

# **Quantitative Characterization of Fracture Frequency Variations Using a Linear Piecewise Regression Analysis and the Akaike Information Criterion\***

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## **Abstract**

We present a new quantitative approach for characterizing fracture frequency variations using a linear piecewise regression (LPR) analysis and the Akaike Information Criterion (AIC). Break points calculated for the LPRs produce linear segments with varying slopes for a cumulative fracture frequency (CFF) curve. An AIC value is calculated for each LPR model in order to determine the optimal number of linear segments that fit the CFF data. The optimal number of segments is obtained by minimizing the AIC value for a single dataset. Results from the statistical analysis produced three CFF slope intervals that define the distribution of possible fracture frequencies unique to the geologic setting from which they were derived. A total of 3678 fracture and vein measurements were collected using scanline, scangrid, and abbreviated methods at 38 sites in the Utica black shale and overlying coarser clastics of the Mohawk Valley in eastern New York State.

To produce a CFF curve, fracture frequency is summed along a transect perpendicular to the strike of the fracture set. The piecewise function in the R package, "Segmented", calculates break points where the slope of the CFF changes. The AIC model selection method produces LPRs with the optimal number of breakpoints and segments by penalizing additional parameters introduced with each new segment. A comparison with the Bayesian Information Criterion (BIC) found that AIC models outperformed the BIC method because the BIC equation over-penalized additional parameters. Segmenting the CFFs produced three unique slope intervals, each with a set of defining characteristics. Background frequencies are defined by an average CFF slope of 8 with no significant changes in slope (including prominent frequency peaks). The average background fracture frequency is 2.4 fractures/m. Transition frequencies exhibit higher CFF slopes, averaging 111, and higher average fracture frequency of 12.3 fractures/m. Fracture intensification domains (including fractures in fault damage zones) are defined by the highest average CFF slope of 1649, produce prominent frequency peaks (>50 fractures/m) and have the highest average fracture frequency of 44.6 fractures/m. Results of the piecewise analysis provide quantified boundaries that can be used to create a fracture frequency framework for a defined geologic setting, aiding in predictions of fracture frequency variations due to local structural features.

### **Selected References**

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O'Hara, Alex P., Robert D. Jacovi, and H. David Sheets, 2017, Predicting the width and average fracture frequency of damage zones using a partial least squares statistical analysis: Implications for fault zone development: Journal Structural Geology, v. 98, p. 38-52.



# Quantitative characterization of fracture frequency variations using a linear piecewise regression and the Akaike Information Criterion

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- Craig Eckert
- Joe Morris
- Travis Warner
- Scott McCallum
- Sandra Cronauer



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**University at Buffalo**

*The State University of New York*



# Problems in fault damage zone research

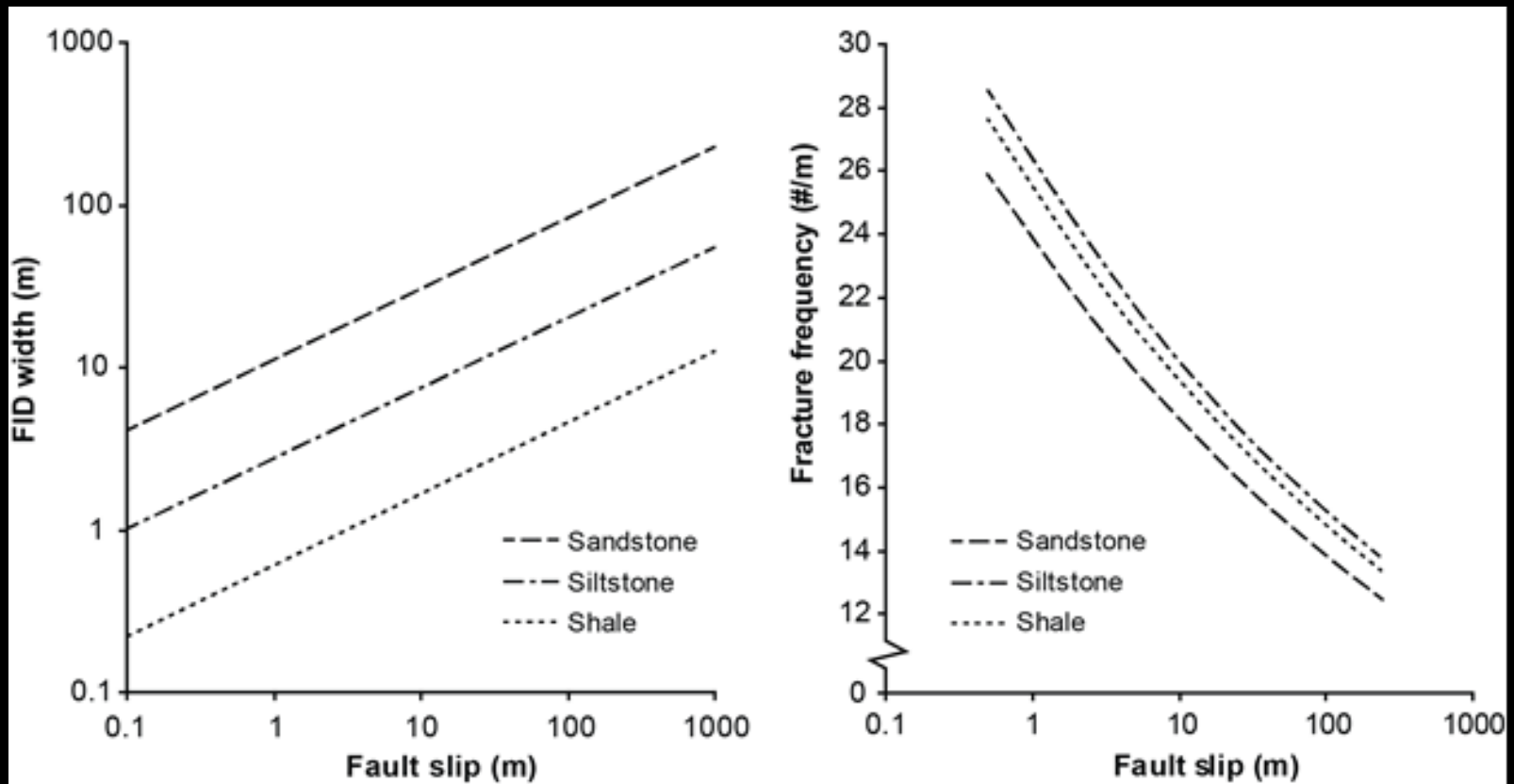
- Subjectivity (Choi et al., 2016)
- Defining “background fracture frequency”
- Error introduced in cross-study comparisons

## Solutions to Subjectivity

- Quantitative determination of fault damage zone width
- Methodology using piecewise analysis and the Akaike Information Criterion

# Application of Quantitative Analysis

Predicting fault damage zone width and average fracture frequency

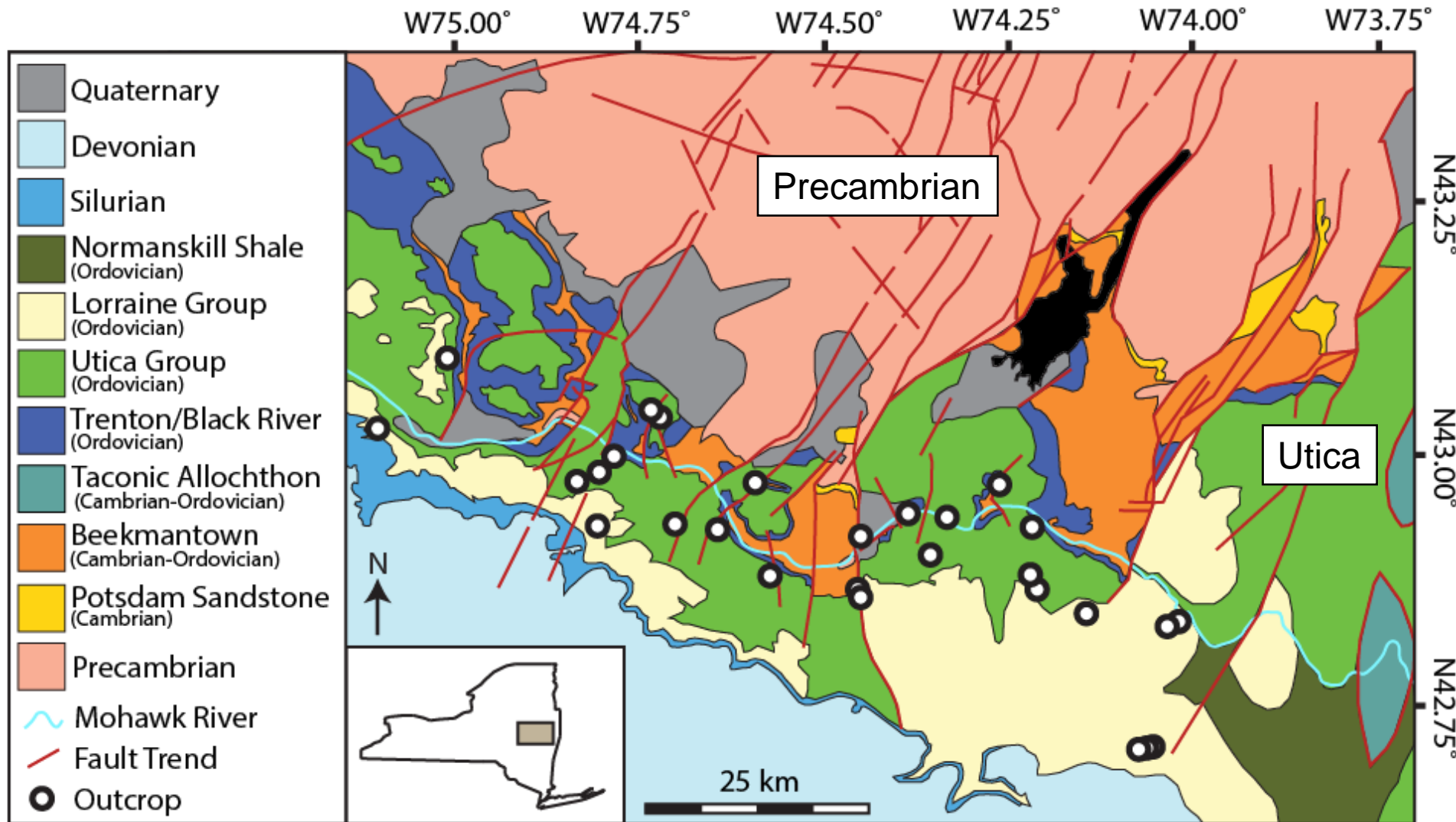


O'Hara et al., 2017

# Importance in developing quantitative analyses of fracture frequency distributions

- Methodology for reliably defining fracture/damage zone boundaries
  - Produce consistent damage zone width values for multivariate statistical analyses
  - Predictable fracture frequency distributions among varying geologic settings
- New hypothesis presented in discussion

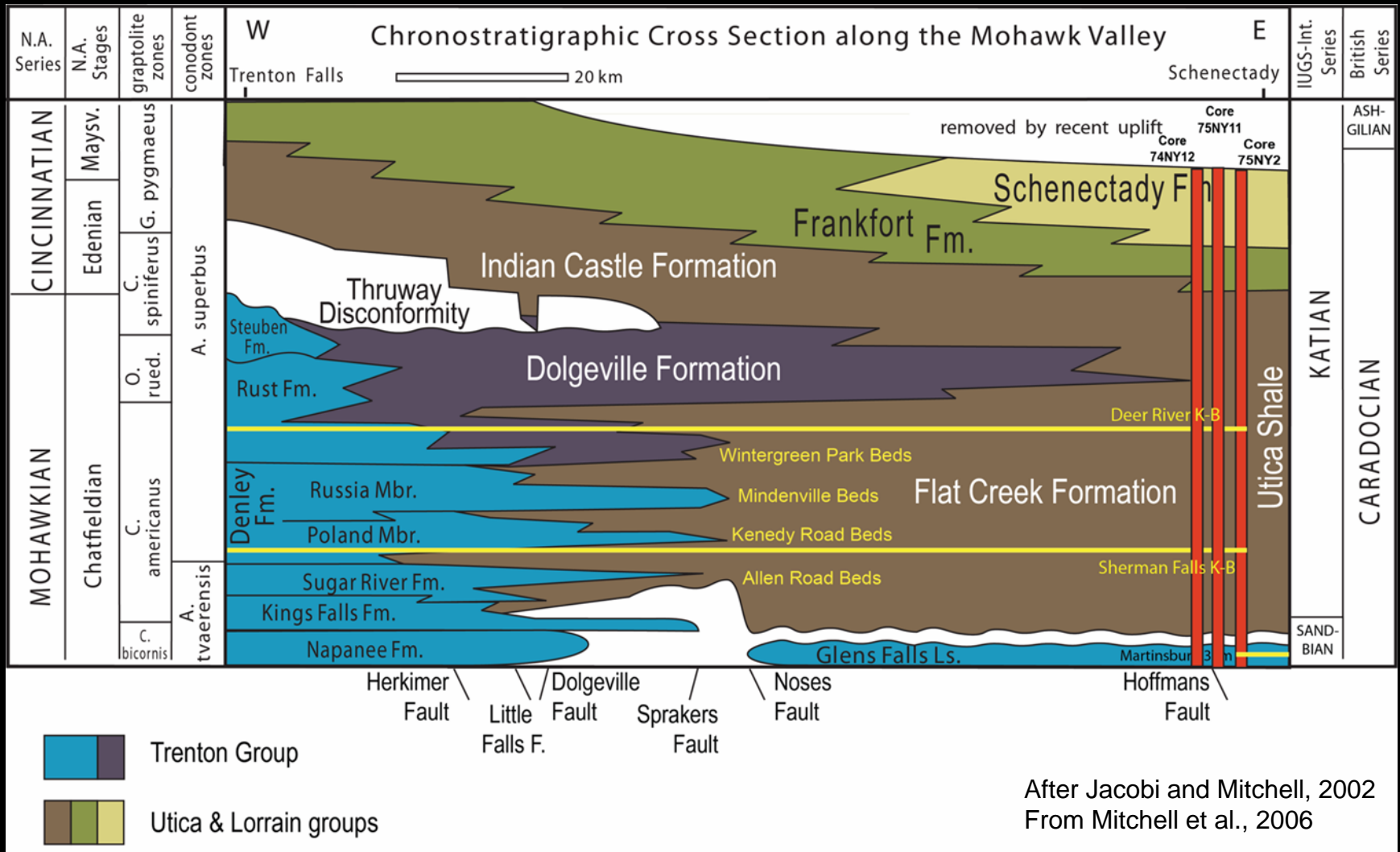
# Study Area



Modified from O'Hara et al., 2017

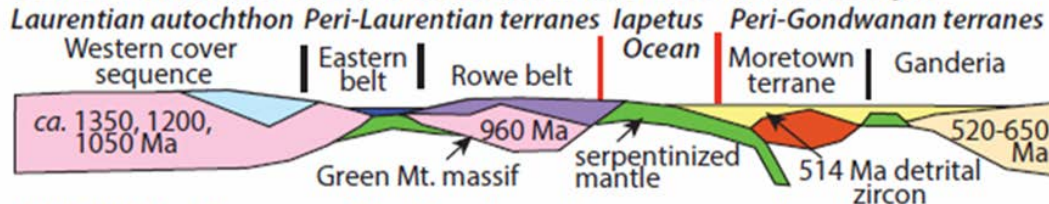


# Study Area: Chronostratigraphy

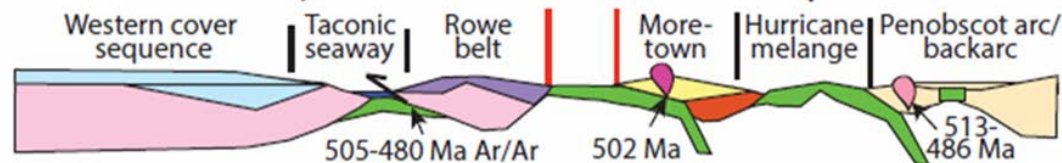


# Study Area: tectonic model

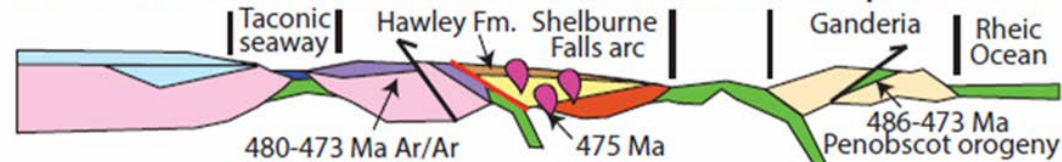
520-505 Ma: Rifted arc system on the active Gondwanan margin



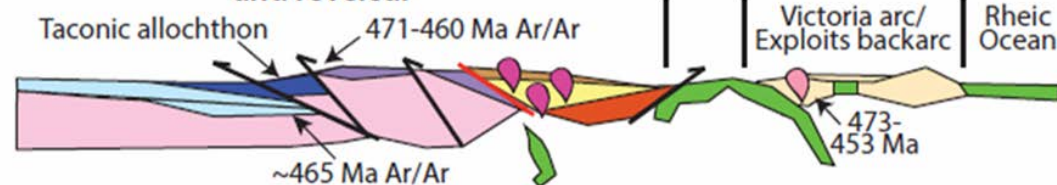
505-480 Ma: Journey of the Moretown terrane across Iapetus Ocean



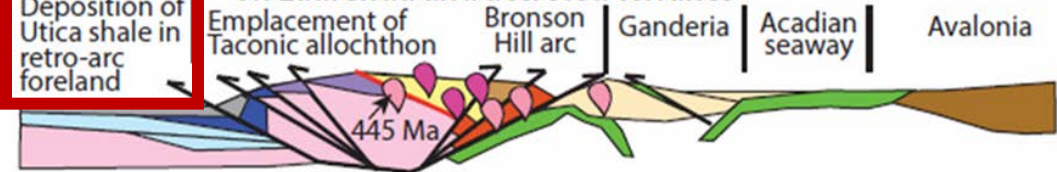
480-470 Ma: Growth of Shelburne Falls arc and closure of Iapetus Ocean



