The Three Elements of Structural Geology*

Terry Engelder¹

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Editorial Note:
The course syllabus consists of 764 pages and is 132 MB in size. Downloading is suggested.

Sessions of Short Course

The course syllabus has been divided into the five groups of sessions, given by the author, as follows:

First Sessions 1.1.1--1.2.5
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The Three Elements of Structural Geology

Sponsored by ExxonMobil

Part. 1

Terry Engelder
Professor of Geosciences, The Pennsylvania State University
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Structural geology evolved as a separate subject in the geological sciences during the latter part of the 19th Century. Origins involved a cross fertilization between the great European masters including the Swiss Alpine geologist Albrecht Heim and their American counterparts. Bailey Willis wrote in his 1923 text, *Geological Structures*, that Heim, “laid the corner-stone of our work in *Das Mechanismus der Gebirgsbildung*”. In North America, the great master was G.K. Gilbert who in 1888 first suggested the study of the mechanical principles underlying structural geology. In 1898, Charles R. Van Hise wrote one of the first compendiums on geological structures, *Principles of North American Pre-Cambrian Geology*, as part of the 16th annual report of the U.S. Geological Survey. In 1913, Charles K Leith published a textbook on *Structural Geology* in which a fault-bend fold is described if not named. 21% of the pages in Willis’s text was devoted to the mechanics of rock deformation. For these early giants of structural geology, mechanics was to key. Stress causes rock strain and strain gave us the rich variety of structures that are the focus of any course on the subject. This short course takes the same path worn in by the early geologist who were guided first by stress, then strain, and then geometric forms that were the result.

Terry Engelder, Boalsburg, PA, June 14, 2016

*Note: Most of the following photos, diagrams, and text are mine. Italics will identify text that is lifted verbatim from other sources. For the most part, figures and photos are also attributed to their source unless unidentified when available on the web.*
Ductile Deformation (l): Crinoid Columnal with Stress Concentration
Brittle Deformation (r): Plumose Morphology indication Cyclic Fracture Propagation

1.1.1 – Prologue

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
On a field trip to the Jura Mountains in 1987 led by the great Swiss structural geologist, Hans Laubscher, he described the Jura as “his garden”. At the time he was considered the ‘expert’ on Jura structural geology. It took a career of 40+ years to accumulate the knowledge that enabled Prof. Laubscher to lead such a great field trip. I like to think of the Appalachian Plateau as my equivalent to Laubscher’s garden. I first presented my ideas, however wildly flawed, about the structural geology of the Appalachian Plateau in a poster back in 1959. We have learned a tremendous amount about structural geology during the following half century.

This short course is shaped by the lessons coming from the Appalachian Mountains (Rodgers, 1970). The objective of this course is to expand that which we have learned while doing field work in these mountains and place these lessons in the context of a global understanding of deformation in the upper crust, bearing in mind that the field of structural geology is so broad that this short course will not be able to touch all the subjects in the discipline.

May 30, 2016
Terry Engelder, State College, PA
For some, choosing the Appalachian Mountains as the template for a short course in structural geology may seem parochial except that these mountains were second only to the Alps as the 19th century cradle for structural geology. Furthermore, the Appalachians gave us the first gas (1829) and oil (the Drake Well – 1859) wells in North America. They were the source of wealth for the company that eventually evolved into ExxonMobil (Standard Oil). Finally the Appalachians is host of the world’s largest unconventional gas field in the Middle Devonian Marcellus gas shale and arguably they gave us the word, *fracking*. 

Terry Engelder
Structural geology offers great opportunities for field work. Such work always starts with identifying the local stratigraphy for the purpose of correlation. Mapping folds and faults is not possible without a thorough understanding where in the stratigraphic section a certain rock unit sits. The stratigraphy of the Appalachian Mountains is largely Paleozoic and reflects three tectonic cycles as marked by clastic outwash from mountain building to the east. Each clastic flux rests on carbonates (vertical white arrows) accumulated during periods of relative quiescence on a passive margin. The three tectonic events (i.e., orogenic cycles) include the Taconic (blue arrow), the Acadian (green arrow) and the Alleghanian (red arrow) (Hatcher et al., 1989).
The deformation of the Alleghanian Orogeny (ca. Permian) is illustrated in this cross section from Bradford County, Pennsylvania, USA. The Appalachian Plateau was folded into broad, open anticlines during slip on a Silurian salt as indicated by salt withdrawal from the decollement which was a thick (> 100 m) bed of salt rather than a single slip surface common of most detachment surfaces (Davis and Engelder, 1985; Scanlin and Engelder, 2003). These are the structures that serve as a template for this short course.
Two structures (Engelder favorites) from the Appalachian Plateau illustrating ductile (l) and brittle (r) deformation.

