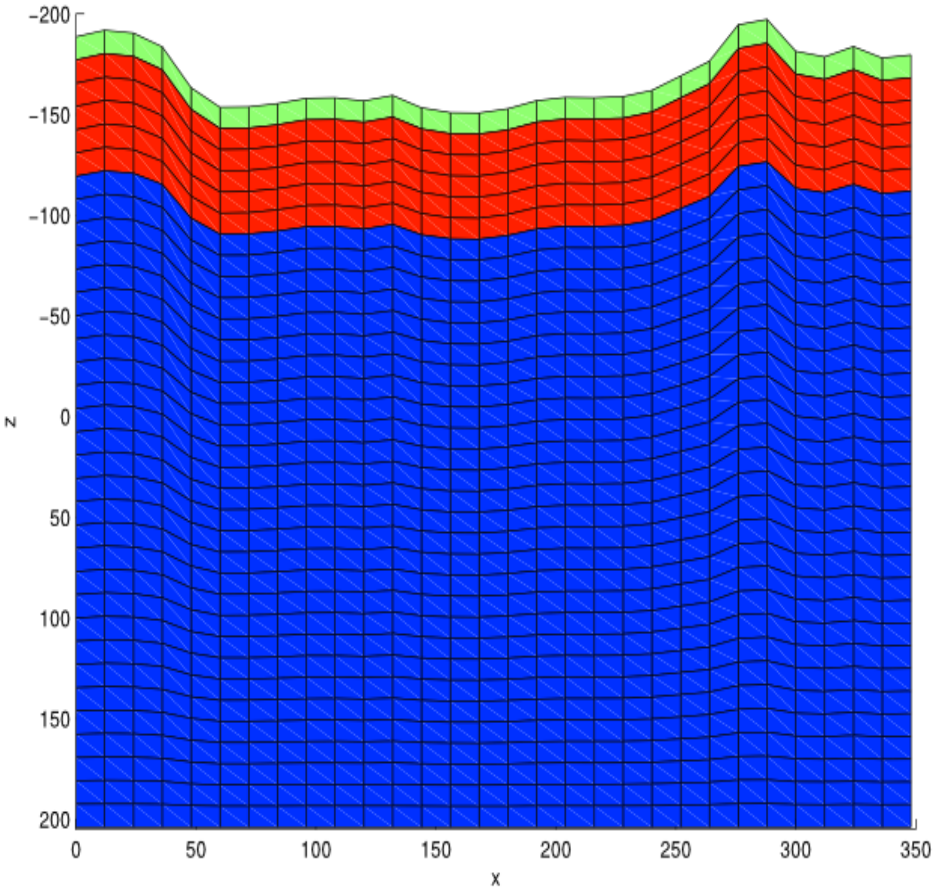
Implicit staircase approach

- Very straightforward solution.
- Easy to implement.
- Less precise.



Deformed (non-cartesian) grid

- Boundary-conforming grid.
- Vertical compression of cells to accommodate media interfaces.
- Vertical spacing for a given column remains constant with depth within a media layer.
- Derivatives in the deformed grid are linearly dependant on the derivatives of the regular grid.



FROM REGULAR TO DEFORMED GRID

Deformed grid mapping

- 2 case study
- Topography
  - One layer domain.
- Bathymetry
  - Two layer domain (water on top, ground on bottom).
- Can be generalized to multi-layered model.

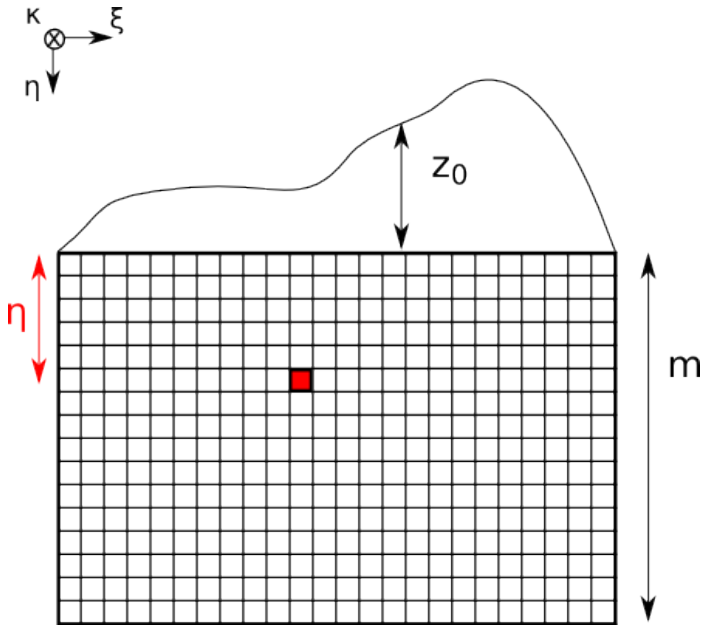
Topography (one layer)