470-460 Ma: Middle Ordovician foreland deposition, slab break-off and reversal



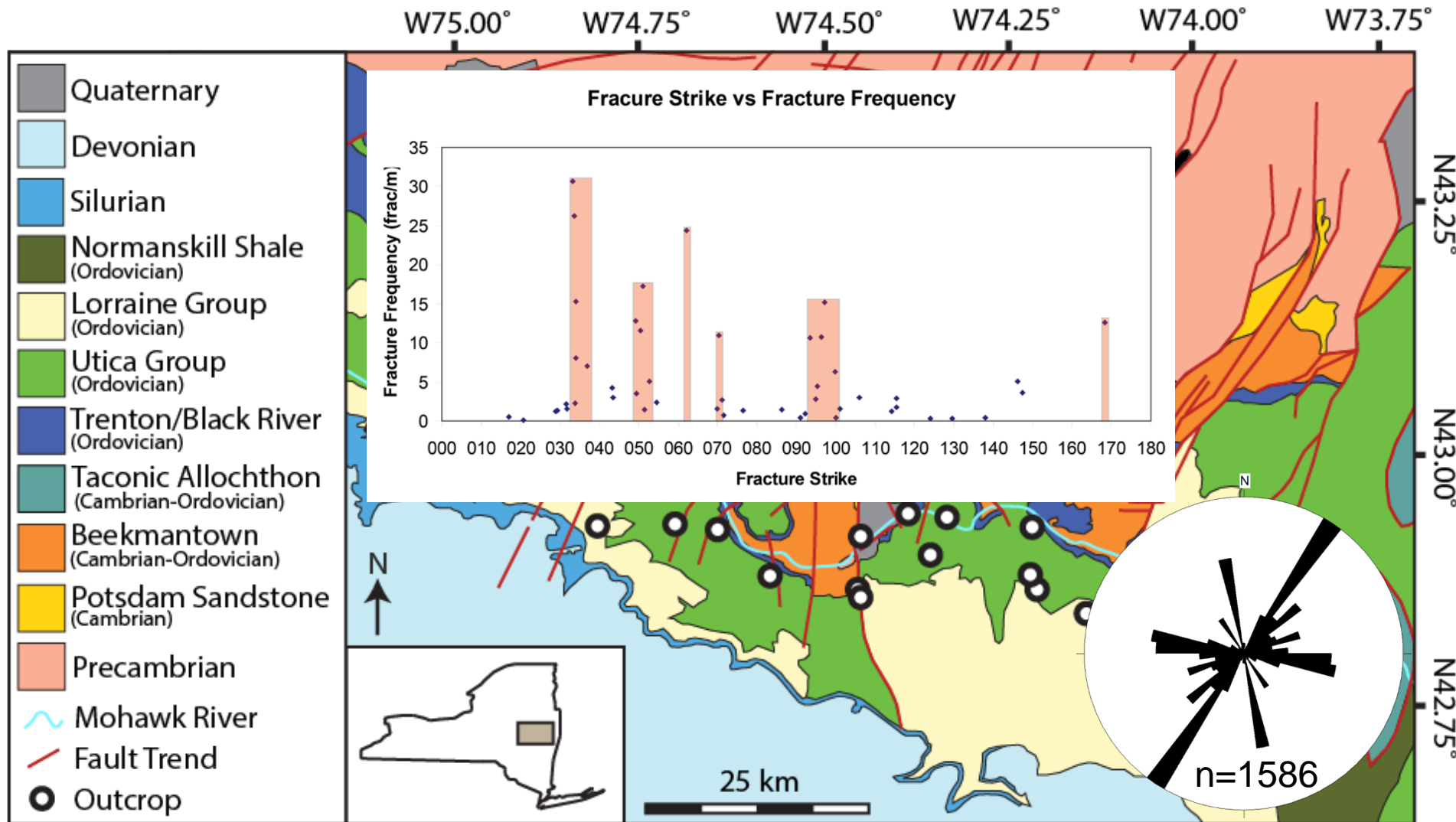
460-445 Ma: Late Ordovician foreland and growth of Bronson Hill arc on Laurentia and accreted terranes



Active  
Mohawk  
Valley  
Faulting

Macdonald et al. 2014

# Study Area



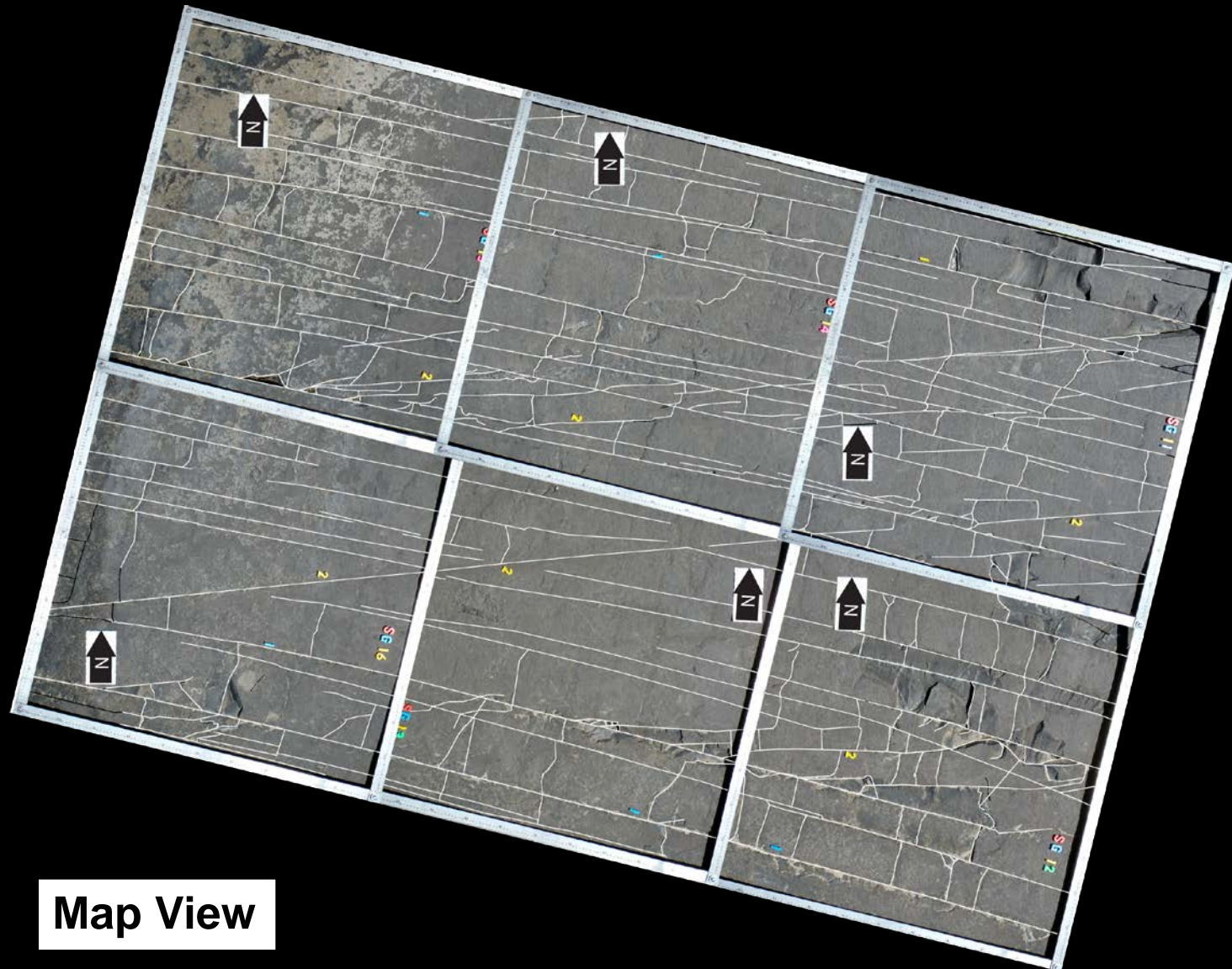
Modified from O'Hara et al., 2017

# Fracture Intensification Domain (FID)

- High-frequency fracture zone
- Do not pre-suppose fault influence or primary slip surface location
- Can be considered a fracture dominated subset of fault damage zones in specific cases (fault(s) present)