Marcellus gas field

Crinoid columnal in thin section

Plumose joint morphology

Seismic section of previous slide

AAPG Highway Map circa 1965

Geological map of the Appalachian Mountains

Valley and Ridge

Terry Engelder
In the deformed crinoid columnals of Devonian Rocks of the Appalachian Plateau the exterior edge is more elliptical than the internal structures of the columnal (Engelder, 1982). This means that part of the external edge has been removed by a process called pressure solution. A critical ingredient for pressure solution is that the rock contain water in its pores and cracks between grains. During pressure solution part of the rock is dissolved into the water within the cracks and pores. While in solution the components of the dissolved rock may diffuse away from the point where they entered solution. Evidently, pressure from the silt matrix surrounding the columnal enhances the solubility of the calcite in contact with the matrix. Enhanced solubility drives calcite into solution at an unusually high rate.

Part of the insides of some of these crinoid columnals have been removed by dissolving into the water that filled microscopic pores in the crinoid's axial canal. This process of internal dissolution is different from the pressure solution on the external rim because the former is enhanced not by the normal pressure from the silt grains within the axial canal, but by an energy associated with internal deformation of the calcite crystal. Straight bands within the calcite crystal mark planes of atoms that have shifted to allow the calcite to deform internally. These deformation bands are known as calcite twin lamellae ("twins"). They disturb the atomic structure of the calcite so that the calcite crystal near the lamellae contains an abnormally high amount of strain energy. It is the strain energy associated with the twin lamellae that increases the rate of dissolution within the axial canal.

Engelder, 1982
The plumose morphology of the surface of a vertical joint contained in a siltstone bed that is isolated in shale at Watkins Glen, New York, USA. The fracture propagated in discrete increments which are numbered in order of development as determined by morphological criteria. Arrest lines as shown mark the position of the fracture front at the termination of each fracturing cycle. (Lacazette and Engelder, 1992)
I have been in science journalism for more than 30 years and I have never seen more scientific disinformation on any topic as fracking. I am amazed at the level of both inadvertent and purposeful disinformation.
FEAR OF FRACKING

A key technique in shale drilling is hydraulic fracturing, aka fracking. A fluid mix of water, sand, and chemicals is pumped down the well at high pressure, creating fissures in the shale that let gas flow into the well. But the whole drilling process may also create pathways that allow gas or chemicals to pollute drinking water.

Leaky ponds
Contaminated wastewater from fracking is often stored in surface ponds, which can overflow or leak, polluting streams or groundwater.

Faulty wells
Wells are reinforced with steel casing and sealed with concrete. But poor cementing can leave gaps that allow methane or fracking chemicals to contaminate drinking-water aquifers.

Fissures
Fracking fissures might connect to natural ones, allowing pollutants to migrate. Whether they'd climb thousands of feet to shallow aquifers isn't clear.
The fracking debate: Terry Engelder at TEDxPSU

https://www.youtube.com/watch?v=BBSVLGf7zPI
Toward A Theory of Impact Crises

Walter Alvarez
Department of Geology and Geophysics.
University of California,
Berkeley, Calif.

Editor's Note: This article is based on a lecture that was given at the Lamont-Doherty Geological Observatory (LDGO), Palisades, N.Y., on September 6, 1985; it also includes new material through early 1986. This is one perspective on a controversial topic in geophysics. Another perspective was presented in the August 19, 1986 Eos.