Coordinate transformation:

$$x = \xi$$

$$y = \kappa$$

$$z = \frac{\eta}{\eta_{max}}(z_0 + m) - z_0 \quad z_0(x, y) \geq 0$$



Partial derivatives transformation:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} + A\frac{\partial}{\partial \eta} \qquad A = \frac{\eta_{max} - \eta}{z_0 + m}\frac{\partial z_0}{\partial \xi}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \kappa} + B\frac{\partial}{\partial \eta} \qquad B = \frac{\eta_{max} - \eta}{z_0 + m}\frac{\partial z_0}{\partial \kappa}$$

$$\frac{\partial}{\partial z} = C\frac{\partial}{\partial \eta} \qquad C = \frac{\eta_{max}}{z_0 + m}$$

Bathymetry (two layers)

Coordinate transformation:

$$x = \xi$$

$$y = \kappa$$

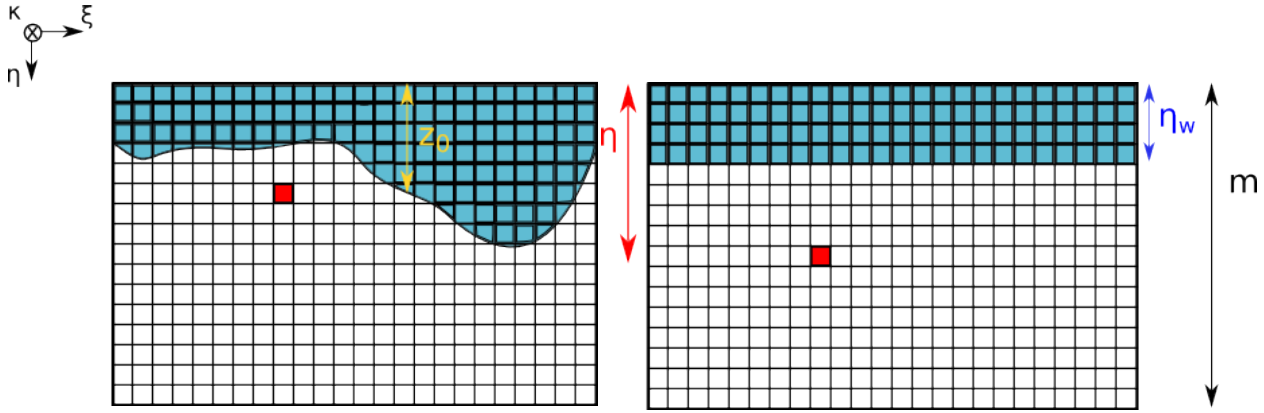
$$z = \begin{cases} \dfrac{-\eta}{\eta_w} z_0 & \text{if } \eta < \eta_w \\ \dfrac{\eta - \eta_w}{\eta_{\max} - \eta_w} (z_0 + m) - z_0 & \text{if } \eta \geq \eta_w \end{cases} \qquad z_0(x, y) \leq 0$$

Partial derivatives transformation:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} + A \frac{\partial}{\partial \eta}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \kappa} + B \frac{\partial}{\partial \eta}$$

$$\frac{\partial}{\partial z} = C \frac{\partial}{\partial \eta}$$



Bathymetry (two layers)

Coordinate transformation:

$$x = \xi$$

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$$z = \begin{cases} \dfrac{-\eta}{\eta_w} z_0 & \text{if } \eta < \eta_w \\ \dfrac{\eta - \eta_w}{\eta_{\max} - \eta_w} (z_0 + m) - z_0 & \text{if } \eta \geq \eta_w \end{cases} \qquad z_0(x, y) \leq 0$$

Partial derivatives transformation:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} + A \frac{\partial}{\partial \eta}$$

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$$\frac{\partial}{\partial z} = C \frac{\partial}{\partial \eta}$$