# Fracture Intensification Domain (FID)



**Map View**

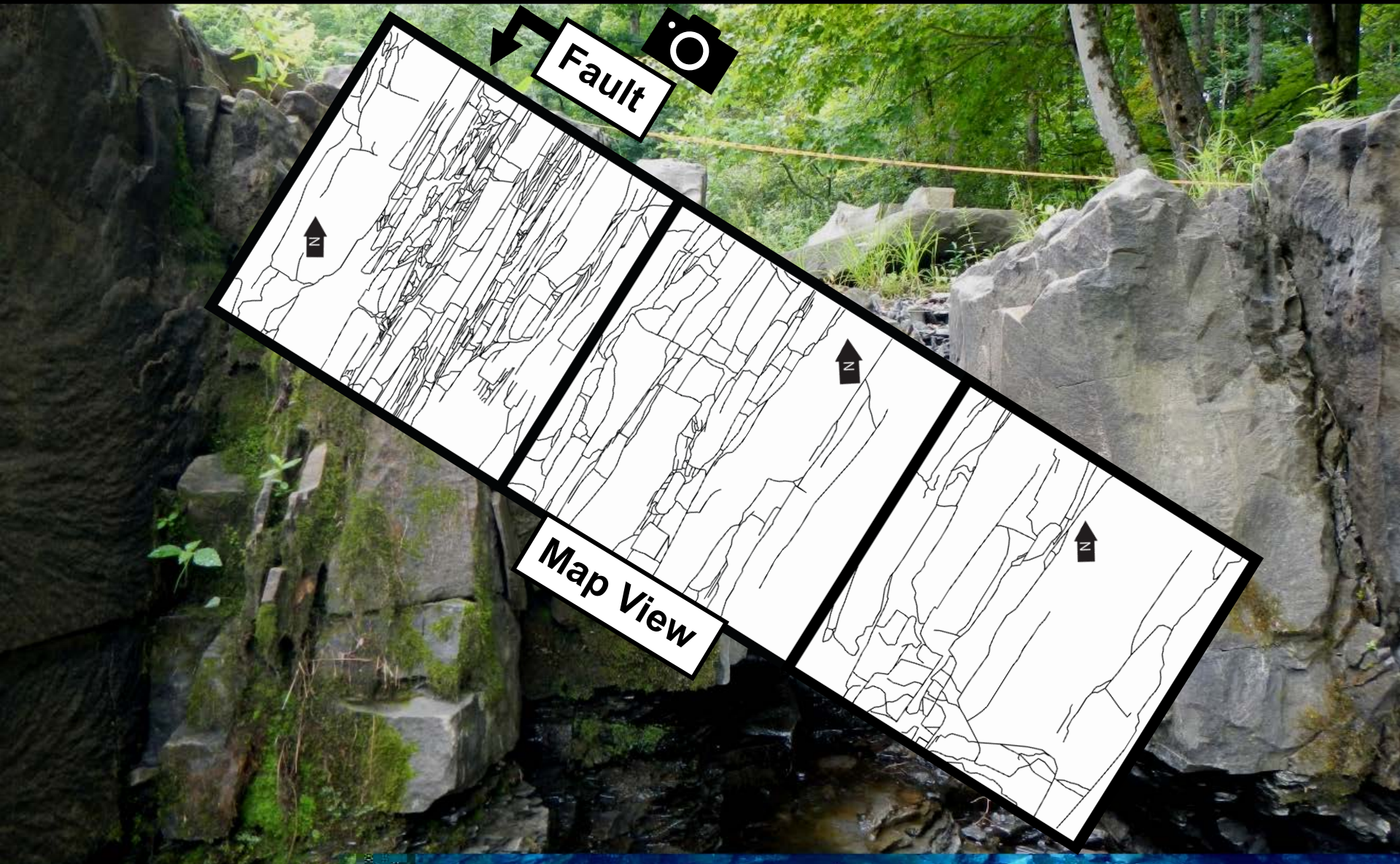


# Fracture Intensification Domain (FID)



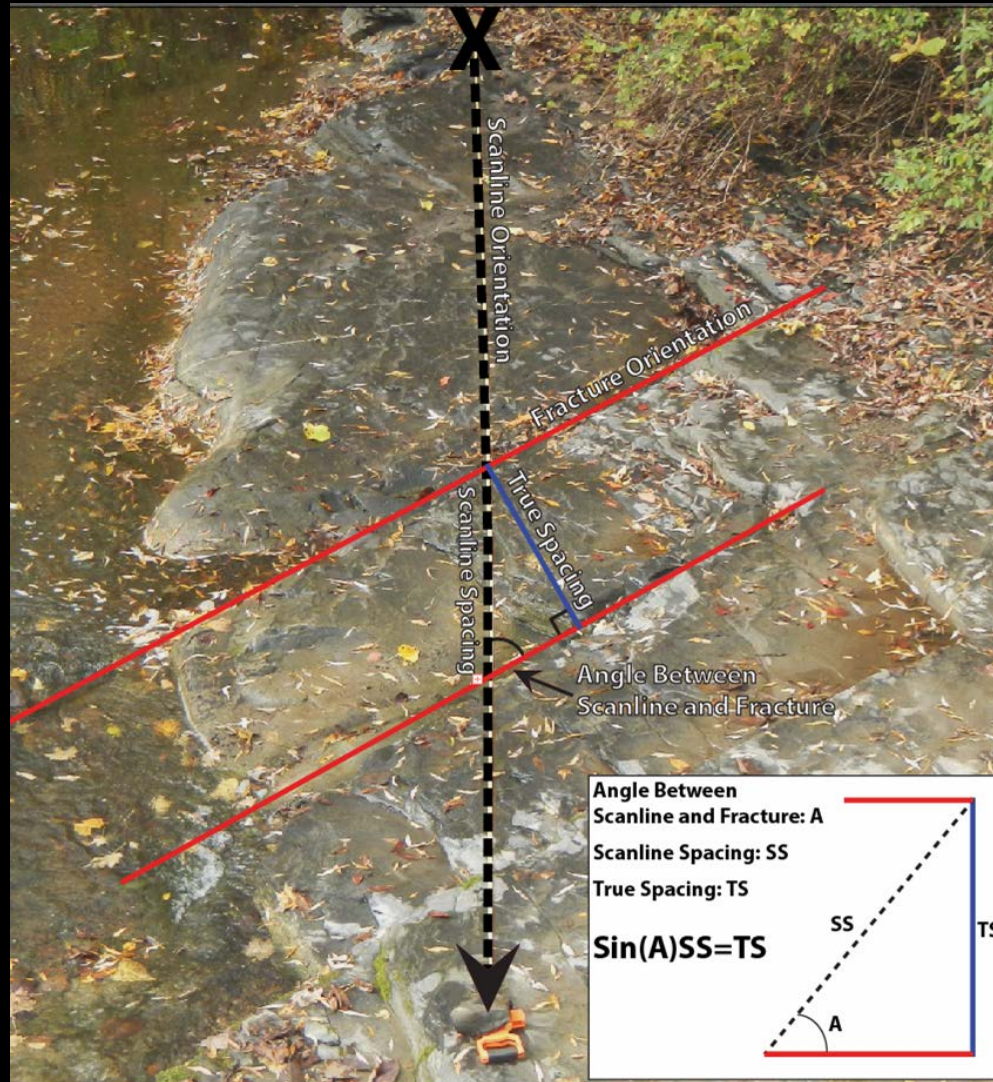


# Fracture Intensification Domain (FID)



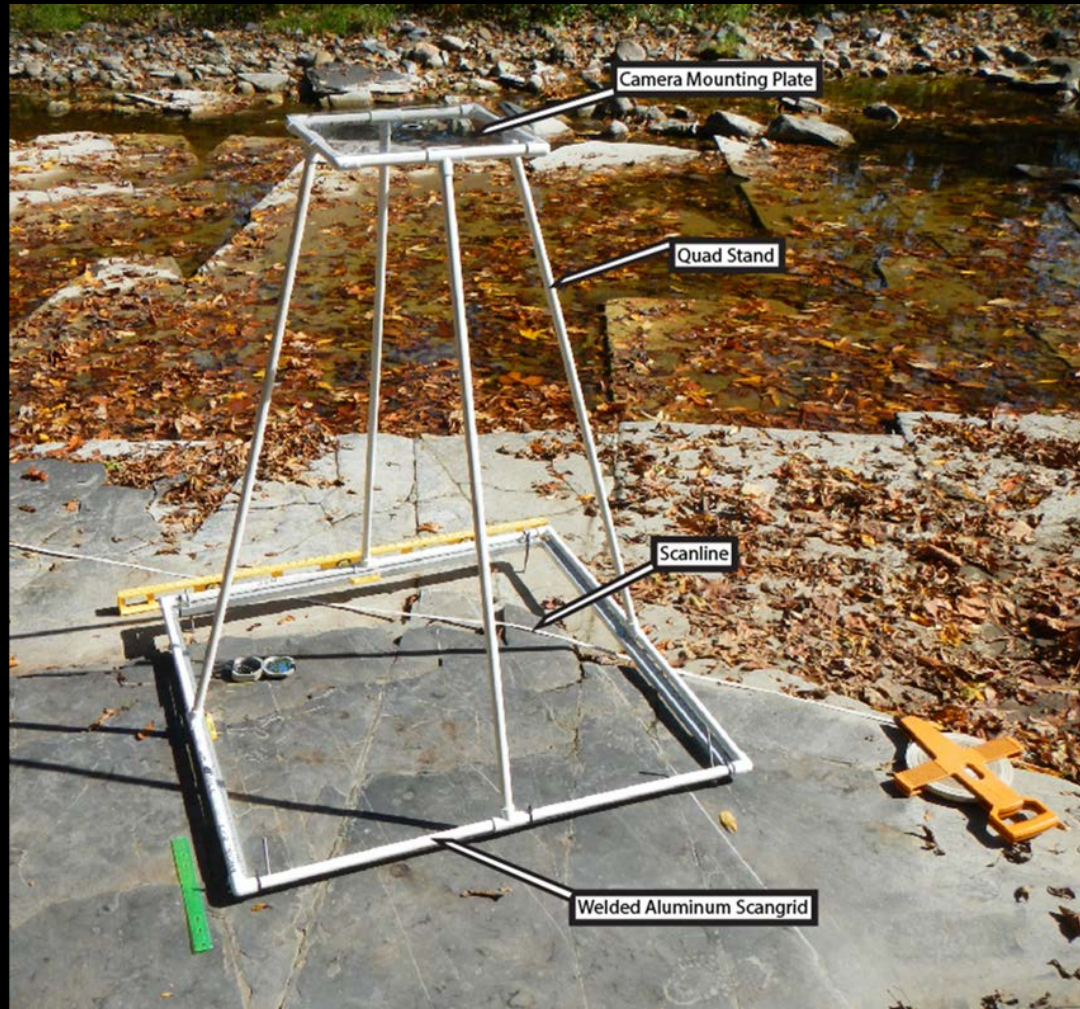


# Field Methods

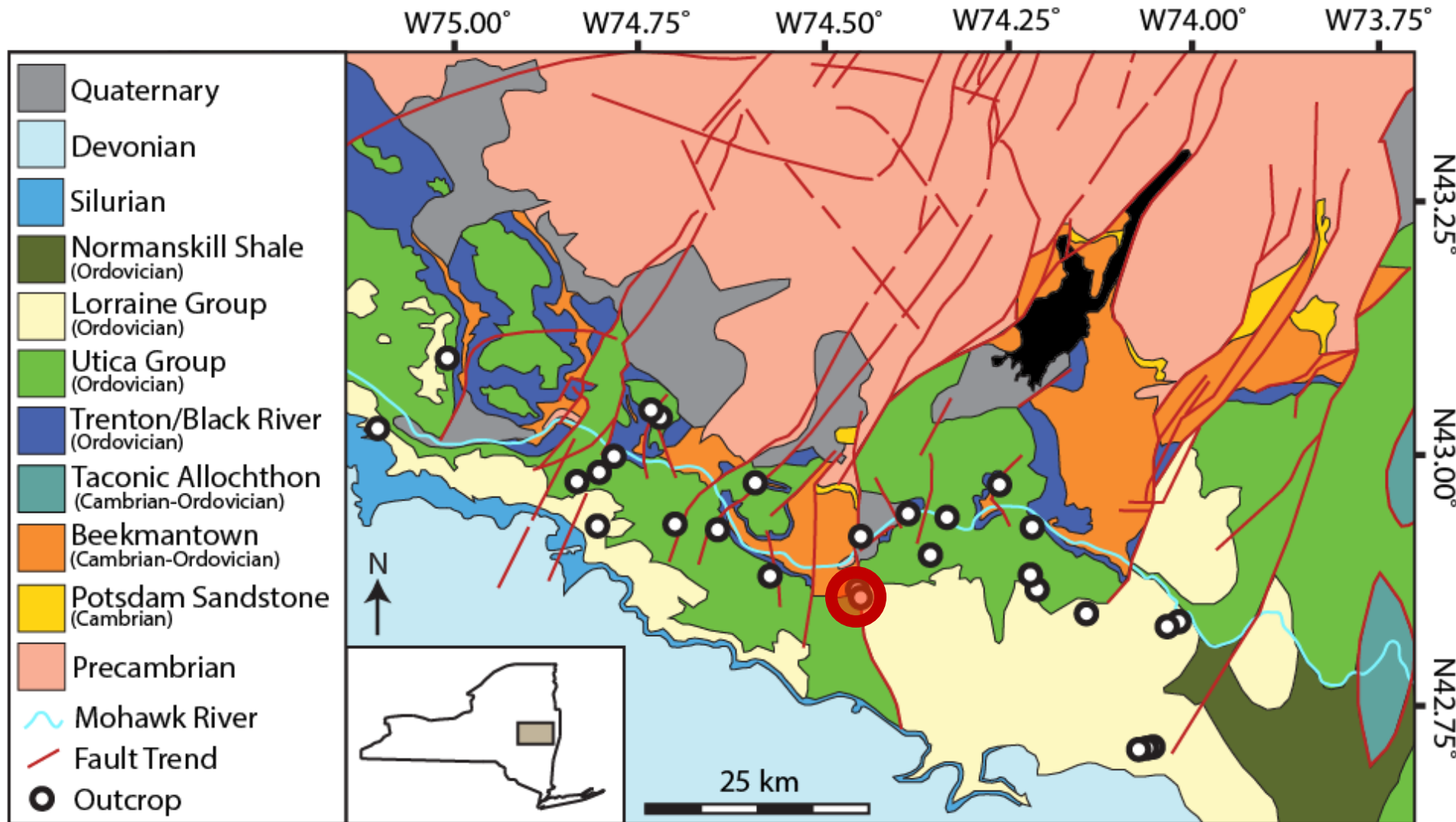




# Field Methods



# Study Area



Background

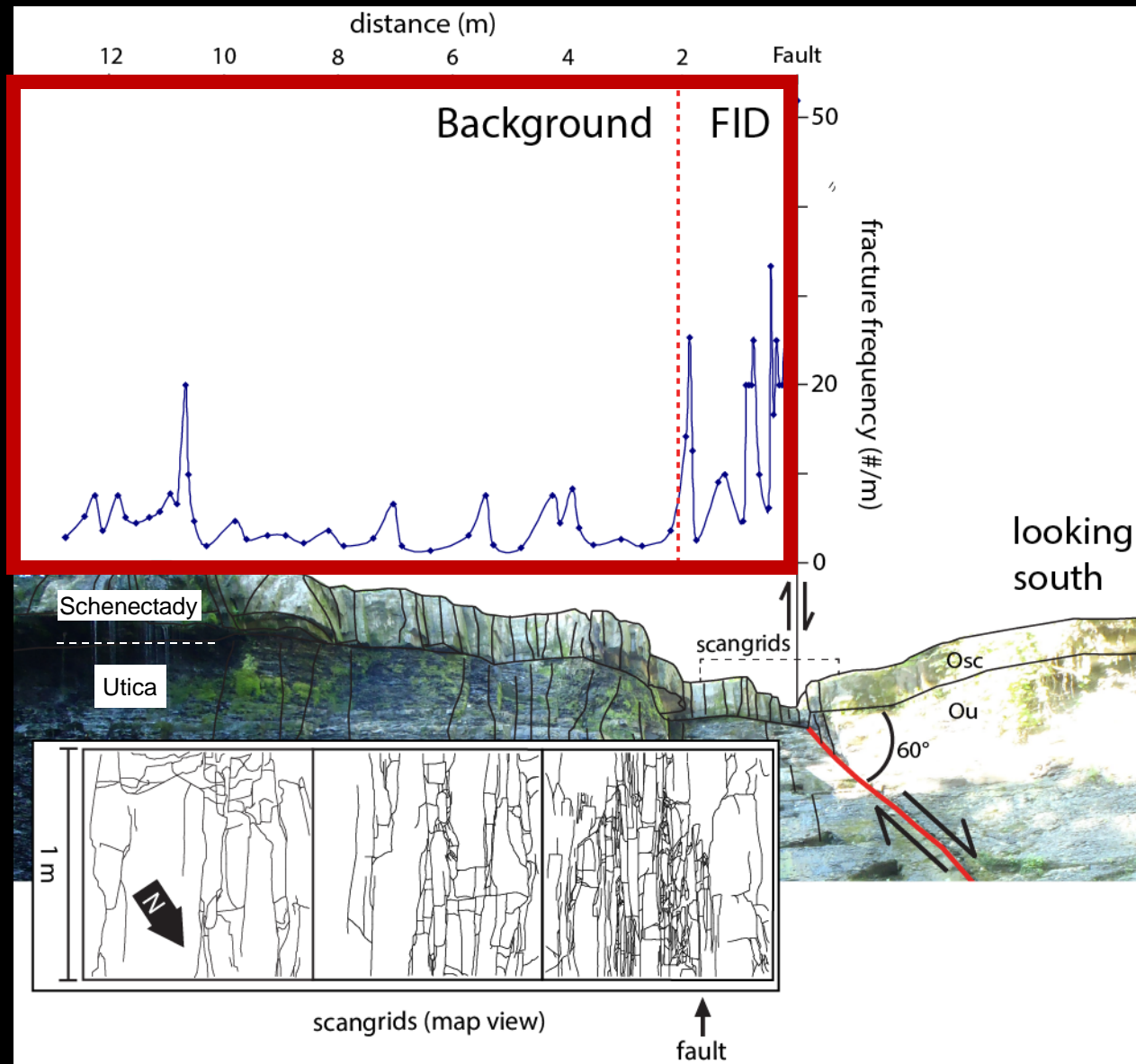
Methods

Results

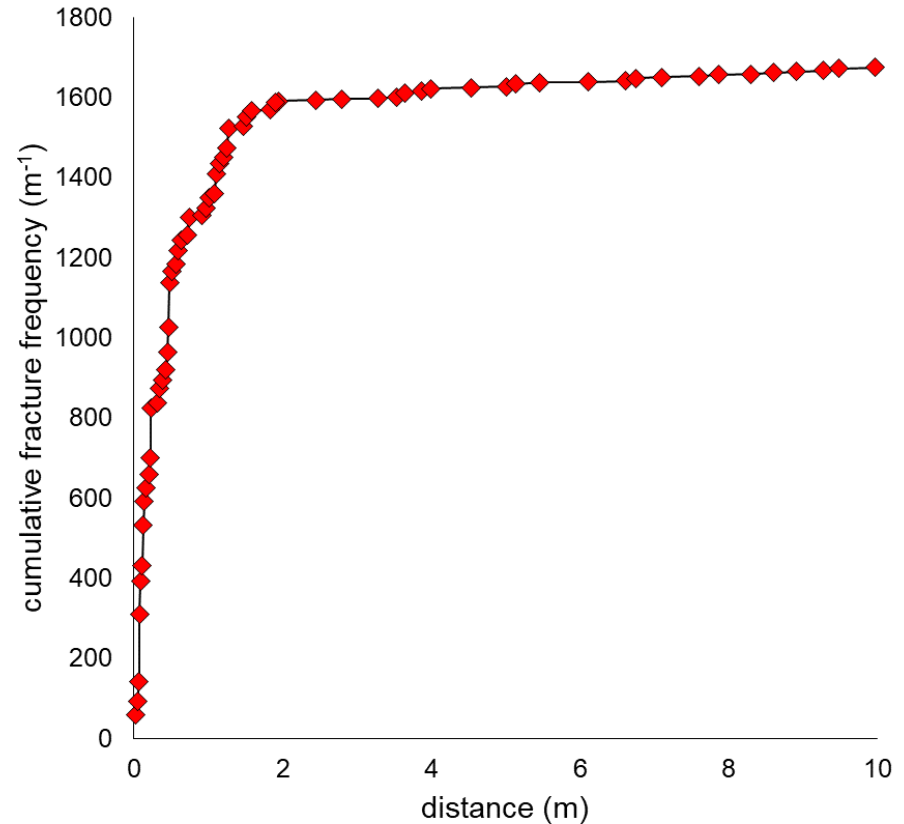
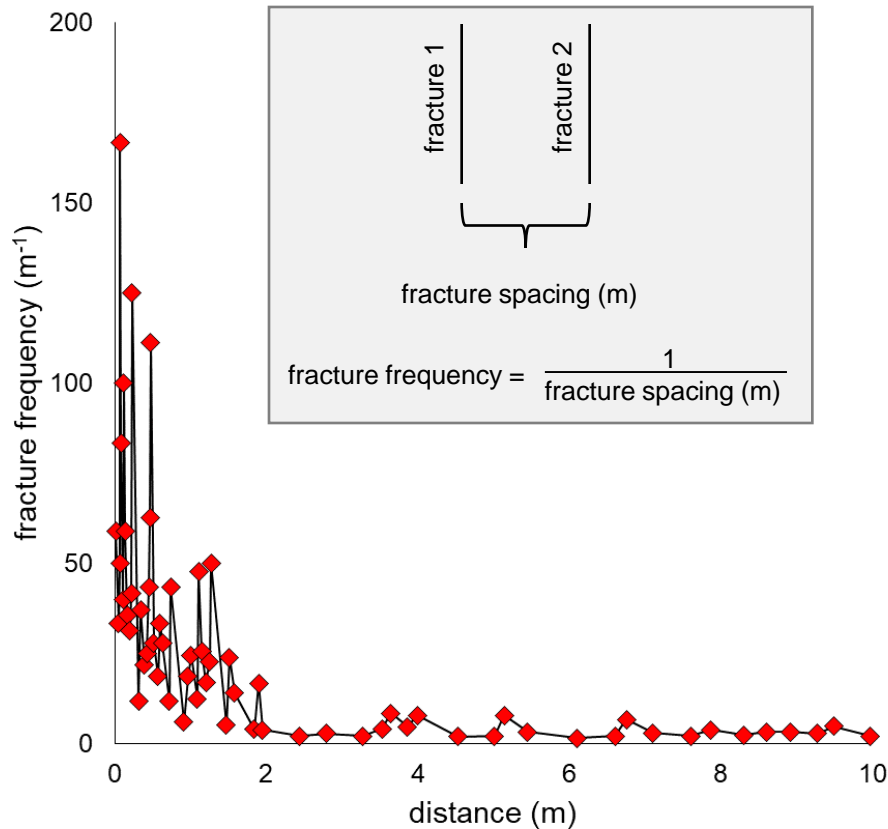
Discussion

Conclusions

# Outcrop Overview

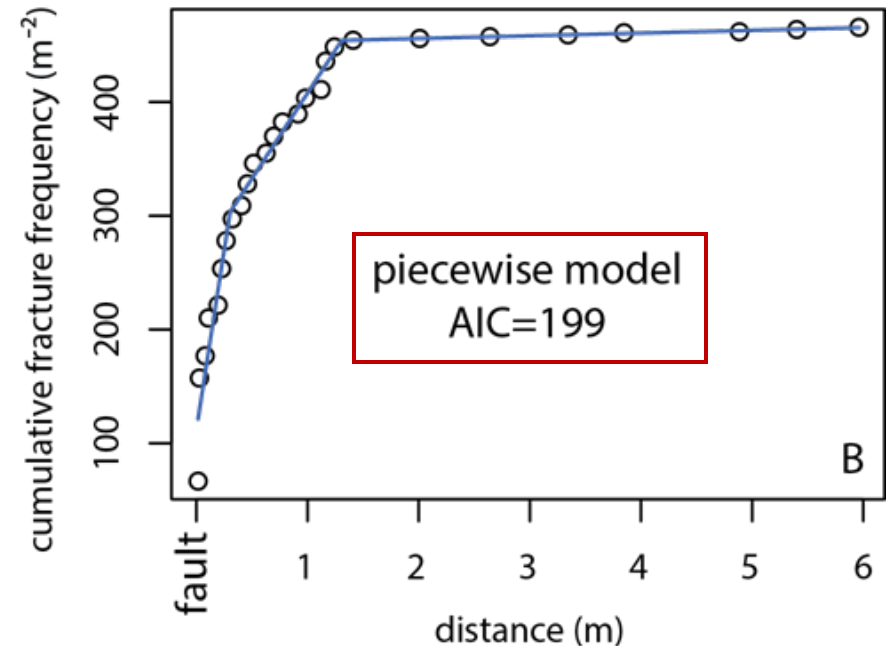
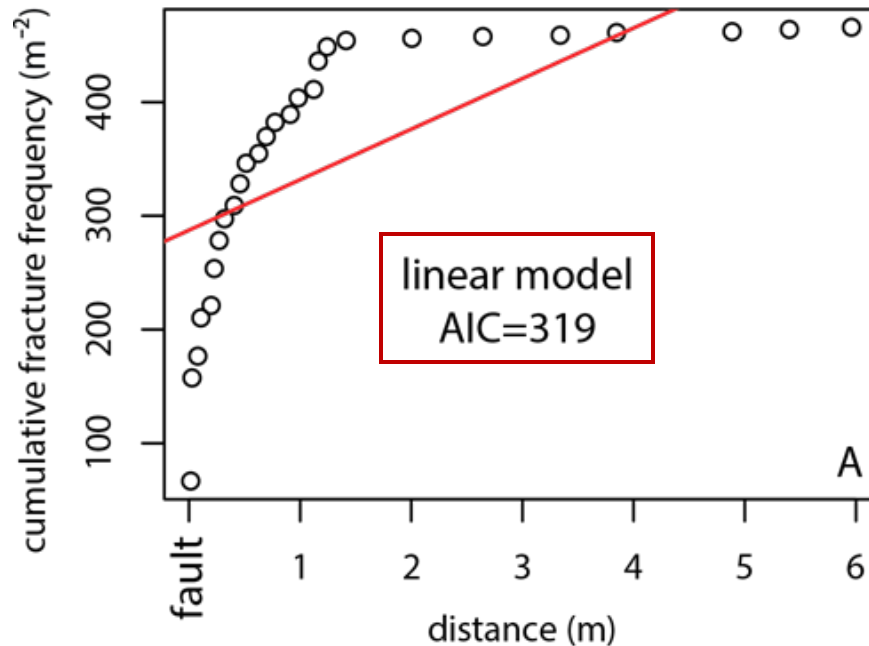