The Terminal Cretaceous Extinction Event

In the summer of 1977, Terry Engelder (LDGO) and I carefully collected some samples of the boundary beds at Gubbio. When I joined the faculty at the University of California, Berkeley, that fall, I started discussing

Robert Frost

At the K/T boundary

Terry Engelder - 1977
In 1977 I worked on the geology of Italy with a geologist named Walter Alvarez. At the time, Walter was puzzled by dinosaurs and why they disappeared!
Alvarez's explanation for this worldwide ash layer

- A large bolide hit the earth
- A comet??
- A meteor??
- An asteroid??

DID COMETS KILL THE DINOSAURS
A New Theory About Mass Extinctions

Doomsday Science
New Theories About Comets, Asteroids and How the World Might End
1.1.2 - The Three Elements of Structural Geology

An AAPG Short Course by
Terry Engelder
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Structural geology boils down to a study of Newton's famous laws of motion as they pertain to the deformation of rocks within the earth. The very presence of faults and folds in the crust suggests that rocks, seemingly at rest, were once subject to forces that changed their original state by motion of one point relative to another. Structural geology is the study of the deformation of rocks. In its simplest form this is a description of present geometries. A study of the motion causing the geometries within rocks is called kinematics. A study of the forces that cause the motion is called dynamics. The mathematics of structural geology are designed to simplify the study of kinematics and dynamics.
The Three Elements:

- Geometry
- Kinematics
- Dynamics

One geologist’s opinion
The premier books presenting each of the three elements of structural geology!
**Geometric analysis** is the descriptive or qualitative portion of structural geology. This element of structural geology is as the name implies: A study of the size, shape, and orientation of structures. This element was covered in classical structural geology courses. However, in this set of lecture notes the study of geometry will be delayed until a good mathematical base is established. In the meantime, many of the lab exercises will be devoted to geometric analysis. One of the most useful tools in geometric analysis is the stereonet which is a qualitative tool that serves the same purpose as vectors within a coordinate system.
The Three Elements of Structural Geology

Worthy books dealing with the geometry of earth structures!

- Geometry
Kinematic analysis requires a mathematical base for a rigorous treatment. Kinematics, as you learned when taking elementary physics, is a mathematical description of the motion of objects. In the case of structural geology, kinematics is the description of the path that rocks took during deformation. It is also the mathematical description of the relative position of two infinitesimal points during the deformation of rocks. Two points can change by translating together, rotating around each other, or changing in distance relative to one another. We shall call such a mathematical description deformation mapping.
The Three Elements of Structural Geology

Worthy books dealing with the kinematics of earth structures!

- Kinematics

- Structural Geology Algorithms
- Vectors and Tensors

- 3-D Structural Geology

- Structural Geology
  The Mechanics of Deforming Metamorphic Rocks
  Volume I: Principles
Dynamics is the study of the forces which caused the deformations studied during kinematic analysis. In the case of structural geology, dynamics includes the study of how rocks react to stress. For every stress the rocks respond with a finite strain. In a sense rock structures would not have formed, if rocks had not been subject to a stress. A study of dynamics starts first in this short course.
The Three Elements of Structural Geology

Worthy books dealing with the dynamics of earth structures!

- Dynamics

1. *Fundamentals of Rock Mechanics*
2. *Reservoir Geomechanics*
3. *Stress Regimes in the Lithosphere*
Structural geology also divides into disciplines that may include field observations (outcrop and well bore), laboratory experiments (including in situ measurements), petrofabric analyses (microscopes and geochemical), analytical modeling, numerical modeling, and a liberal integration of physical theory. An understanding of structural geology includes the incorporation of lessons from each of these disciplines. While geometric descriptions may arise from raw field observations and some kinematic analysis possible from these field observations, dynamic analysis cannot proceed without experiments. This is largely because stress can’t be seen in the field and, in fact, is not measured directly in the laboratory. But, careful laboratory experiments allow inferences about rock stress. One of the most informative derivatives of laboratory experiments was this schematic representation of the spectrum from brittle fracture to ductile flow (Griggs and Handin, 1960).

Here, $\sigma_1$, $\sigma_2$, and $\sigma_3$ are the maximum, intermediate, and minimum principal stresses, respectively, with compression given a positive sign.
Geological structures range across the spectrum from brittle fracture to ductile flow. The crust of the earth may also be divided in the same fashion with the **schizosphere** being that part of the outer crust where confining stress governs crustal behavior (Scholz, 2002). Below is the **plastosphere** where temperature governs behavior virtually independent of confining stress. Throughout this range of behavior, but particularly in the schizosphere, crack propagation has a universal presence. Crack propagation is governed by the elasticity properties of the rock.