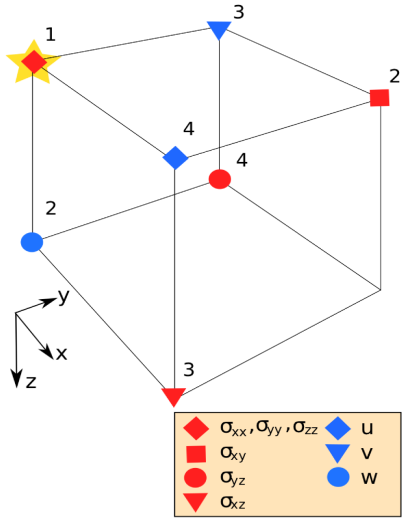
$$A = \begin{cases} \dfrac{-\eta}{z_0} \dfrac{\partial z_0}{\partial \xi} & \text{if } \eta < \eta_w \\ \dfrac{\eta_{\max} - \eta}{z_0 + m} \dfrac{\partial z_0}{\partial \xi} & \text{if } \eta \geq \eta_w \end{cases}$$

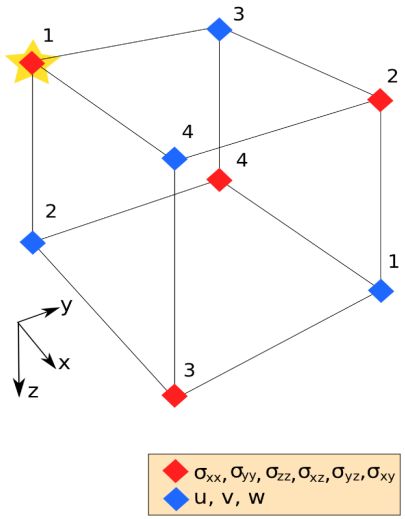
$$B = \begin{cases} \dfrac{-\eta}{z_0} \dfrac{\partial z_0}{\partial \kappa} & \text{if } \eta < \eta_w \\ \dfrac{\eta_{\max} - \eta}{z_0 + m} \dfrac{\partial z_0}{\partial \kappa} & \text{if } \eta \geq \eta_w \end{cases}$$

$$C = \begin{cases} \dfrac{-\eta_w}{z_0} & \text{if } \eta < \eta_w \\ \dfrac{\eta_{\max} - \eta_w}{z_0 + m} & \text{if } \eta \geq \eta_w \end{cases}$$

Deformed grid considerations

- Things to consider:
  - Standard Staggered Grid lacks data points needed for the deformed derivatives. We use a Lebedev grid instead with increased memory and computational cost.





$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} + A\frac{\partial}{\partial \eta}$$
$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \kappa} + B\frac{\partial}{\partial \eta}$$
$$\frac{\partial}{\partial z} = C\frac{\partial}{\partial \eta}$$

- Alternatively, interpolation might be performed to obtain the missing data points.

Deformed grid considerations

- Things to consider:
  - Deformed vertical discretization is lesser or equal to regular discretization.
    - Adding cells to the grid in the vertical direction might be necessary.

$$n_z^w = \lceil (-\min(z_0))/\Delta z \rceil$$
$$n_z^s = \lceil (m + \max(z_0))/\Delta z \rceil + 1\,.$$

  - On multi-layer domains, vertical discretization is not continuous.

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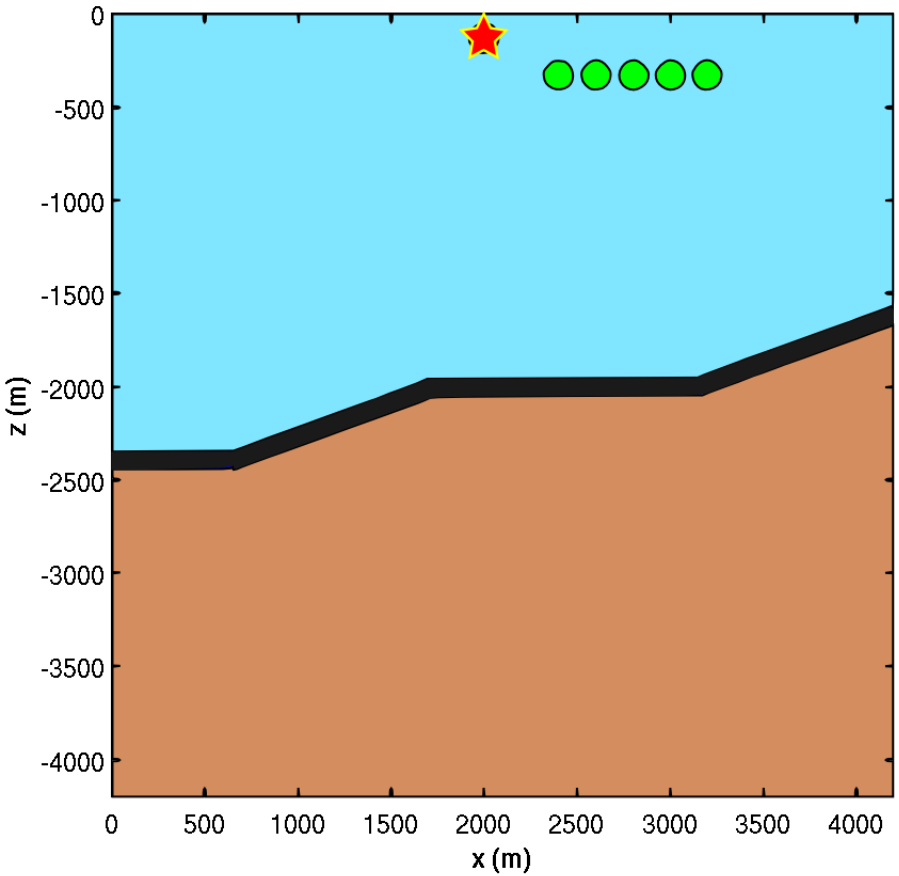
EXAMPLES