# Cumulative Fracture Frequency (CFF)





# Piecewise Regression



# Optimizing Model Complexity



Background

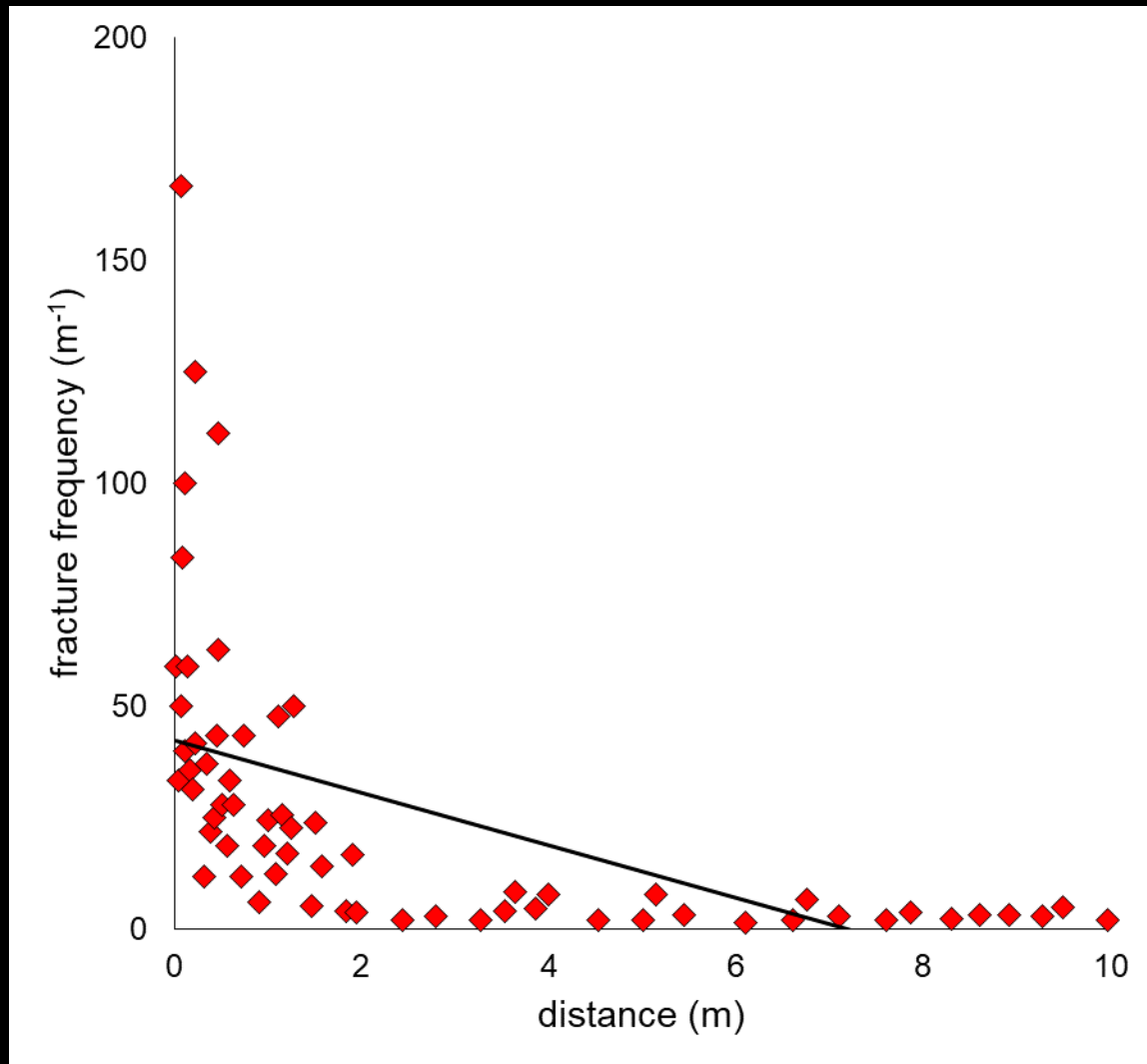
Methods

Results

Discussion

Conclusions

# High Bias

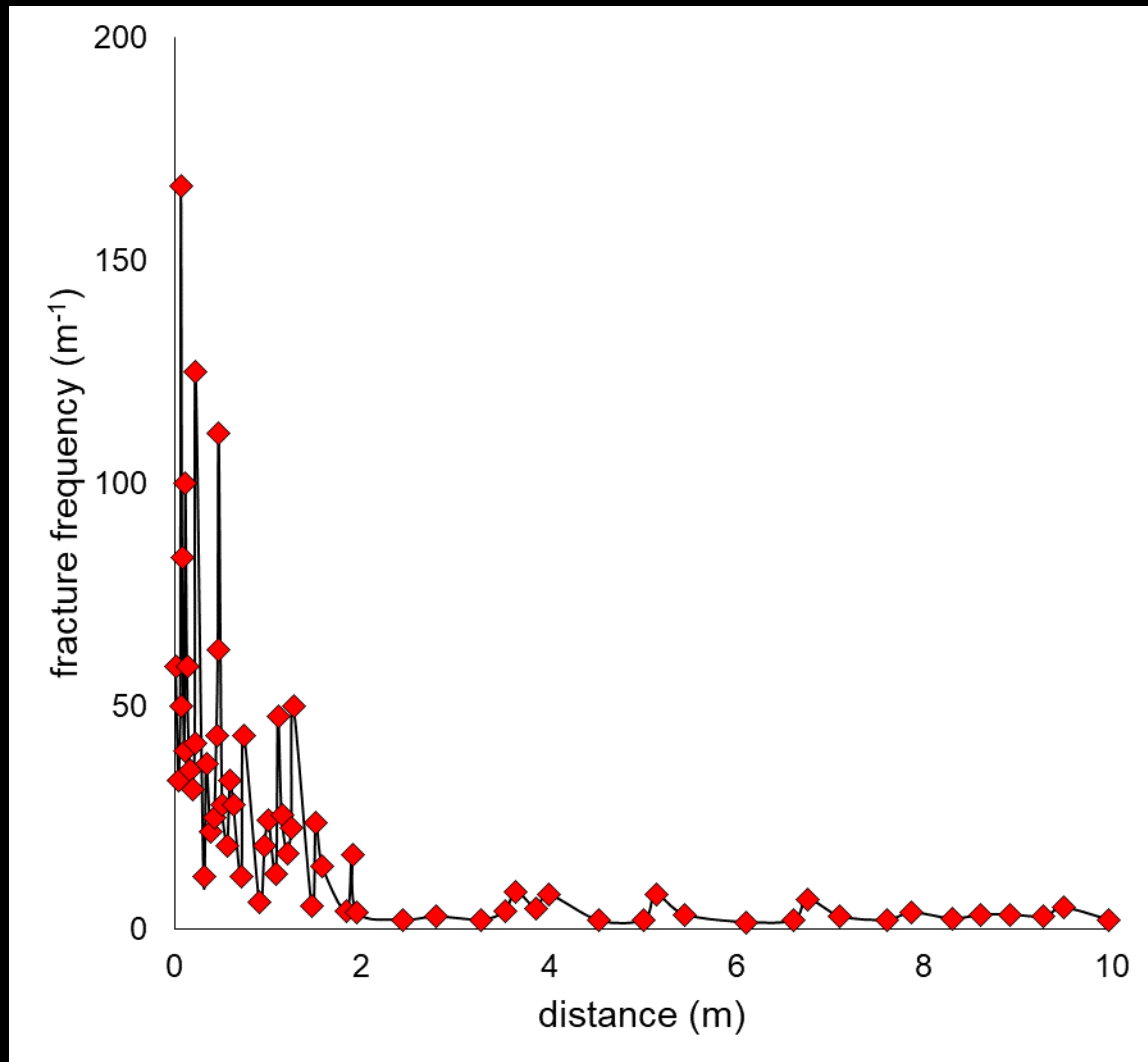


# Optimizing Model Complexity





# High Variance



# Optimizing Model Complexity



Background

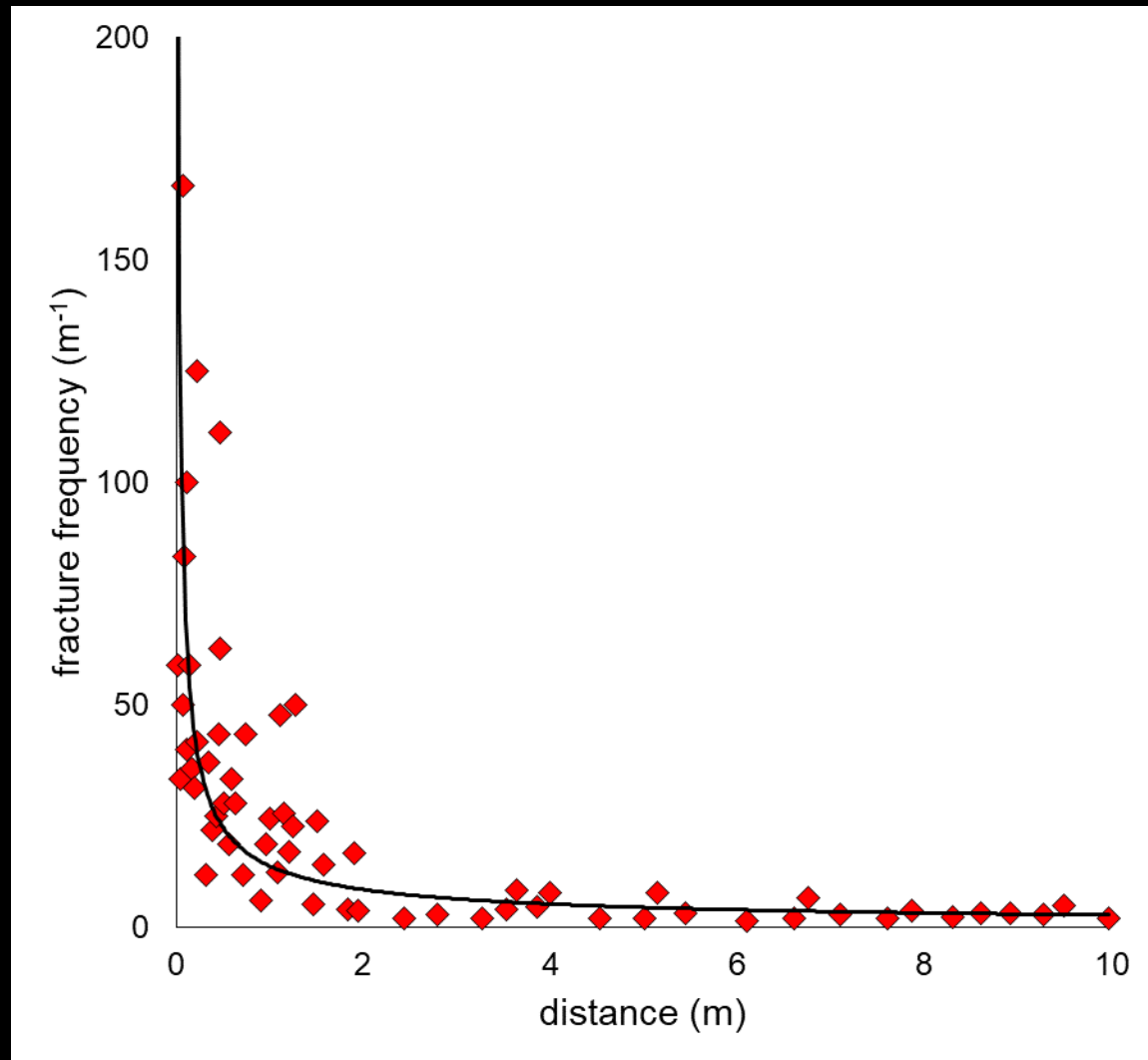
Methods

Results

Discussion

Conclusions

# Optimum Model Complexity



# Akaike (AIC) - Bayesian (BIC) Information Criterion

$$\text{AIC} = \boxed{n + n \log(2\pi)} + \boxed{n \log(\text{RSS}/n)} + \boxed{2(p + 1)}$$

constant

residual sum  
of squares

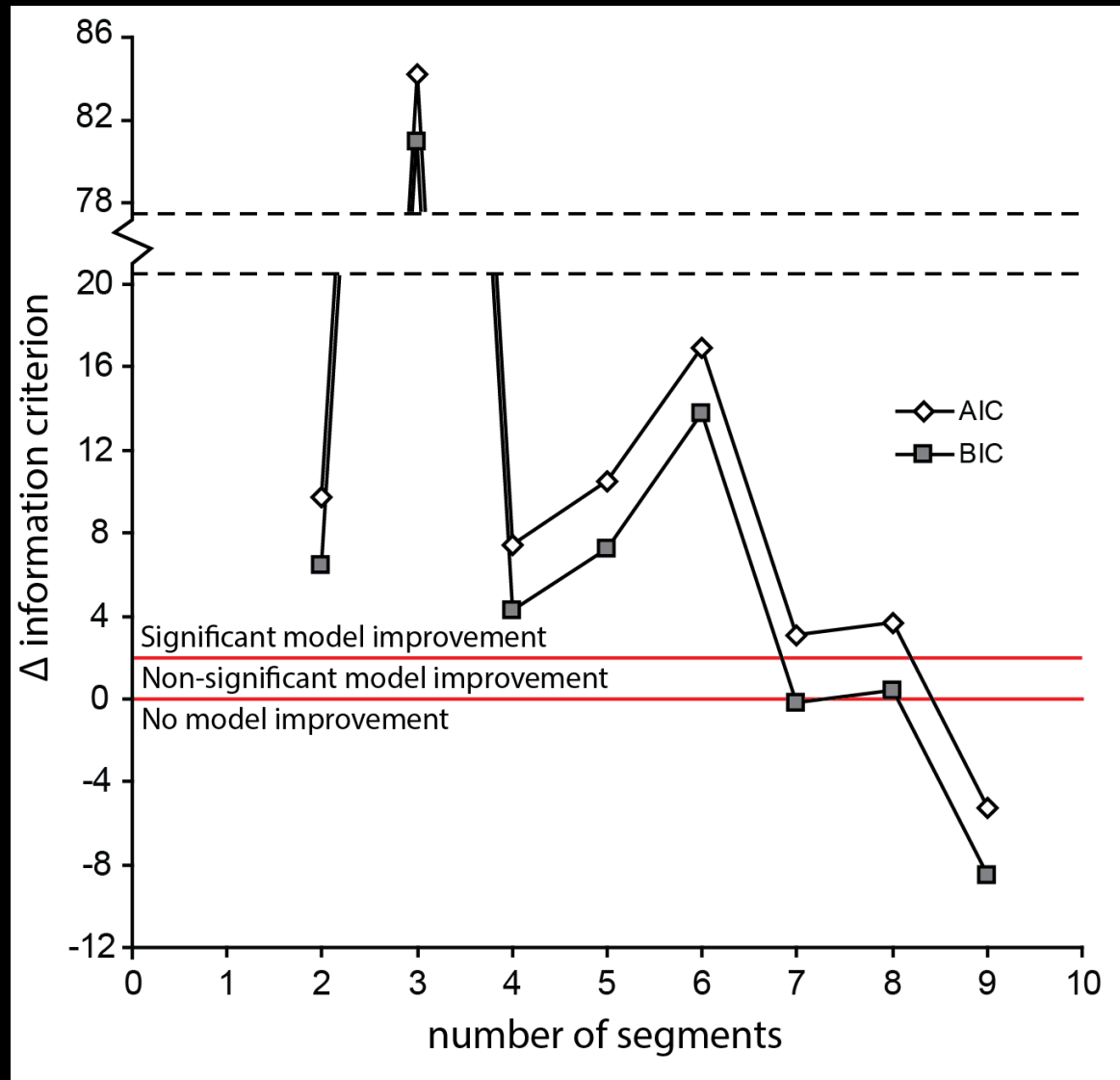
# of parameters

$$\text{BIC} = \boxed{n + n \log(2\pi)} + \boxed{n \log(\text{RSS}/n)} + \boxed{(\log n)(p + 1)}$$

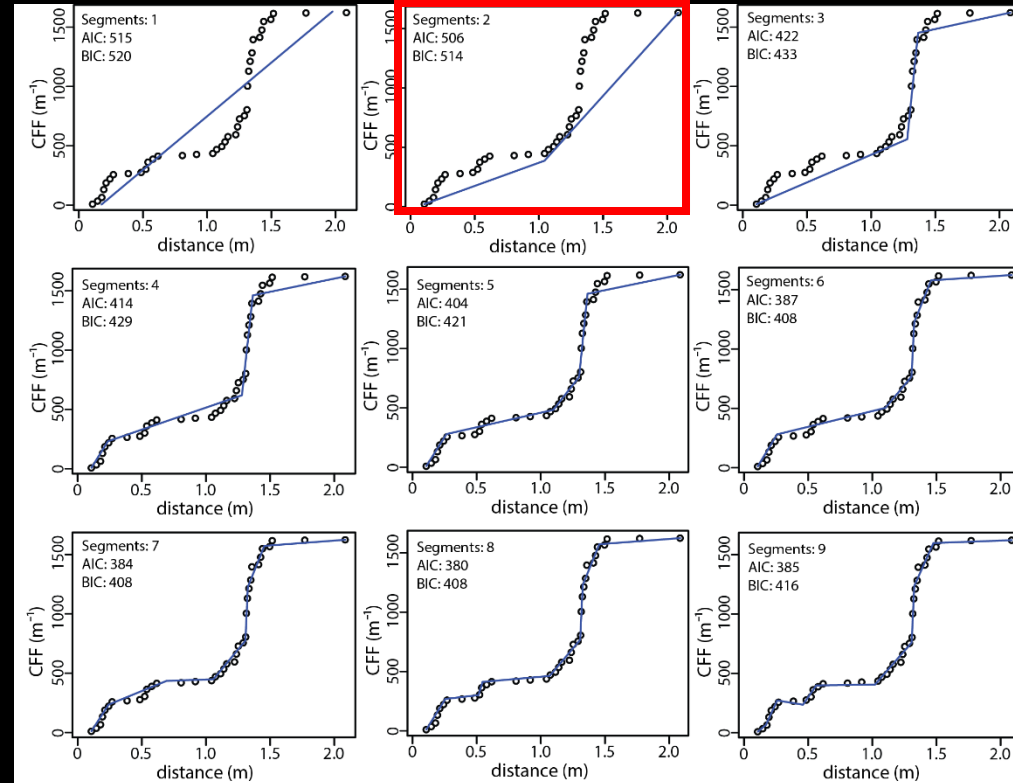
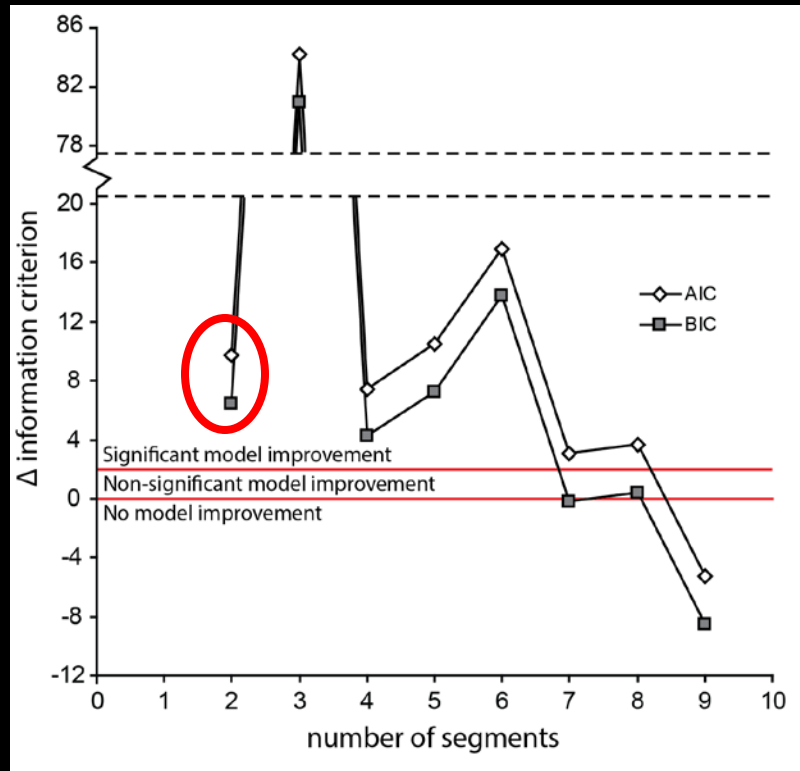
- Minimize AIC/BIC values among potential models
- Use change in AIC/BIC values between 2 models
- Significant change in AIC/BIC values  $\geq 2$



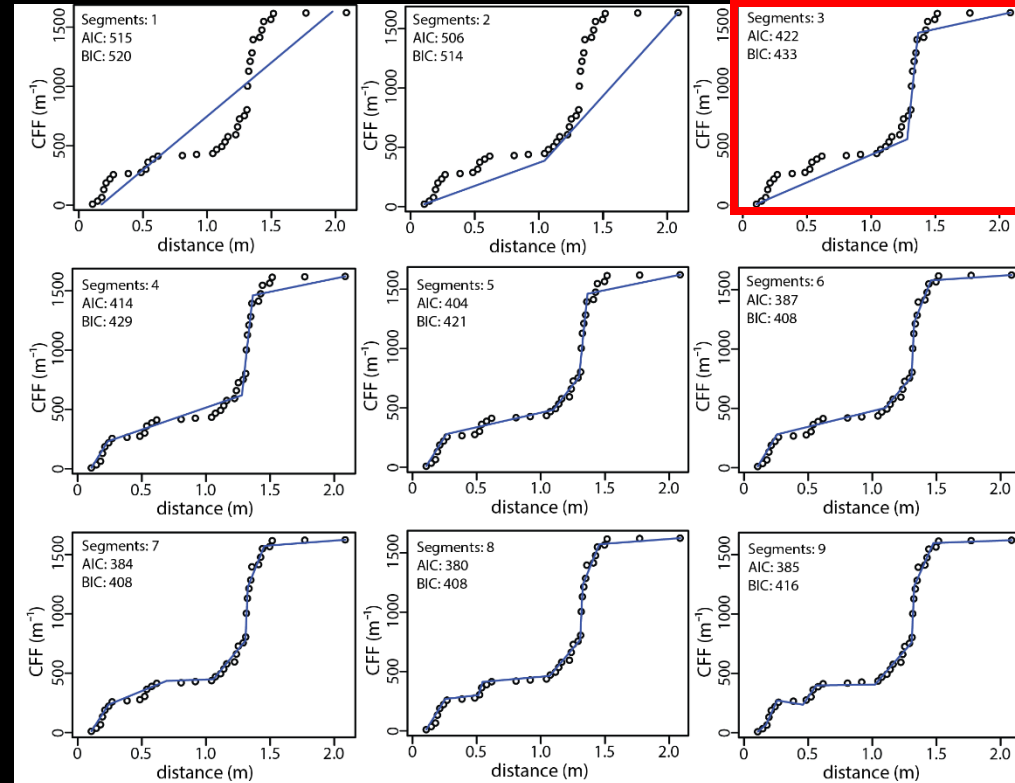
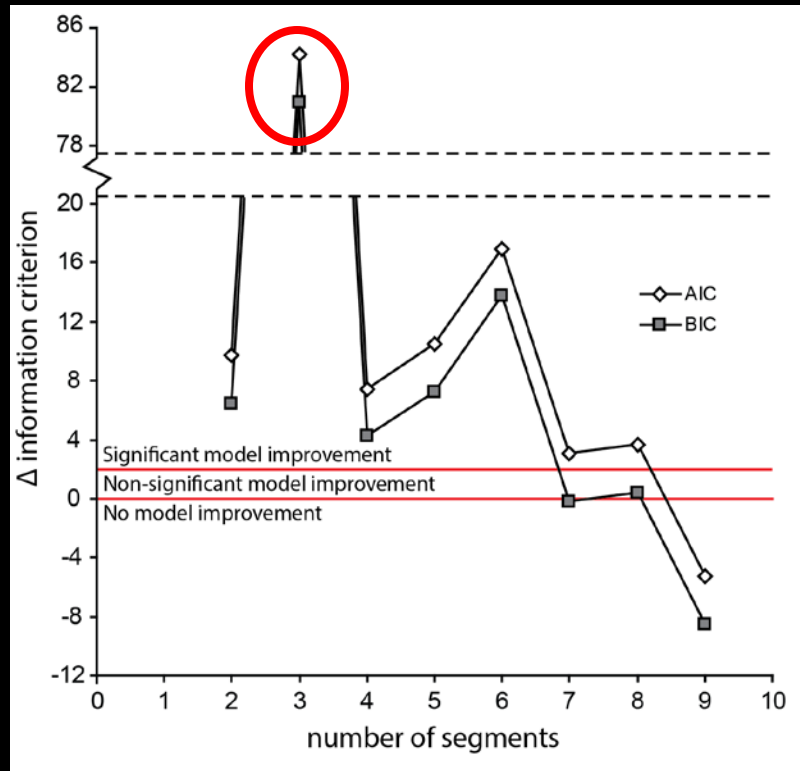
# $\Delta$ Information Criterion