A laboratory experiment in plexiglass where a ductile shear zone is about to develop with en echelon cracks guiding the evolution of this shear zone. In cross section the crack opens most in the center and narrows to a sharp tip. In plan view, the crack grows as a circular, ‘penny’, hence the name, penny-shaped crack.
When progressing through the three elements of structural geology (geometry, kinematics, dynamics) the concepts become increasing abstract. This reaches a culmination with stress which is a non-intuitive subject. However, part of the difficulty with the abstract is that the twelve different books featured above use different descriptors (i.e., variables) for the same concepts. The components of stress are particularly confusing because of the different way that the variables are represented. This is a reference table to aid in placing this short course in context with six of the premier books on structural geology.

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<td>Sign for Compressive Stress</td>
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1.1.3 - The Mathematics of Structural Geology

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
**Scalar** - A physical quantity expressed by a single number (i.e., temperature); a tensor of zero rank. Gradient of a Scalar - The direction at which a quantity increases the fastest.

\[ \text{grad } \phi = \nabla \phi \]

**Vector** - A physical quantity expressed by a three numbers; a tensor of rank one. Coordinate system for a vector: RIGHT HANDED - looking up the $x_3$ axis from the origin in positive direction, the $x_1$ axis will rotate into the $x_2$ axis in 90° in a clockwise direction. The components of a vector, $\mathbf{F} = F$, are:

Within this text a vector is represented by variables printed in bold letters (e.g. $a$). A knowledge of direction implies that a coordinate system is known. For structural geology a rectangular or Cartesian coordinate system represented by the direction of three positive unit vectors $i, j, k$ will do (e.g. $i = 1$, etc). Any vector can then be represented with scalar components $f_1, f_2, f_3$ which represent distances along the axes denoted by the three unit vectors.

\[ \mathbf{f} = f_1 \mathbf{i} + f_2 \mathbf{j} + f_3 \mathbf{k} \]

Magnitude of a vector is

\[ |\mathbf{F}| = \sqrt{f_1^2 + f_2^2 + f_3^2} \]

The dot product of two vectors is the projection of vector $\mathbf{A}$ on vector $\mathbf{B}$: \[ \mathbf{A} \cdot \mathbf{B} = AB \cos \theta \]

where $q$ is the angle between the two vectors. An example of the dot product is the work done by a force. Consider a rock on which a constant force $\mathbf{f}$ acts. Let the rock be given a displacement $\mathbf{d}$. Then the work $W$ done during the displacement of the rock is defined as the product of $|\mathbf{d}|$ and the component of $|\mathbf{f}|$ in the direction of $\mathbf{d}$, that is,

\[ W = |\mathbf{f}| |\mathbf{d}| \cos \alpha = \mathbf{f} \cdot \mathbf{d} \]
Vector Analysis (Isotropy vs. anisotropy)

**Scalar product or Dot product** of a vector - magnitude of one vector times the magnitude of projection of 2nd vector on the first vector. This number of a scalar.

\[ \mathbf{F} \cdot \mathbf{G} = |\mathbf{F}| |\mathbf{G}| \cos \theta \]

\[ \mathbf{F} \cdot \mathbf{G} = f_1 g_1 + f_2 g_2 + f_3 g_3 \]

The **cross product** of two vectors is the magnitude of A times B times the sine of the angle between then. The direction (\(\mathbf{u}\)) is perpendicular to the plane of \(\mathbf{A}\) and \(\mathbf{B}\)

\[ \mathbf{A} \times \mathbf{B} = AB \sin \theta \mathbf{u} \]

where \(\theta\) is the angle between \(\mathbf{A}\) and \(\mathbf{B}\). According to plate tectonics theory, as continental plates move over the mantle a **moment of force** is generated. The moment of force is also known as a torque \(\tau\) which is a vector quantity.

**Einstein Summation** using Heat flow, \(q\) (cal cm\(^{-2}\) sec\(^{-1}\)), in homogeneous, isotropic rock is indicated by a thermal gradient

\[ q = -\kappa \frac{\partial T}{\partial x_j} \]

where \(\kappa\) is the thermal conductivity.