2D bathymetry example

2D water-ground interface



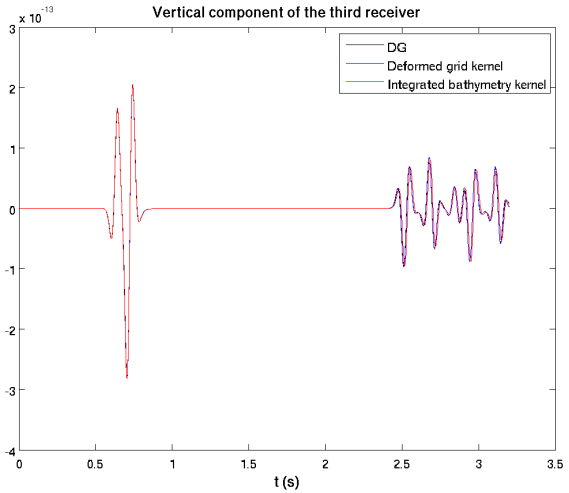
10 Hz Ricker wavelet

$$Vp_{water} = 1500\, m/s$$
$$\rho_{water} = 1000\, kg/m^3$$

$$Vp_{ground} = 2000\, m/s$$
$$Vs_{ground} = 1000\, m/s$$
$$\rho_{ground} = 1600\, kg/m^3$$