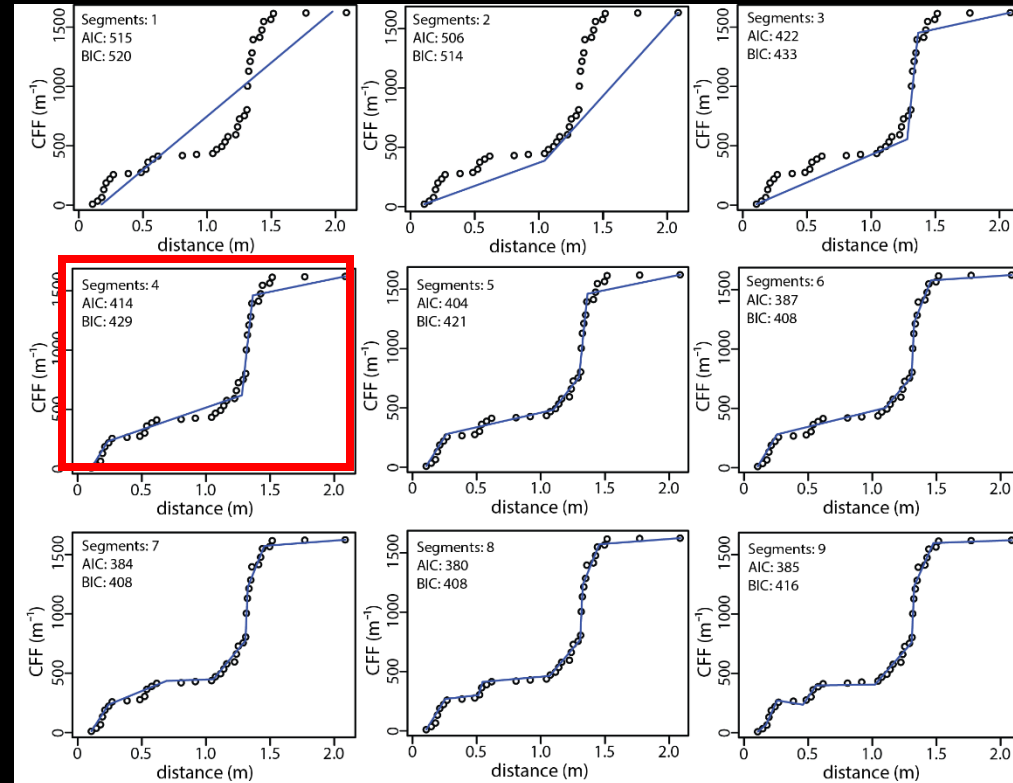
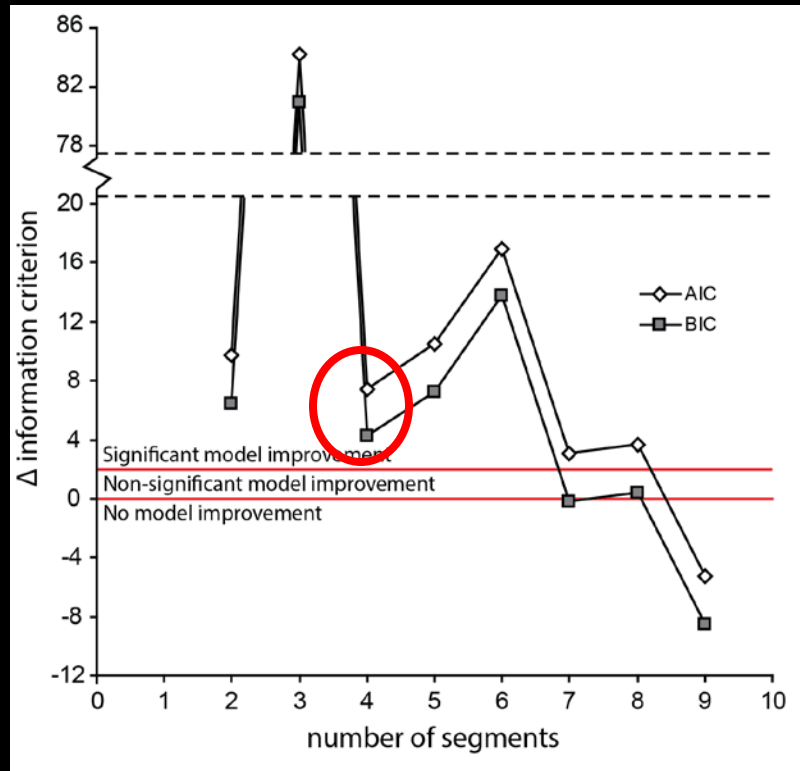
# Model Selection



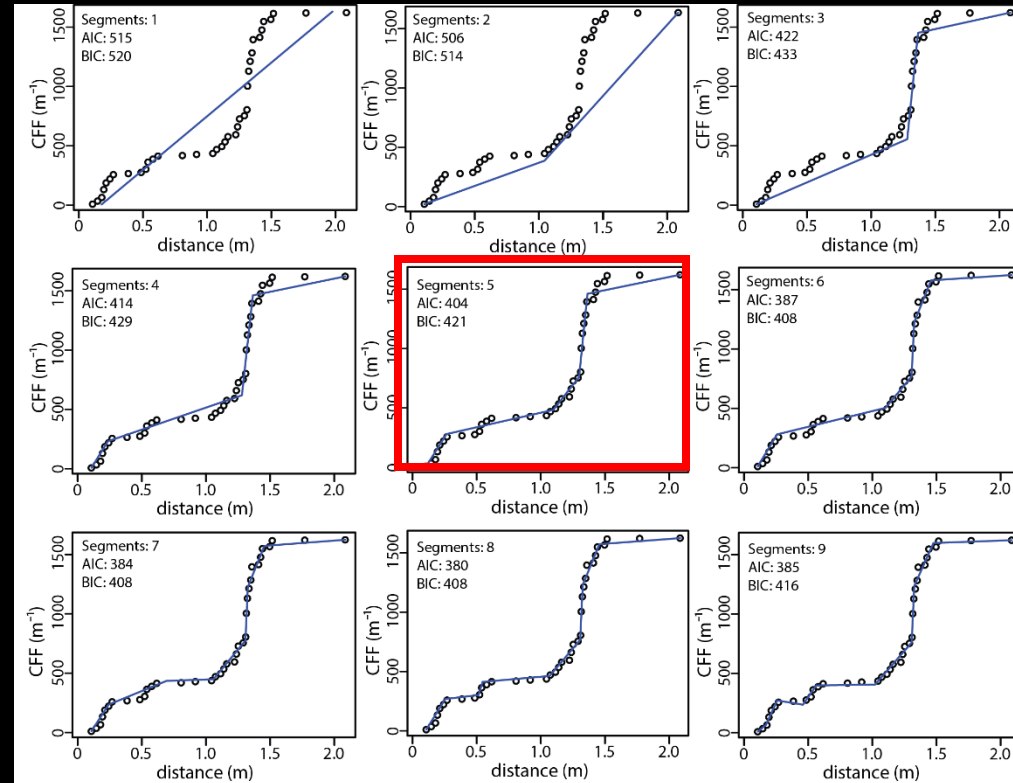
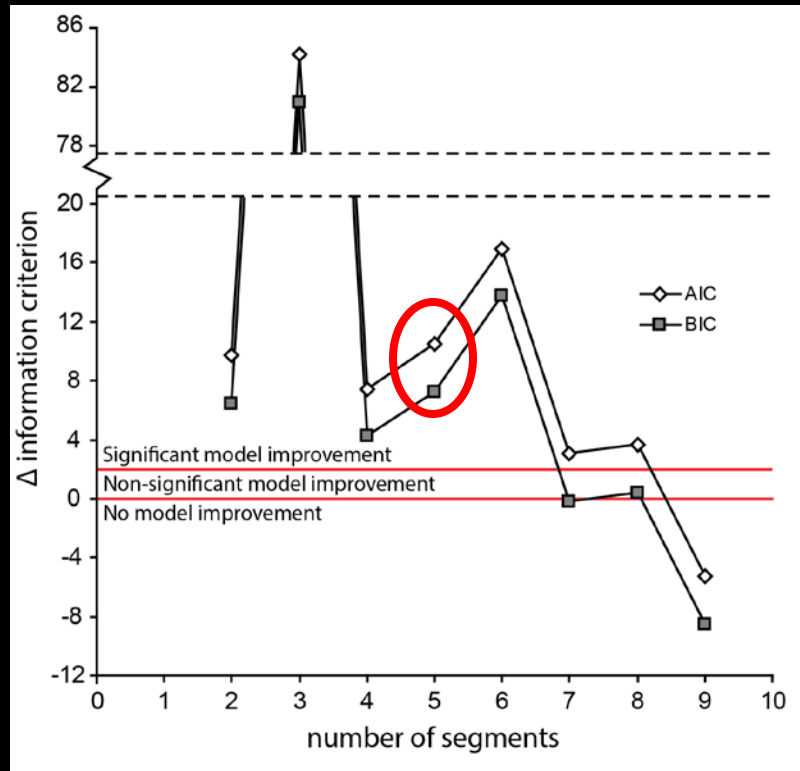
# Model Selection



# Model Selection

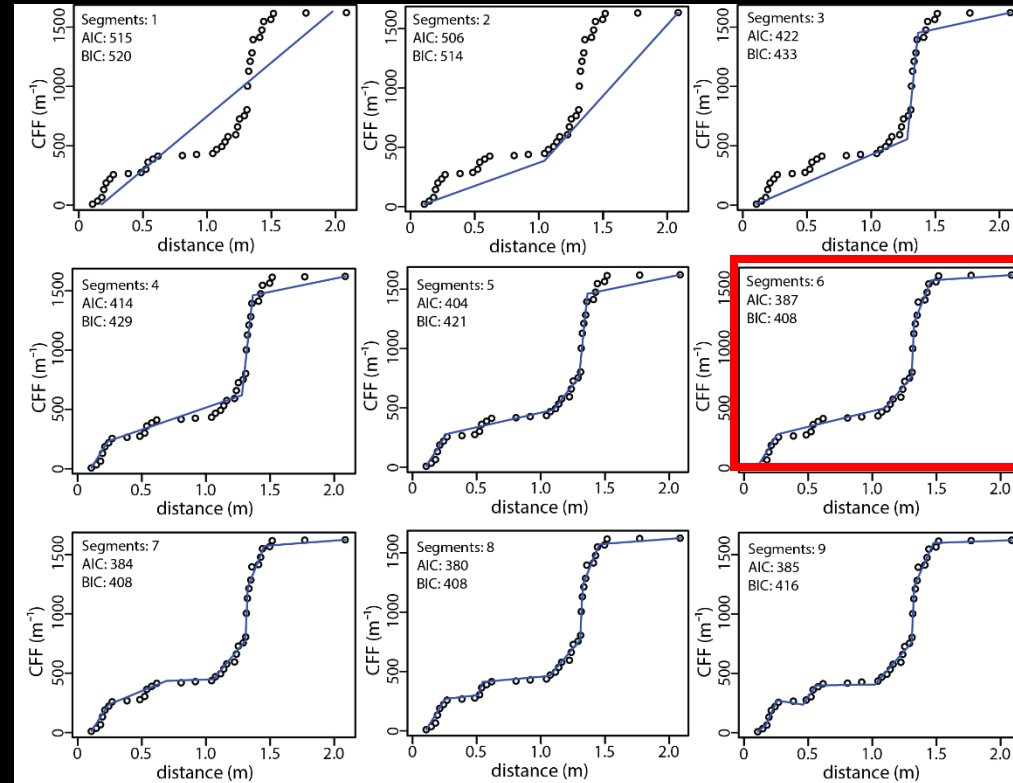
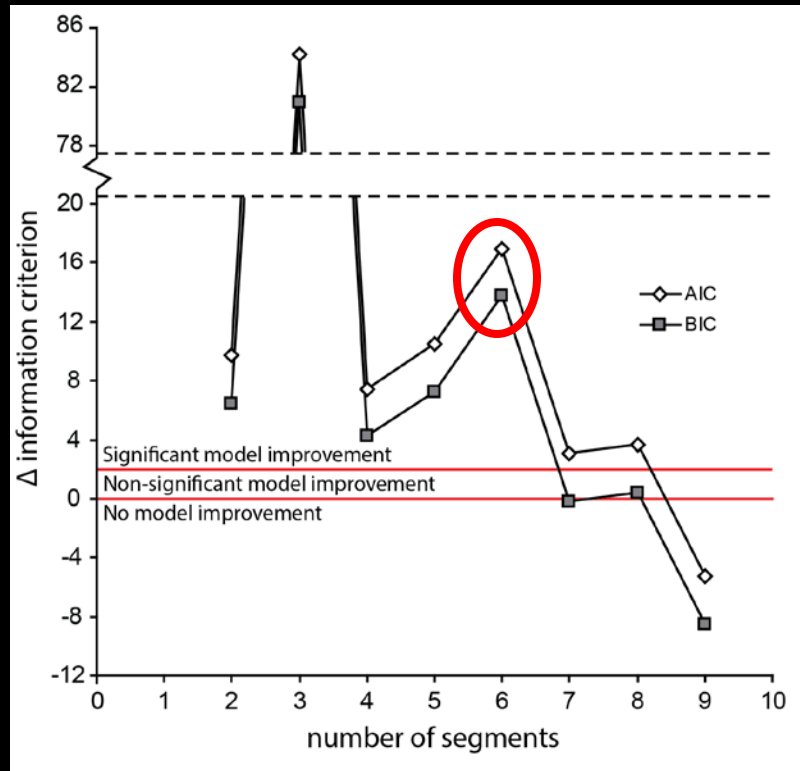


# Model Selection

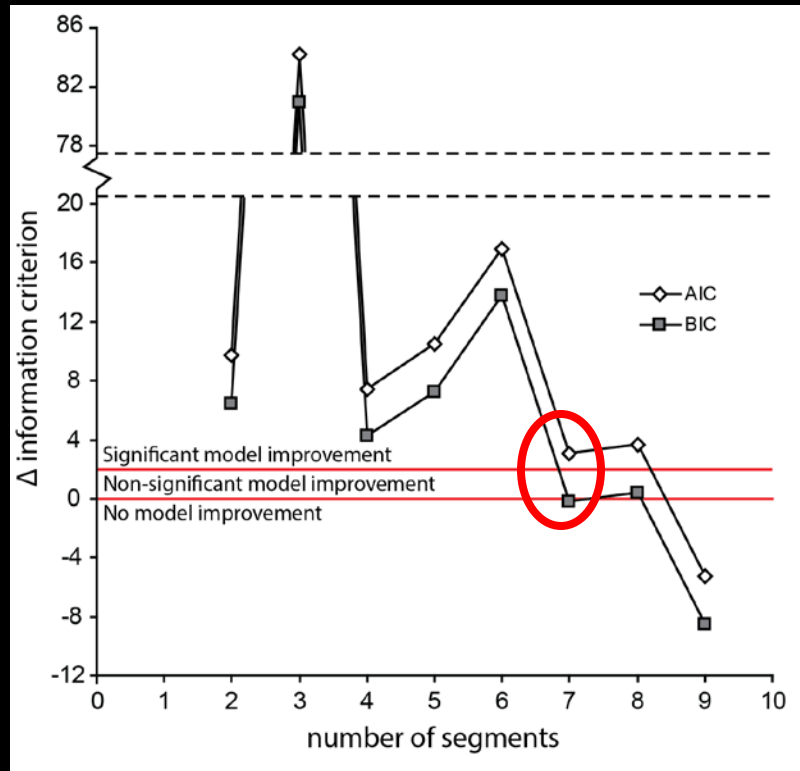




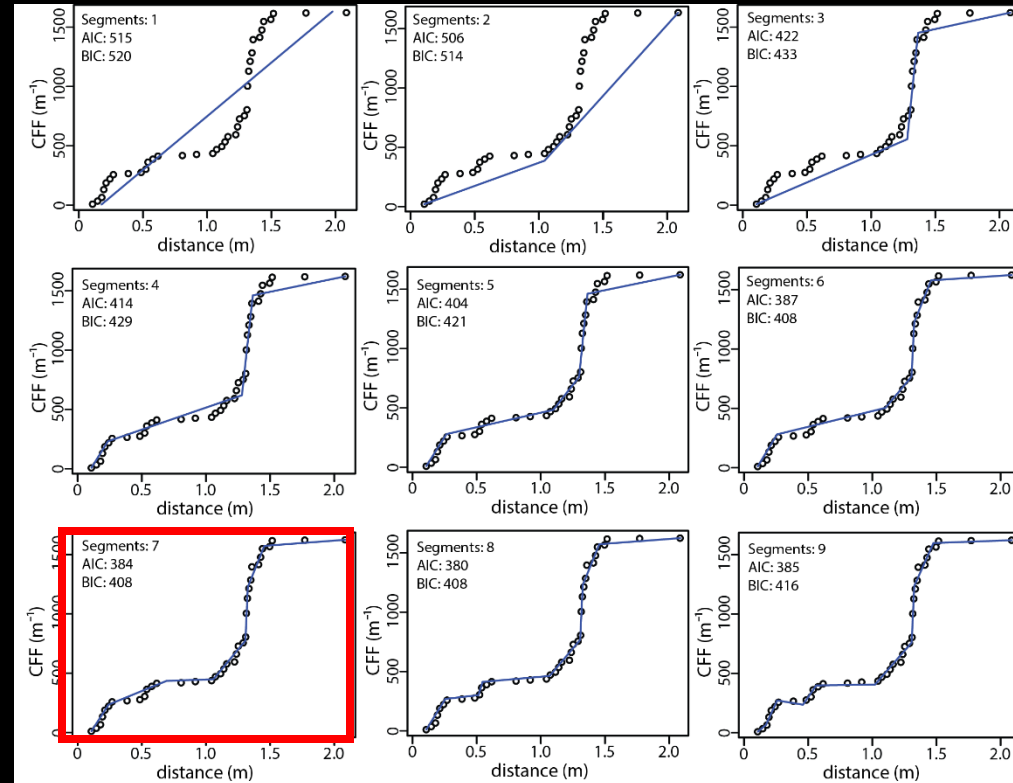
# Model Selection



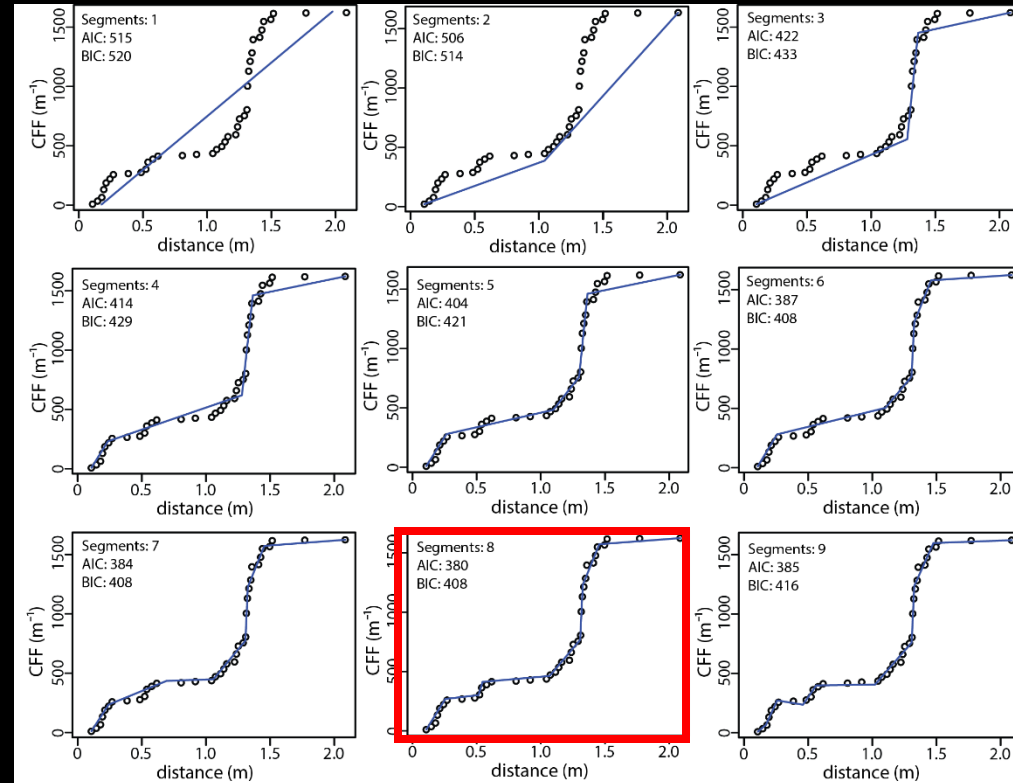
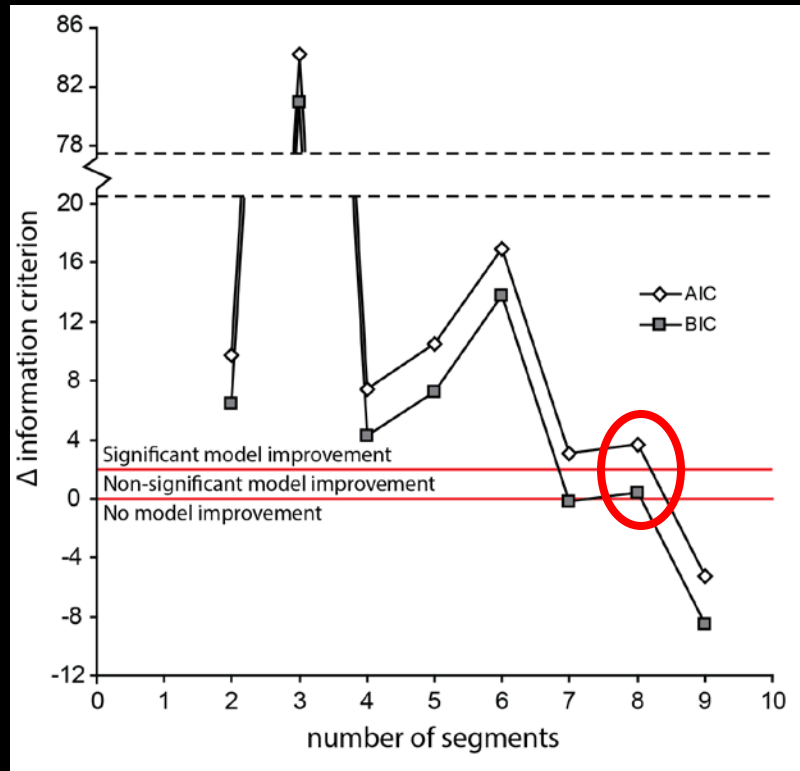
# Model Selection