Heat flow is parallel to a thermal gradient so it can be expresses in a vector form where for an isotropic homogeneous rock.

If the rock is **anisotropic** then the thermal conductivity is not the same in each of the three directions. Then

\[ q_1 = \kappa_{11} a_1 + \kappa_{12} a_2 + \kappa_{13} a_3 \]

\[ q_2 = \kappa_{21} a_1 + \kappa_{22} a_2 + \kappa_{23} a_3 \]

\[ q_3 = \kappa_{31} a_1 + \kappa_{32} a_2 + \kappa_{33} a_3 \]
If we consider heat flow through a cube, then the first subscript relates to the direction of the heat flow and the second subscript relates to the plane on which the heat flow operates. Remember that the ‘1″ plane is defined by the 2- and 3- axes of the coordinate system defining the planes. If the temperature gradient is parallel to the $x_1$ axis, then

$$\frac{\partial T}{\partial x_j} = \left[ \frac{\partial T}{\partial x_1}, 0, 0 \right]$$

but heat flow still occurs in three directions according to

Because of anisotropy of the rock, there is heat flow in all three directions of the rock even though the gradient was applied parallel to the $x_1$ axis.

$\kappa_{ij}$ is a tensor of second rank represented by the matrix

For a temperature gradient which is not parallel to a coordinate axis in an anisotropic material

$$q_i = \sum_{j=1}^{3} \kappa_{ij} \left[ \frac{\partial T}{\partial x_j} \right]$$

for $i = 1, 3$

The expression for heat flow in an anisotropic material may be written in its full form as
Einstein Summation and Tensors

Partial differential equations are time consuming to write so a convention called Einstein summation is used. To illustrate Einstein summation lets consider a property of a rock that relates two vectors. A position within a rock $\mathbf{x}$ is related to a displacement $\mathbf{u}$ by displacement gradients $\mathbf{E}$, which is a second-rank tensor. Position and displacement are both vectors with components $x_1, x_2, x_3$ and $u_1, u_2, u_3$, respectively. $\mathbf{x}$ and $\mathbf{u}$ are related in the following way:

$$
\begin{align*}
    u_1 &= E_{11}x_1 + E_{12}x_2 + E_{13}x_3 \\
    u_2 &= E_{21}x_1 + E_{22}x_2 + E_{23}x_3 \\
    u_3 &= E_{31}x_1 + E_{32}x_2 + E_{33}x_3
\end{align*}
$$

Rather than writing out all of these equations we can shorten then in the following manner:

$$
\begin{align*}
    u_1 &= \sum_{j=1}^{3} E_{1j}x_j \\
    u_2 &= \sum_{j=1}^{3} E_{2j}x_j \\
    u_3 &= \sum_{j=1}^{3} E_{3j}x_j
\end{align*}
$$

or more compactly as:

$$
    u_i = \sum_{j=1}^{3} E_{ij}x_j
$$

We now leave out the summation sign:

$$
    u_i = E_{ij}x_j
$$

and introduce the Einstein summation convention: When a letter suffix occurs twice in the same term, summation with respect to that suffix is to be automatically understood. In the above example the suffixes can be any letters. $i$ is the free suffix and $j$ is the dummy suffix. By this we mean that $j$ can take any variable.

$$
    u_i = E_{ij}x_j = E_{ik}x_k.
$$

In addition to those referenced in the introduction (1.1.2), books by Nye (1985), Means (1976) and Oertel (1996) are excellent sources for understanding tensor analysis in the study of structural geology.
Einstein summation convention -- Here the Einstein summation convention applies and the components $\sigma_{ij}$ relate two vectors $f_i$ and $l_j$ in a linear manner.}

The components $\sigma_{ij}$ form a 3 x 3 matrix:

$$
\begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix}
$$

In the study of vector analysis we have introduced three sorts of quantities: the scalar; the vector; and then a nine component quantity which relates two three-component vectors. These three quantities are all called tensors where:

- **Tensor of zero rank** = Scalar -- This is a single number unrelated to any axes of reference.
- **Tensor of first rank** = Vector -- This is specified by three numbers or components, each of which is associated with one of the axes of reference.
- **Tensor of second rank** = (e.g. stress) -- This is specified by nine numbers, or components, each of which is associated with a pair of axes (taken in a particular order).