2D bathymetry example

- 2D water-ground interface



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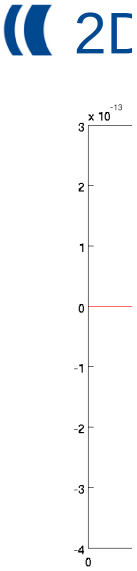
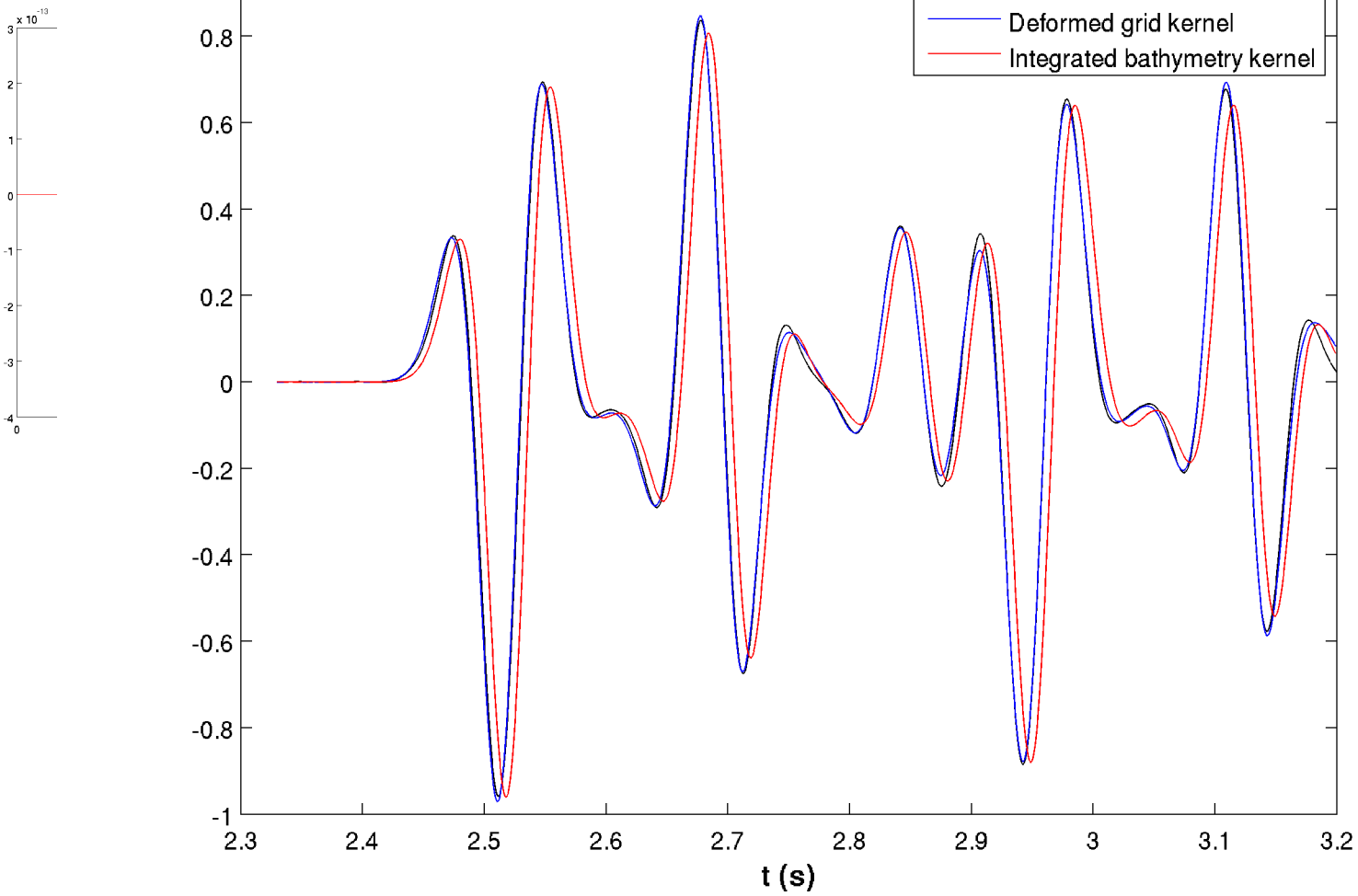
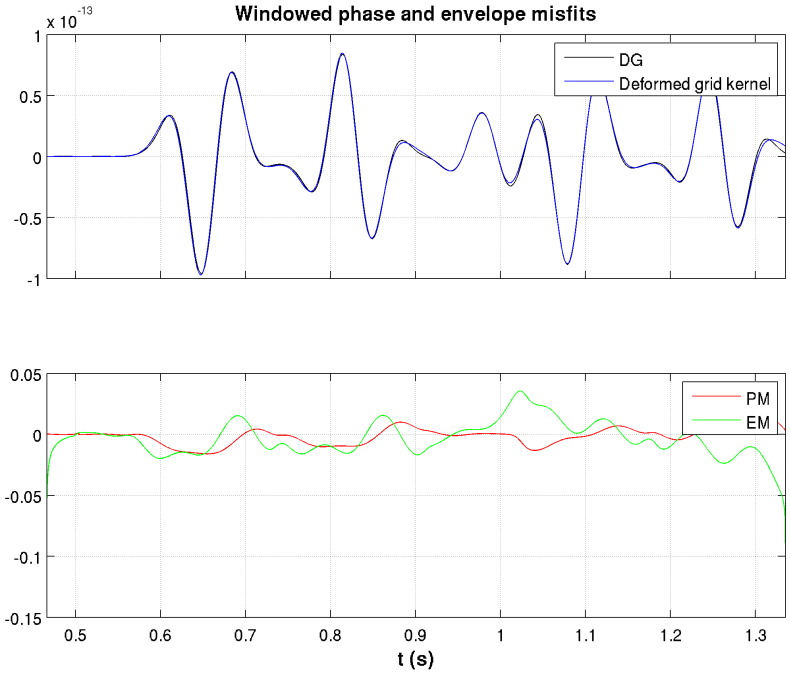