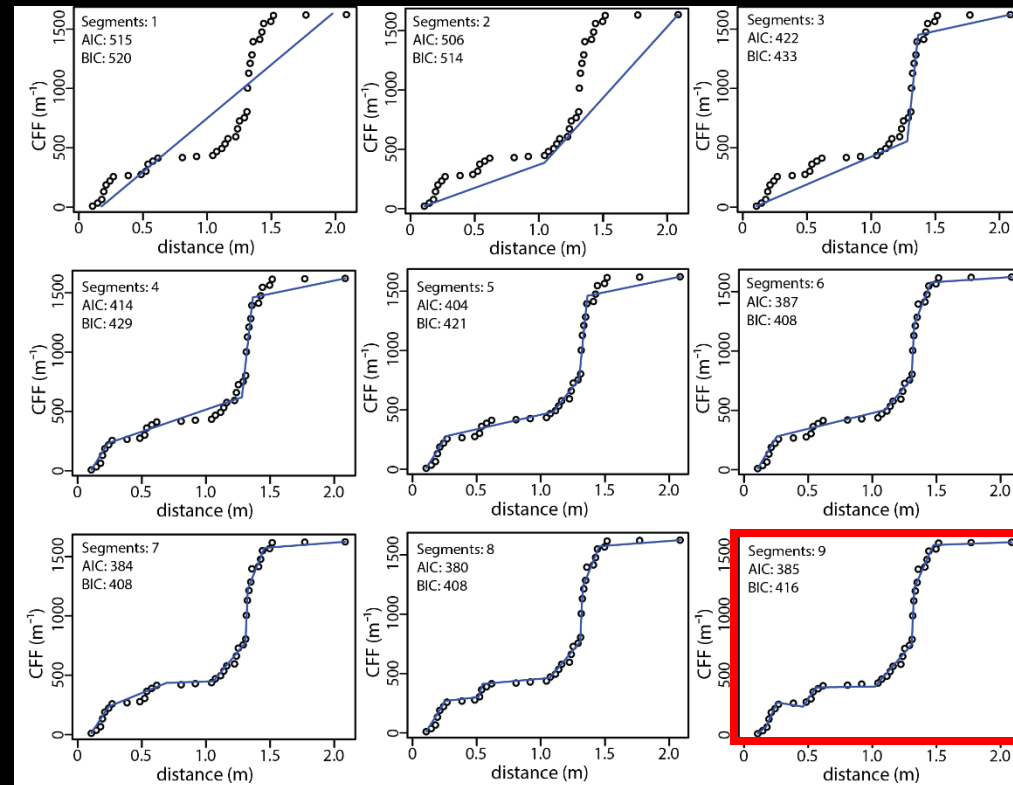
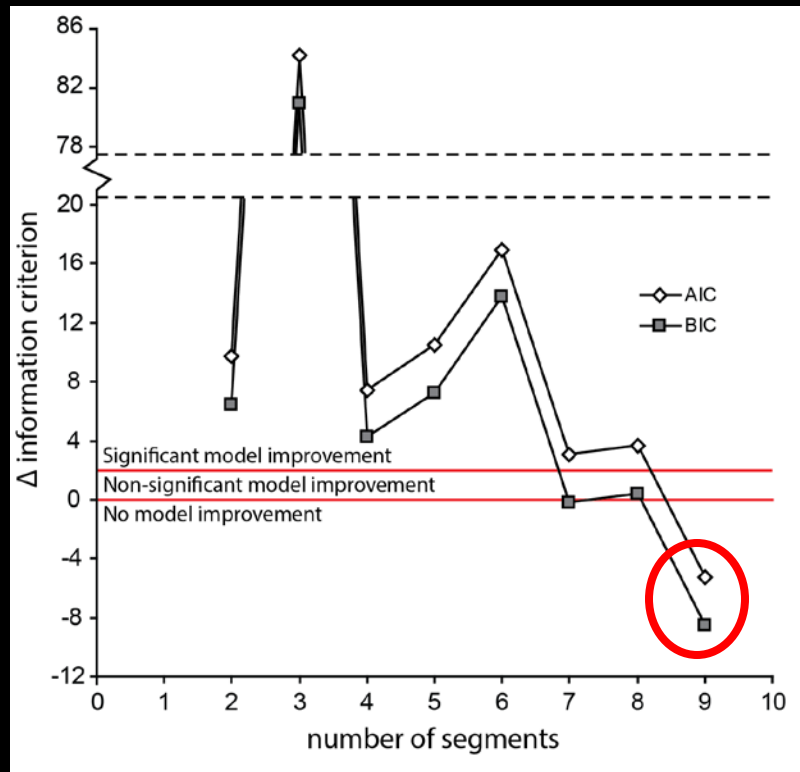
BIC not significant



# Model Selection

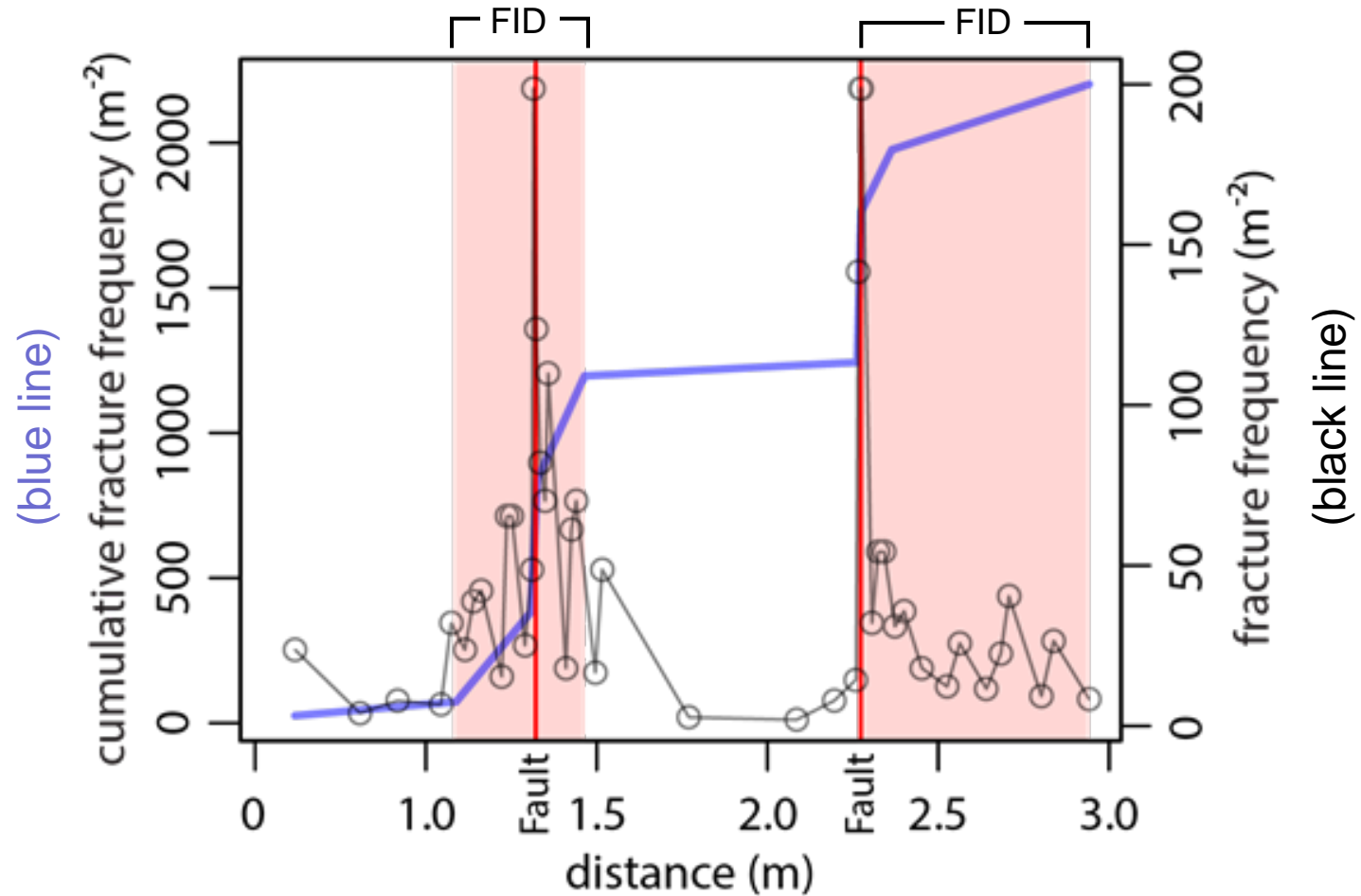


# Model Selection



AIC not significant

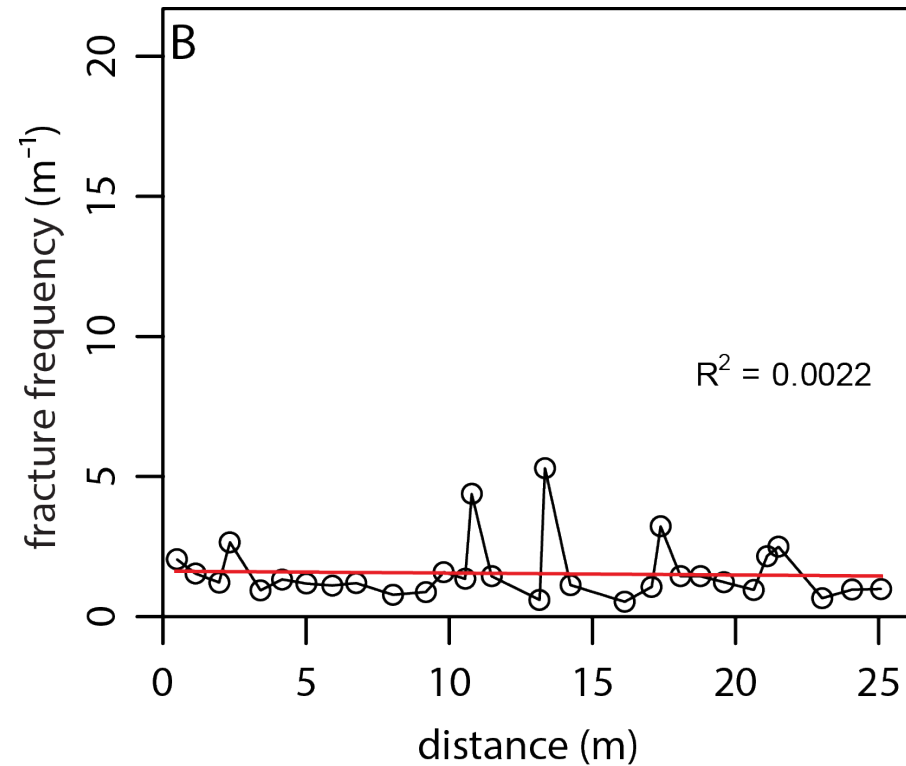
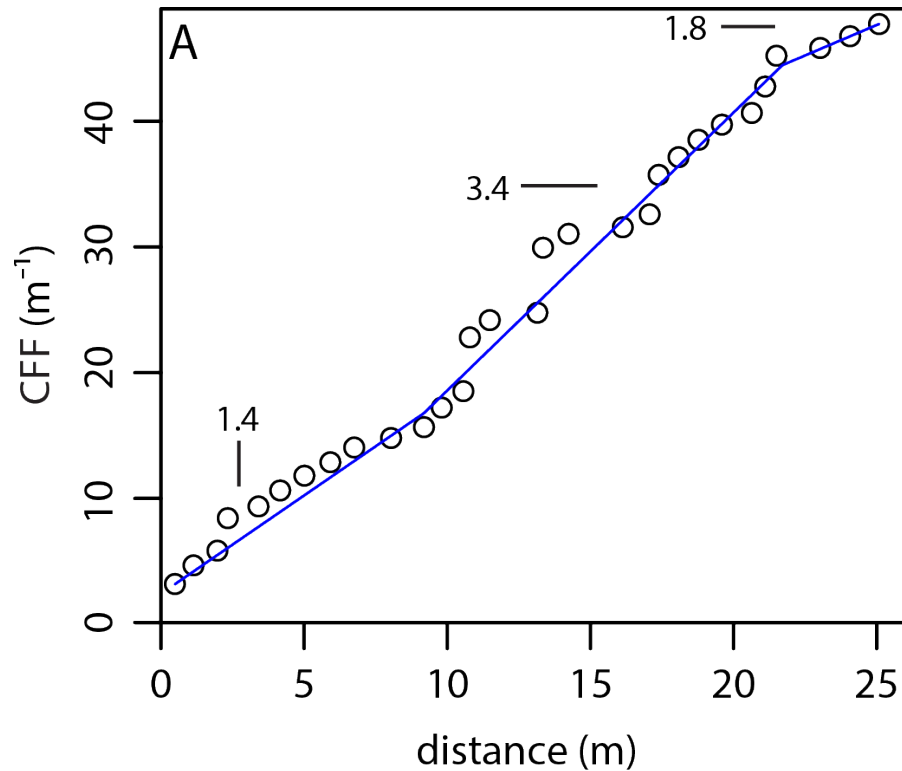
# Model Selection



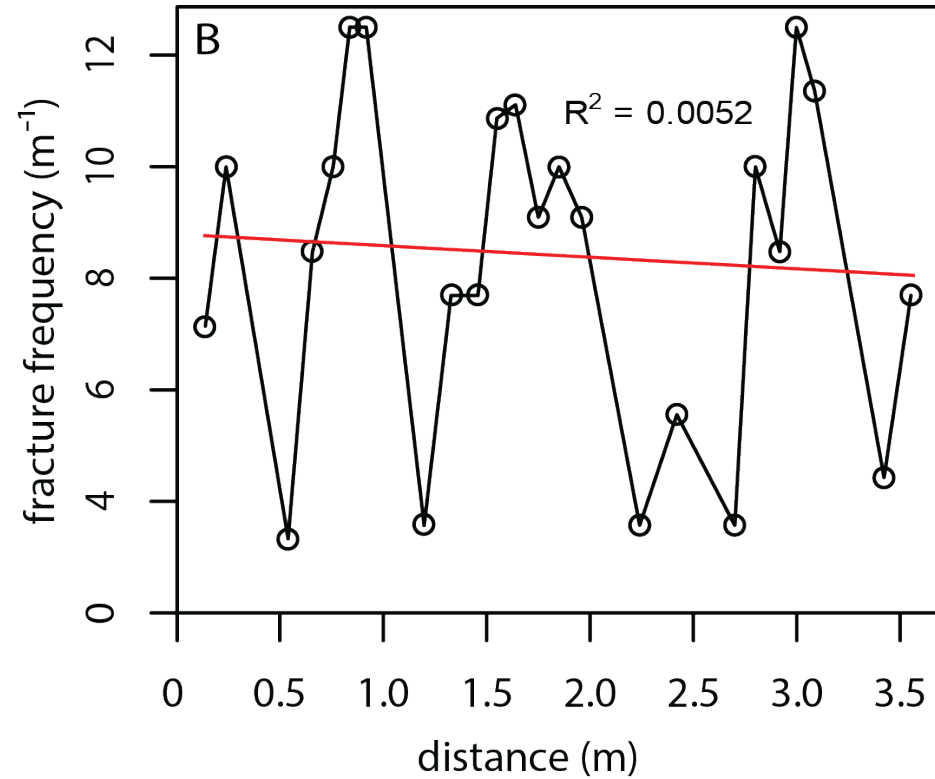
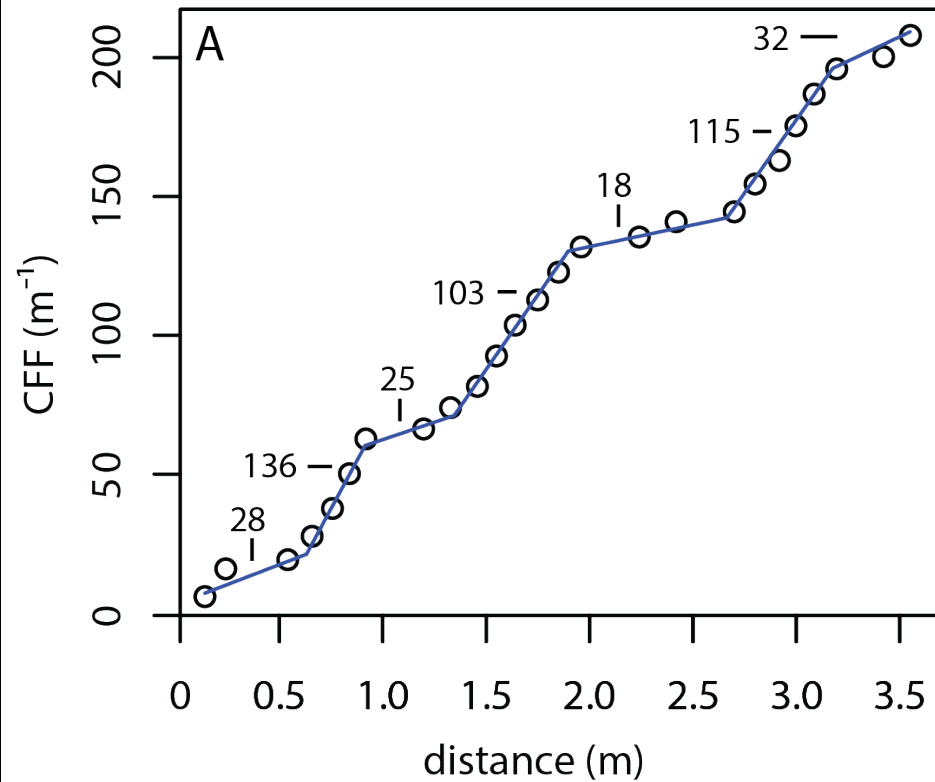
Breakpoints checked against true number of faults observed in outcrop