Tensor transformation -- The physical quantity, such as thermal conductivity, $k_{ij}$, is the same regardless of the set of axis chosen. This is the chief characteristic of a tensor. Transformation from one coordinate system to another depends on the direction cosine relating the axes of each of the coordinate systems. $a_{ij}$ are the components of the transformation matrix where $a_{12}$ is the cosine of the angle between $x_1$ of the old coordinate system and $x_2'$ of the new coordinate system. The complete transformation matrix is

$$
\begin{bmatrix}
x_1' & x_2' & x_3' \\
x_1 & a_{11} & a_{12} & a_{13} \\
x_2 & a_{21} & a_{22} & a_{23} \\
x_3 & a_{31} & a_{32} & a_{33}
\end{bmatrix}
$$

The first subscript in the a's refers to the 'new' axes and the second to the 'old'. There are only three independent direction cosines of the nine given in the transformation matrix. The coordinates of a point $(x_1, x_2, x_3)$ can also be transformed to the coordinates $(x_1', x_2', x_3')$ using direction cosines in the same manner.
Likewise the equations for the transformation of a vector from one coordinate system to another resemble the equations for transforming a point.

\[ p'_i = a_{ij} p_j \]

where \( p'_i \) are the components of the vector in the new coordinate system and \( p_j \) are the components of the vector in the old coordinate system. Here the free suffix is \( i \) and the dummy suffix, \( j \), is together in the right-hand term. Written in its full form the equation becomes

\[
\begin{align*}
p'_1 &= a_{11} p_1 + a_{12} p_2 + a_{13} p_3 \\
p'_2 &= a_{21} p_1 + a_{22} p_2 + a_{23} p_3 \\
p'_3 &= a_{31} p_1 + a_{32} p_2 + a_{33} p_3
\end{align*}
\]

We can reverse the transformation process to find the old vector in terms of the new vector. Here the dummy suffix, \( j \), is separated by the free suffix, \( i \).

\[ p_i = a_{ji} p'_j \]
Relations between direction cosines - Since each row of the array (3-1) represents three direction cosines of a straight line with respect to a coordinate system, \( x_1, x_2, x_3 \), the following equations hold true:

\[
\begin{align*}
    a_{11}^2 + a_{12}^2 + a_{13}^2 &= 1 \\
    a_{21}^2 + a_{22}^2 + a_{23}^2 &= 1 \\
    a_{31}^2 + a_{32}^2 + a_{33}^2 &= 1
\end{align*}
\]

We can combine the orthogonality relations into one equation where the symbol, \( \delta_{ij} \), is called the Kronecker delta:

\[
a_{ik} a_{jk} = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}
\]

Second rank tensor -- In general, if a property, \( T \), relates two vectors \( P = [p_1, p_2, p_3] \) and \( Q = [q_1, q_2, q_3] \) in such a way that

\[
\begin{align*}
    p_1 &= T_{11} q_1 + T_{12} q_2 + T_{13} q_3 \\
    p_2 &= T_{21} q_1 + T_{22} q_2 + T_{23} q_3 \\
    p_3 &= T_{31} q_1 + T_{32} q_2 + T_{33} q_3
\end{align*}
\]

where the components, \( T_{11}, T_{12}, \ldots \) are constants, then \( T_{11}, T_{12}, \ldots \) are said to form a second-rank tensor:

\[
\begin{bmatrix}
    T_{11} & T_{12} & T_{13} \\
    T_{21} & T_{22} & T_{23} \\
    T_{31} & T_{32} & T_{33}
\end{bmatrix}
\]
The values of the coefficients $T_{11}$, $T_{12}$, .....depend on the orientation of the coordinate axes $x_1, x_2, x_3$. Now suppose we choose a new set of coordinate axes $x_1', x_2', x_3'$ related to the old axes by the direction cosines, $a_{ij}$. If so, the vectors P and Q have new components $p_i'$ and $q_i'$. Next we find $P'$ in terms of $Q'$ the series of equations shown below (The arrow means "in terms of").