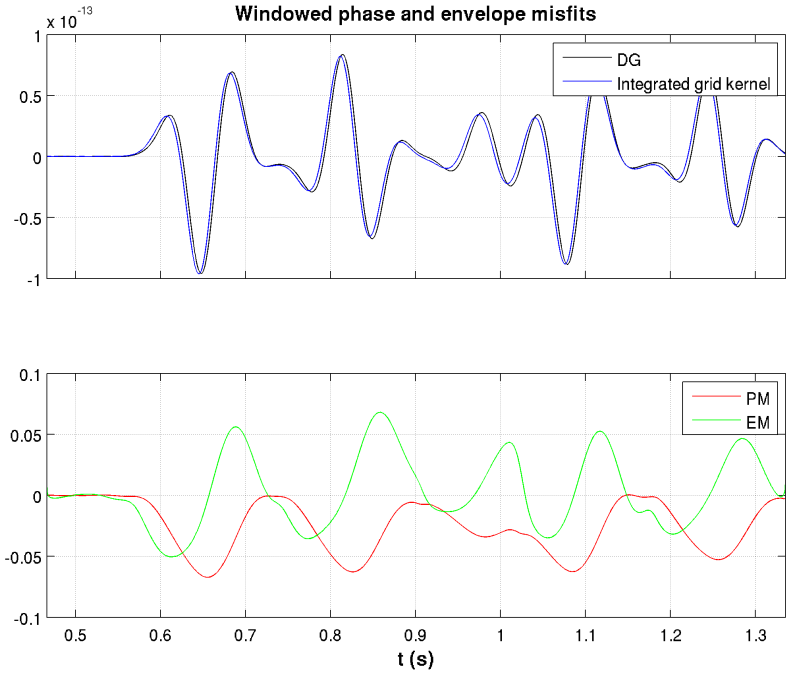
2D bathymetry example

2D water-ground interface



$$S_{phase} = 1.47\%$$

$$S_{envelope} = 3.03\%$$

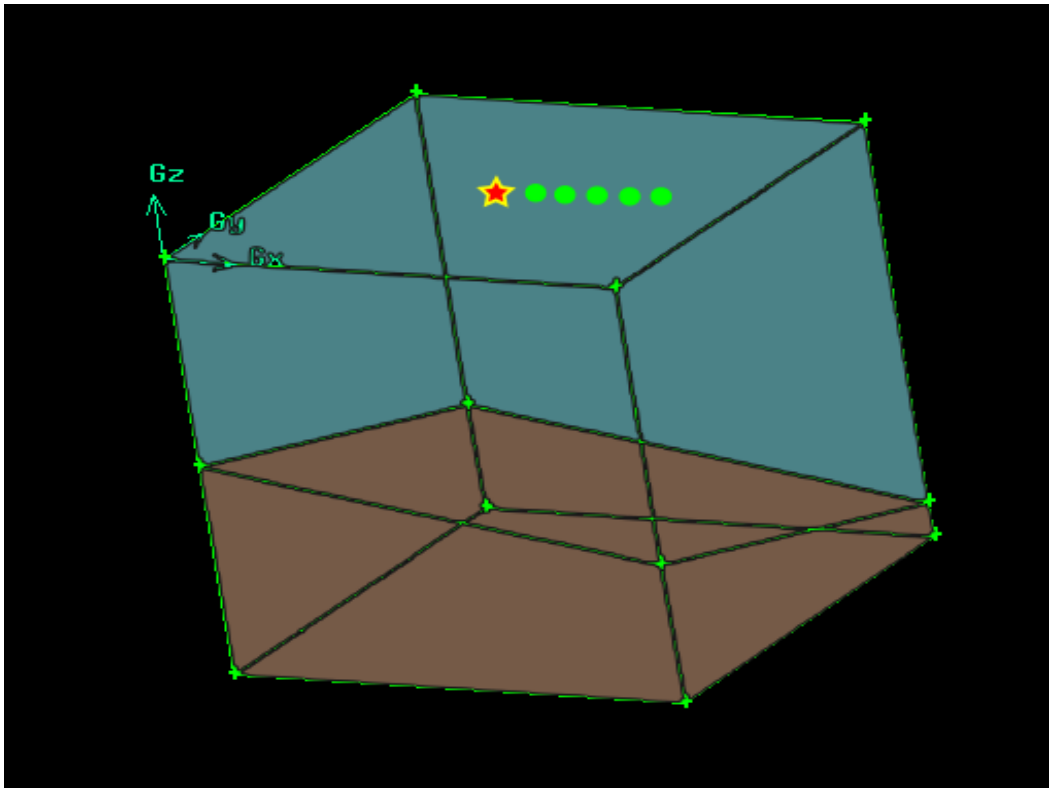


$$S_{phase} = 6.94\%$$

$$S_{envelope} = 6.15\%$$