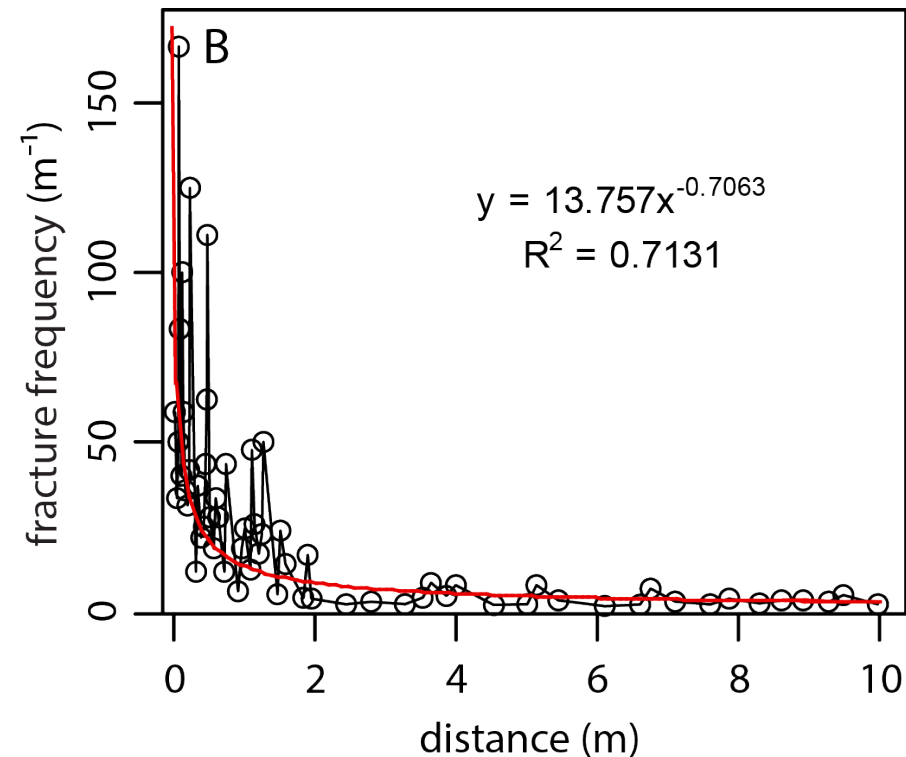
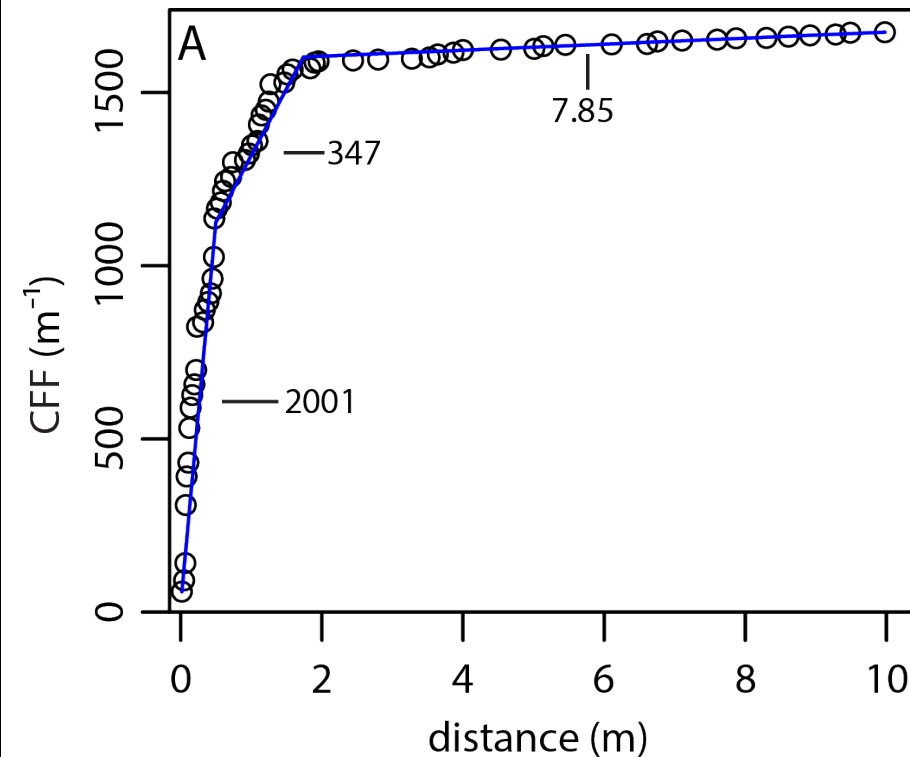
# Fracture Frequency variations



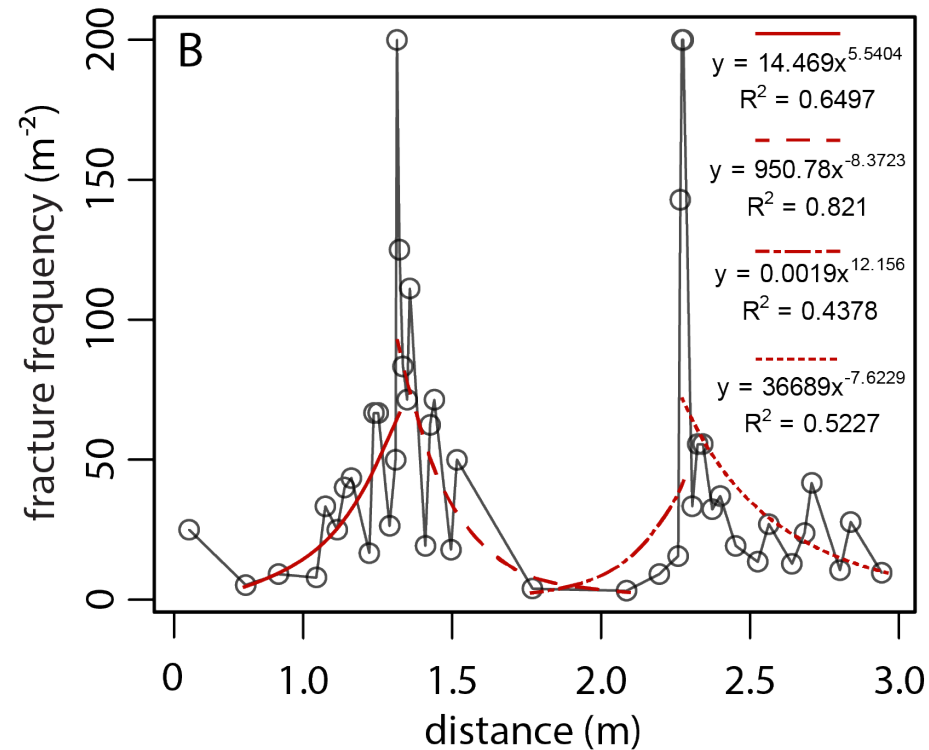
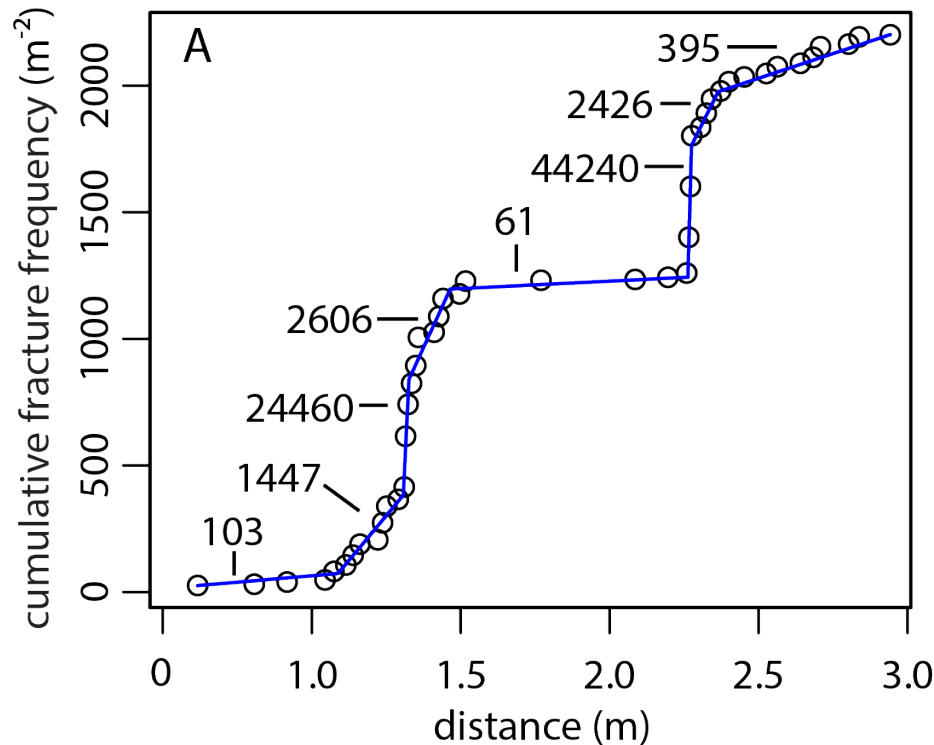
# Fracture Frequency variations



# Fracture Frequency variations

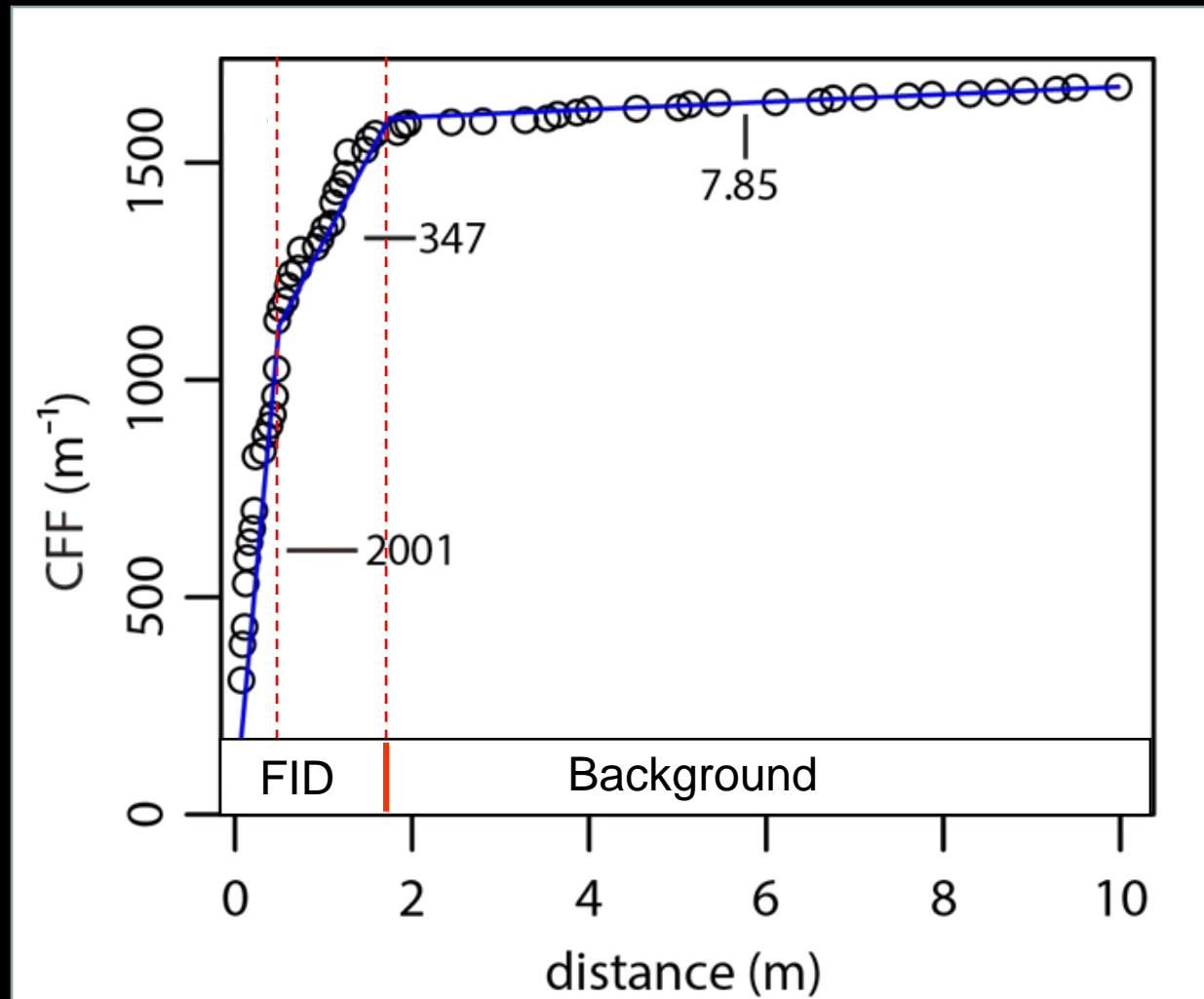


# Fracture Frequency variations

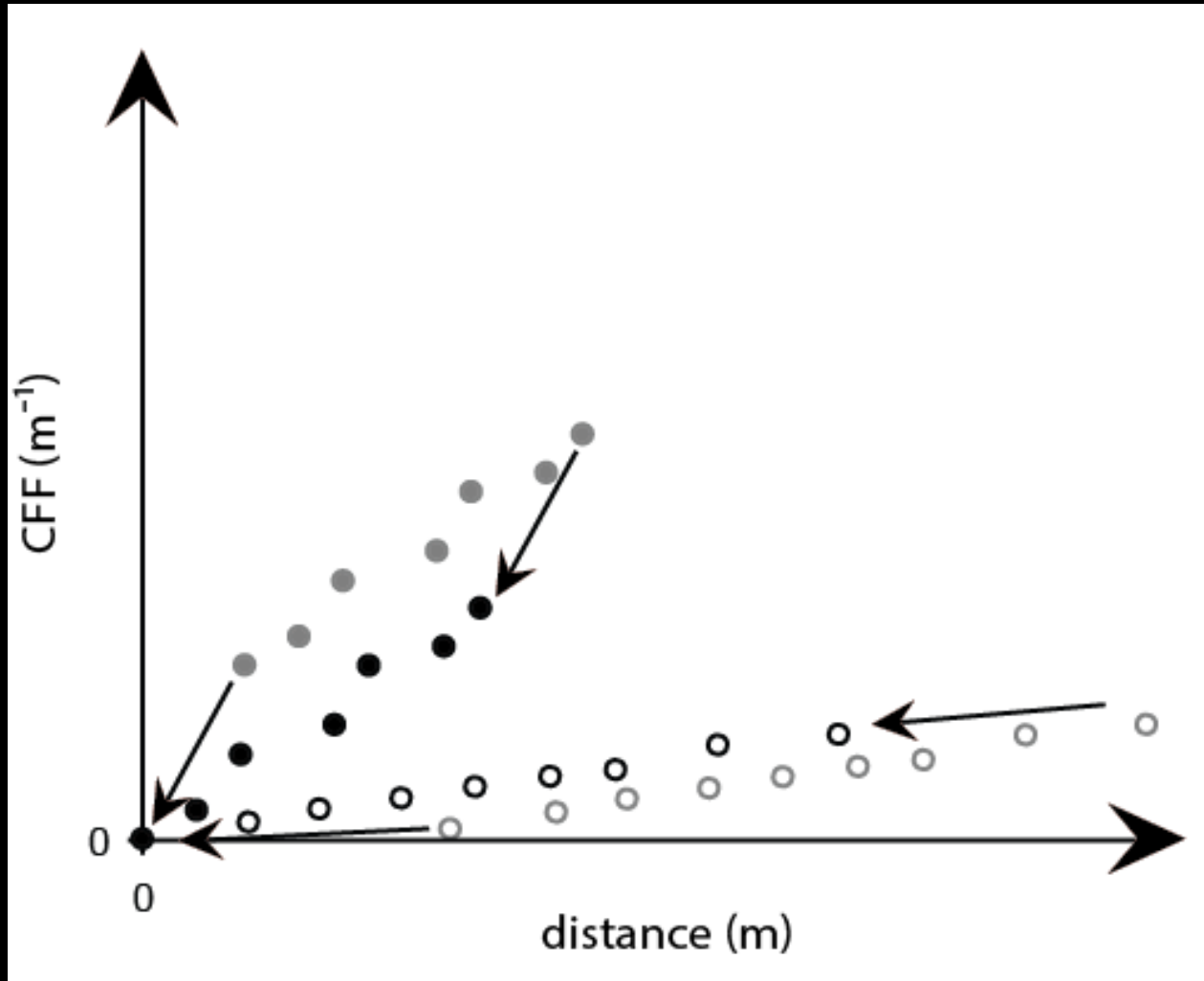




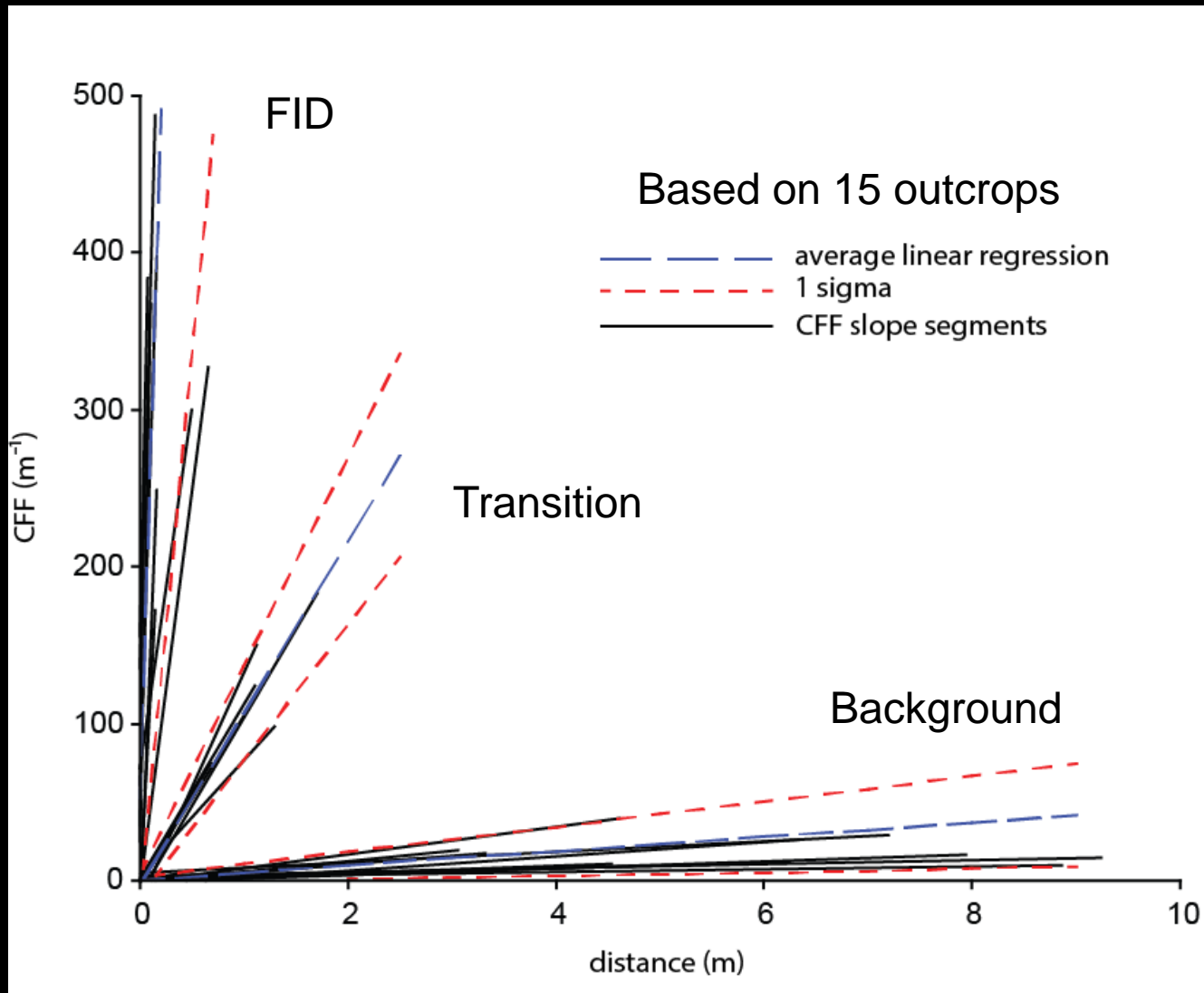
# Cumulative fracture frequency (CFF) segment comparison