\[ P' \rightarrow P \rightarrow Q \rightarrow Q' \]

The following equations permit the transformation

\[
p'_{i} = a_{ik}p_{k}
\]

\[
p_{k} = T_{kl}q_{l}
\]

\[
q_{l} = a_{jl}q'_{j}
\]

\[
p'_{i} = a_{ik}T_{kl}a_{jl}q'_{j}
\]

\[
p'_{i} = T'_{ij}q'_{j}
\]

So

\[
T'_{ij} = a_{ik}a_{jl}T_{kl}
\]

This is the Transformation Law for a tensor of second rank. Written out in full form each of its nine equations are rather long.
At first we were taught that pressure is defined as force per unit area. Stress has the same definition. The distinction between pressure and stress in structural geology is based on the nature of the material on which the force is acting. The distinction is made depending on whether the material in question has a shear strength. Materials such as rock are said to have a shear strength because they maintain their shape when placed unsupported on a table. If we are dealing with a rock that has a shear strength (fluids such as water and gases do not), then we say that it exerts a stress on its surroundings. Materials which have a shear strength can exert different stresses in different directions. In contrast, water, without a shear strength would proceed to run over the table top seeking the lowest spot. If we wish to describe the force per unit area that a liquid or gas is exerting on its container, we use the term pressure. Water in the pores in rocks exerts a pressure on the grains surrounding the pore.

The most useful equations for teaching the concept of stress are the equations for normal ($\sigma_n$) and shear ($\tau$) stress in terms of principal stresses ($\sigma_1$, $\sigma_3$). The derivation of these equations is based on a force-balance problem which assumes that a body subject to forces is in equilibrium which means that all forces in any direction add to zero.

Consider the forces ($\mathbf{N}$ and $\mathbf{S}$) acting parallel to the front face of a triangular solid. These forces are acting in a plane with the forces on the bottom and left faces so that this force-balance problem is two dimensional. If the area of the hypotenuse face is unity (i.e. 1), then the area of the bottom face is unity times $\sin \theta$ whereas the area of the left face is unity times $\cos \theta$. Forces acting on the three faces are stresses multiplied by the area of the face [$\text{Force} = (\text{force/area}) \times (\text{area})$]. Summing the forces in the horizontal and vertical directions (the forces add to zero in both directions), we obtain:

$$\sum F_h = 0 = F_x - S \sin \theta - N \cos \theta$$

$$\sum F_v = 0 = F_y - N \sin \theta + S \cos \theta$$
The Force-Balance Equilibrium Wedge

Rewriting the force terms using components of stress, we obtain the normal and shear stresses acting on a plane whose normal is at an angle of $\theta$ to $\sigma_1$, the maximum compressive principal stress.

\[
\sigma_1 \cos \theta - \sigma_n \cos \theta - \tau \sin \theta = 0
\]
\[
\sigma_3 \sin \theta - \sigma_n \sin \theta + \tau \cos \theta = 0
\]

In many geological applications the plane of interest is a fault plane. Now we can solve these two equations for $\sigma_n$ and $\tau$

\[
\sigma_n = \sigma_3 \cos^2 \theta + \sigma_1 \sin^2 \theta
\]
\[
\tau = (\sigma_1 - \sigma_3) \cos \theta \sin \theta
\]

Using trigonometry

\[
\sin 2\theta = 2 \sin \theta \cos \theta \quad \cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta) \quad \text{and} \quad \sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)
\]

the famous equations for the Mohr’s Circle are derived

\[
\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta
\]
\[
\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta.
\]

Note that in these equations $\theta$ is the angle between $\sigma_1$ and the normal to the fault plane. If $\theta$ is the angle between $\sigma_1$ and the fault plane the sign convention for normal stress becomes

\[
\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta
\]