3D bathymetry example

3D water-ground interface



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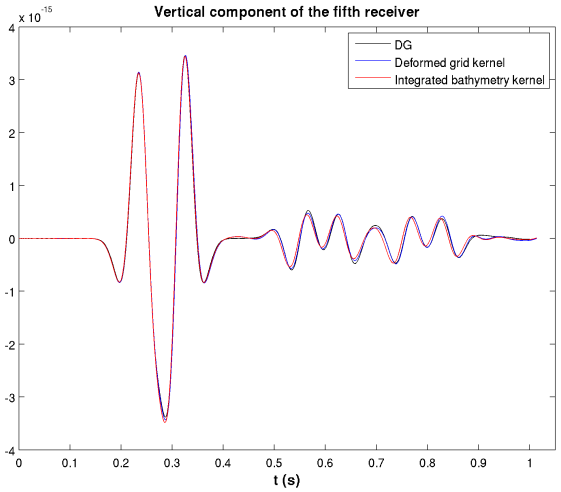
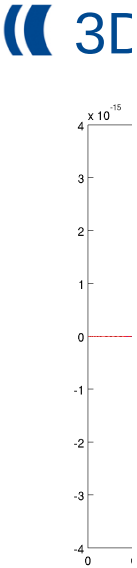
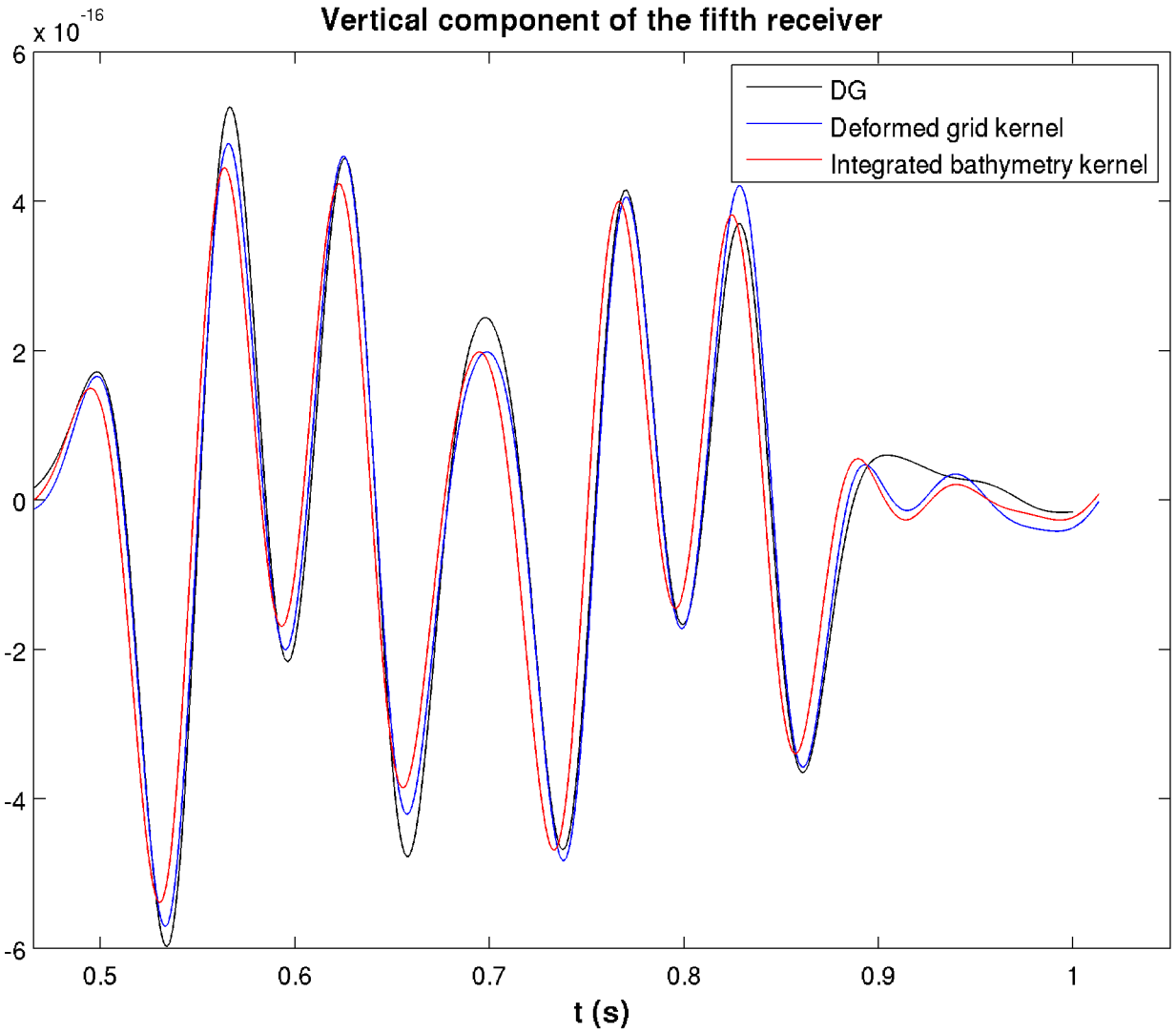
3D bathymetry example