# Cumulative fracture frequency (CFF) segment comparison



# Fracture Frequency Intervals

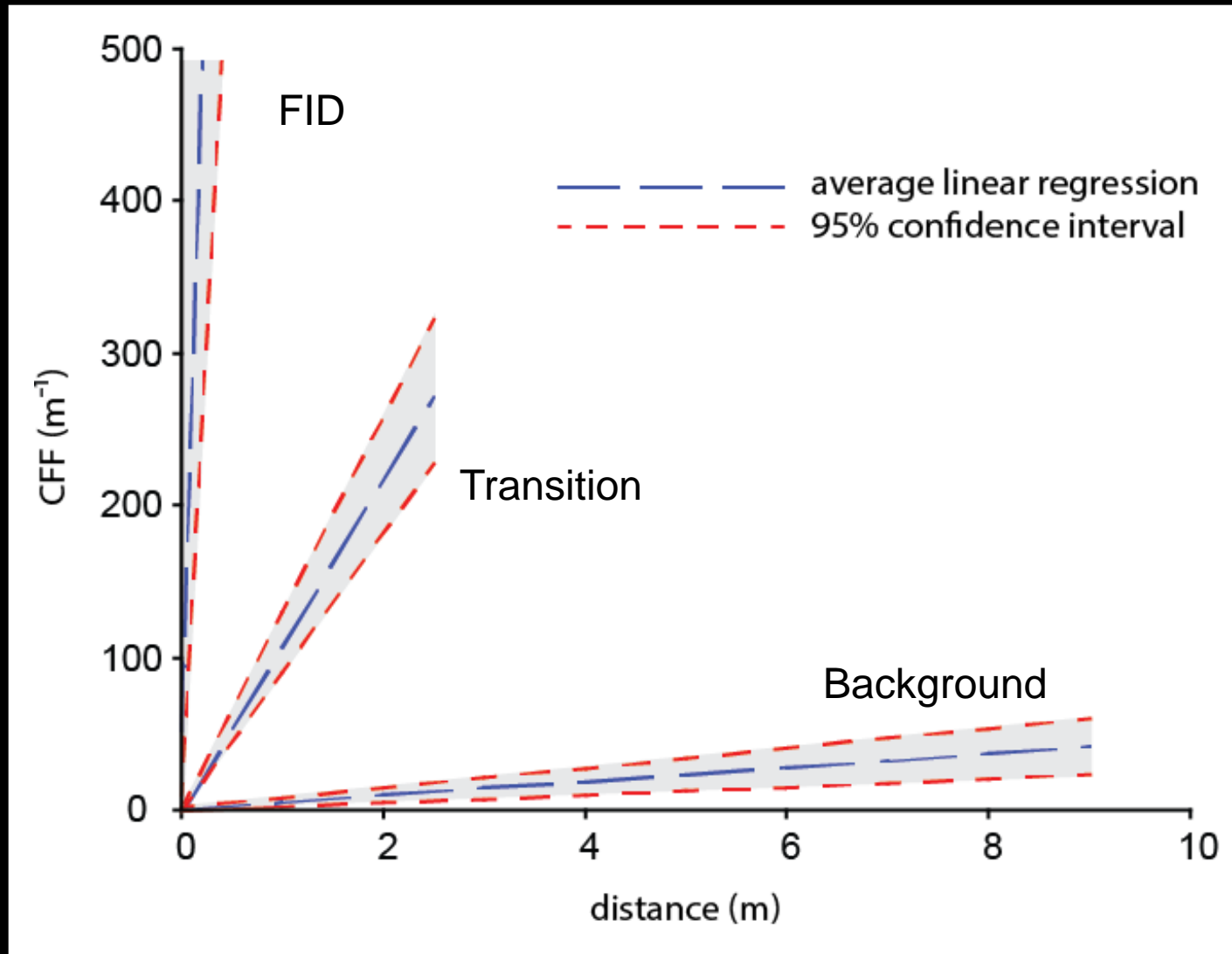


# Fracture Frequency Intervals

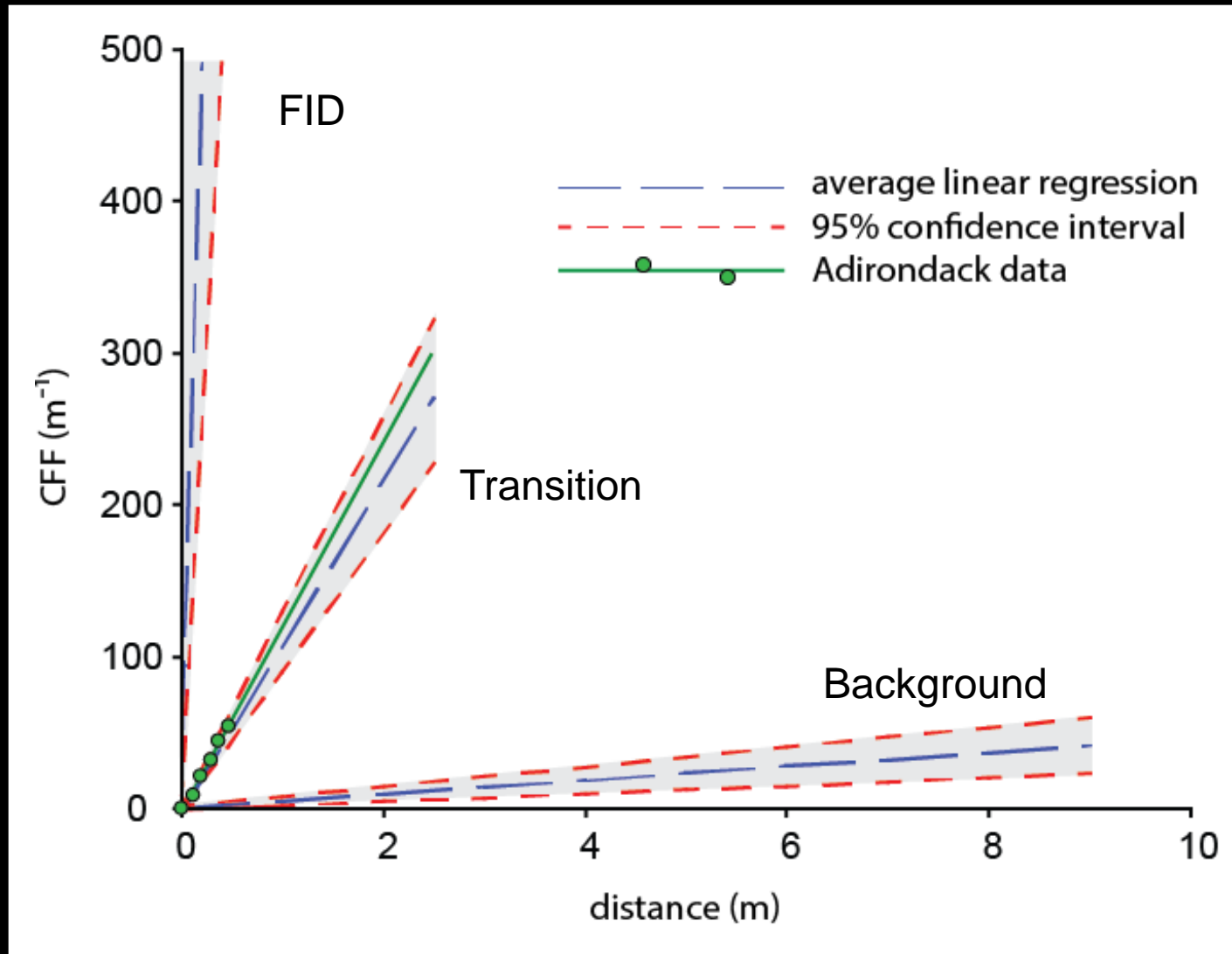
	background slope	background y intercept	transition slope	transition y intercept	FID slope	FID y intercept
	0.9065	1.8498	73.169	3.7616	475.18	71.74
	1.255	3.015	106.23	-1.3595	502.61	-4.3833
	1.759	0.7198	108.52	0.1476	1189.2	-5.0608
	2.1351	-0.5716	111.33	1.2871	1691	-24.602
	2.373	0.2051	112.69	-8.8958	2090.4	61.267
	3.3435	4.4701	146.12	-13.934	2301.1	135.1
	4.4894	-2.876			3014.400	155.25
	5.1641	0.0024			3409.8	24.08
	5.384	-0.1425			5200	33.542
	6.7791	-1.2958				
	8.772	-1.0865				
	12.403	-0.2261				
avg	4.564	0.339	109.677	-3.166	2208.188	49.659
stdev	3.431	1.982	23.177	6.795	1511.846	62.665
avg+stdev	7.995	2.321	132.854	3.629	3720.034	112.324
avg-stdev	1.133	-1.643	86.499	-9.960	696.342	-13.006
std error	0.990	0.572	9.462	2.774	503.949	20.888
ci+	6.505	1.460	128.222	1.460	3195.927	90.601
ci-	2.622	-0.783	91.131	-0.783	1220.448	8.718



# Fracture Frequency Intervals

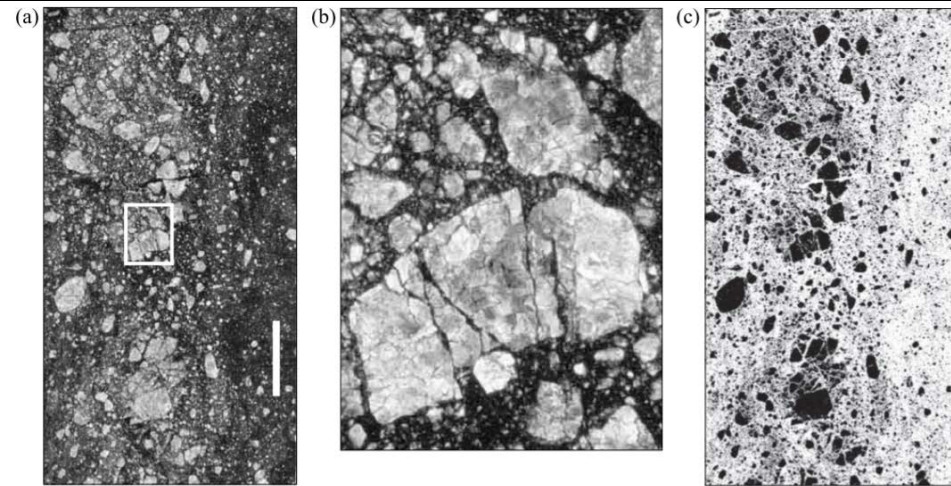


# Fracture Frequency Intervals



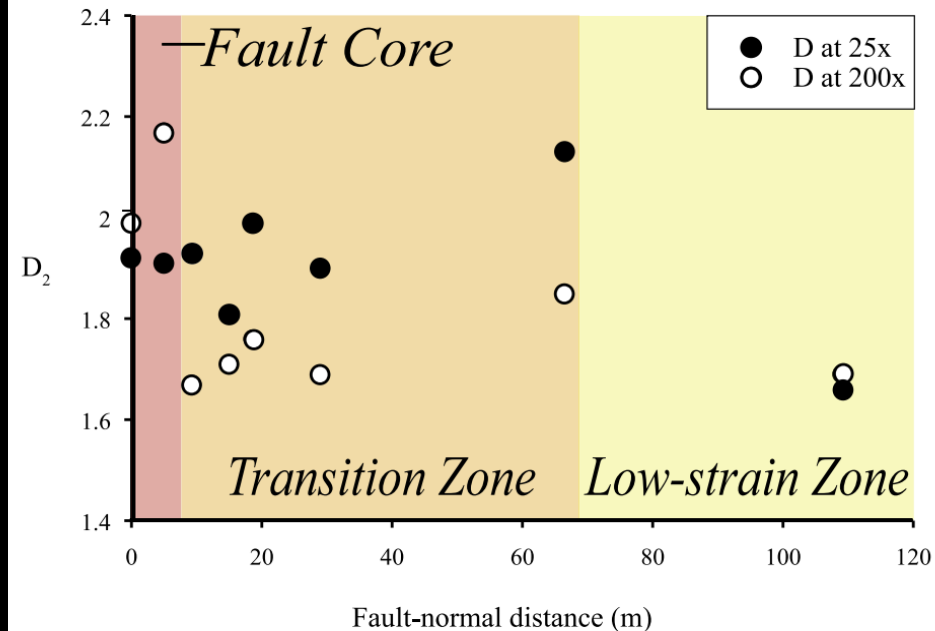
# Strain Localization

## Grain Distribution



**Figure 4.** (a) A thin section of dolomite taken ~5 m into our sampling transect, shown at 25 $\times$  magnification, with a scale bar of 4mm. (b) Magnifying the box highlighted in Figure 4a, we see evidence that grains are composed of distinct smaller particles. This suggests the dolomite may have experienced episodes of fracture and healing, in keeping with predictions from the dynamic model of fault zone evolution. (c) Grains are resolved and analyzed by calculating the optimal gray-scale threshold value via Otsu's [1979] method in Image J. The Otsu [1979] threshold is the value that minimizes the gray-scale variance within the foreground and background.

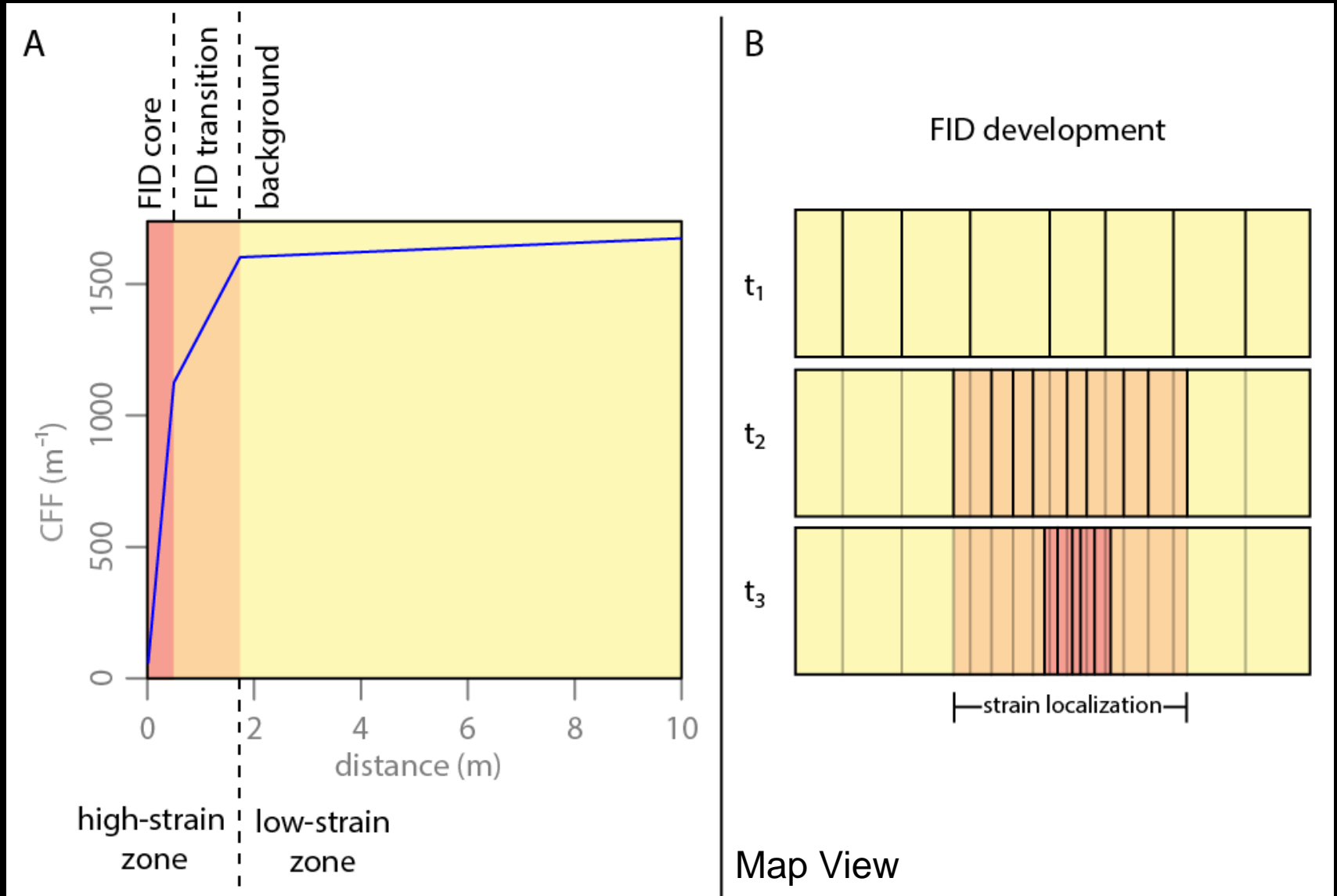
## Fault Damage Zone Boundaries



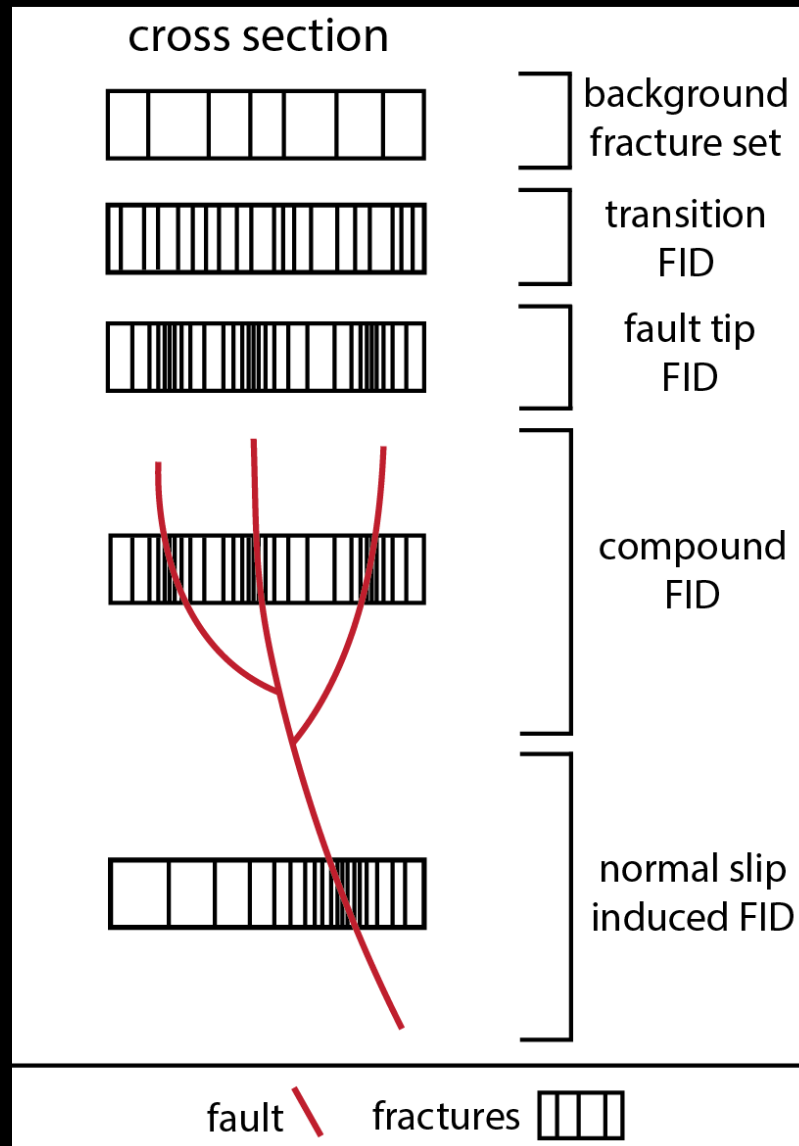
Grain Size Distribution ( $D_2$ )  
 $D_2$  = grain size/grain density  
 $D_2 \sim 2$  (fault core)

Frost et al., 2009

# Strain Localization



# Fracture variations on a fault





# Conclusions

- Research produced a methodology for reliably defining fracture/damage zone boundaries using linear piecewise regressions and AIC
- Background, transition and FID fracture sets produce unique CFF slope responses
- Predictable fracture frequency distributions among varying geologic settings
- Strain localization controls fracture formation during fault initiation