- 3D water-ground interface



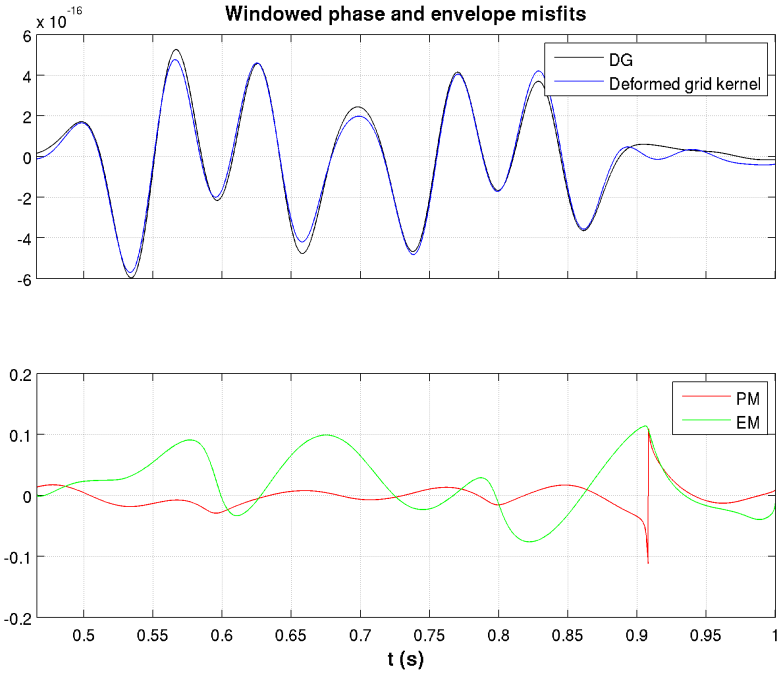
3D bathymetry example





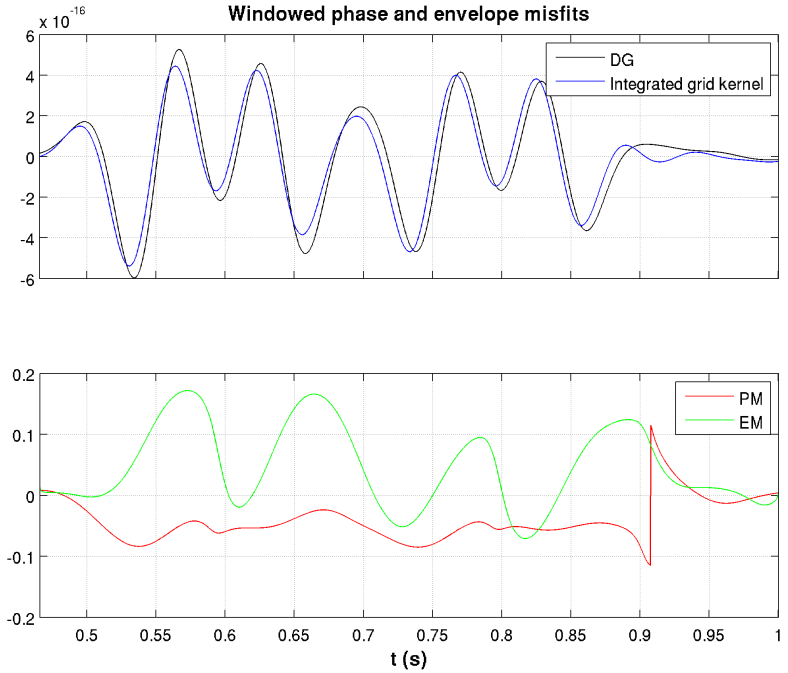
3D bathymetry example

3D water-ground interface



$$S_{phase} = 3.25\%$$

$$S_{envelope} = 9.59\%$$



$$S_{phase} = 9.91\%$$

$$S_{envelope} = 15.04\%$$

CONCLUSIONS AND FUTURE WORK

Conclusions and future work

- In summary:
  - Finite differences generally offer greater performance at the cost of lower accuracy.
  - Deformed grids can help mitigate this with little increasing of computational cost.
- Future work:
  - Generalized multi-layer domains.
  - Adding mimetic operators to boundaries between layers to increase precision near the interfaces.
  - Measure instability conditions.

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Thank you!

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