Note also that equilibrium does not require equal stress on the back and bottom sides. This leads to a differential stress ($\sigma_d = \sigma_1 - \sigma_3$) within the solid as a consequence of one surface traction which is divided into components of a normal and shear traction.
The general force-balance problem is shown below where the back and side faces of the unit triangle are subject to both normal and shear stresses. In other words, the coordinate system x-y is not parallel to the principal stress directions. The general case is given to illustrate a property of stress called invariance with respect to coordinate system. In two dimensions we define stress as a force per unit of line length, in contrast to the three dimensional situation where stress is a force per unit area. Consider a coordinate system $O_{xy}$ with an arbitrary line AB cutting the x and y axes such that the normal to the line AB makes an angle $\theta$ with the y-axis. This gives a right triangle $AOB$ with sides $OA$ (parallel to Ox) and $OB$ (parallel to Oy) and a hypotenuse $AB$. Across the line AB a stress vector $p$ can be applied making an angle $\theta$ with the x-axis. Remember that $p = \delta f / \delta A$ when $\delta A \rightarrow 0$, so a stress vector can represent stress at a point. Otherwise stress is defined on a line (2-D) or a surface (3-D). The stress vector $p$ can be resolved into components parallel to the x and y axes: $p = p_x + p_y$. Even though $P$ is called a vector it still has units of stress (force/length in two dimensions). Because the triangle ABO is in equilibrium the sum of the force-vectors on all sides balance. In 2-D stress multiplied by line length will give a force vector. So

$$p_{xAB} = \sigma_x OB + \tau_{yx} OA.$$ 

If length $a = AB$, then $a \times \cos \theta = OB$ and $a \times \sin \theta = OA$. If we divide by length $a$, then

$$p_x = \sigma_x \cos \theta + \tau_{yx} \sin \theta, \quad p_y = \sigma_y \sin \theta + \tau_{xy} \cos \theta.$$ 

Now consider a normal stress $\sigma_n$ and shear stress $\tau$ across AB in terms of the components of the stress vector

$$\sigma_n = p_x \cos \theta + p_y \sin \theta \quad \tau = p_y \cos \theta - p_x \sin \theta.$$ 

Substituting for $p_x$ and $p_y$ and remembering the trig functions from earlier

$$\sigma_n = \sigma_x \cos^2 \theta + 2 \tau_{xy} \sin \theta \cos \theta + \sigma_y \sin^2 \theta$$ 

$$\tau = 1/2(\sigma_y - \sigma_x) \sin 2\theta + \tau_{xy} \cos 2\theta.$$
**Force-Balance Equilibrium**

These are the general equations for a stress system in which the orientation of the principal stresses are unknown. Let us look at the specific case where the principal stresses are parallel to the coordinate axes. Now we consider the effect of rotating the coordinate system to parallel a normal and shear stress. To accomplish this coordinate rotation let $\sigma_n$ and $\tau$ be $\sigma_n'$ and $\tau'_{x'y'}$ where $O_{x'y'}$ is rotated $\theta$ from $O_{xy}$. From a previous equation we have

$$
\sigma'_{x} = \sigma_x \cos^2\theta + 2\tau_{xy} \sin\theta \cos\theta + \sigma_y \sin^2\theta
$$

Now we must find $\sigma_y'$ in the new coordinate system. This is accomplished by replacing $\theta$ by $\theta + 1/2\pi$ and we get

$$
\sigma'_{y} = \sigma_x \sin^2\theta - 2\tau_{xy} \sin\theta \cos\theta + \sigma_y \cos^2\theta
$$

If we add $\sigma_{x}'$ and $\sigma_{y}'$ and remember that $\sin^2\theta + \cos^2\theta = 1$, we get

$$
\sigma_{x}' + \sigma_{y}' = \sigma_x + \sigma_y.
$$

This shows that the sum of the normal stresses is **invariant** or unchanged by rotation of the coordinate system. Likewise

$$
\tau_{x'y'} = 0.5(\sigma_y - \sigma_x) \sin2\theta + \tau_{xy} \cos2\theta.
$$

Principal stresses are found in planes containing no shear stress. If in these planes, $\tau_{x'y''} = 0$, then from the previous equation

$$
\tan2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}
$$

where $\Theta$ is the one angle between the coordinate system $Ox''y''$ and $Oxy$ where the shear stresses vanish along the directions $Ox''$ and $Oy''$. In this coordinate system the only stresses are the normal stresses $\sigma_x''$ and $\sigma_y''$. This coordinate system contains the principal stress axes and the components, $\sigma_{x''}$ and $\sigma_{y''}$, are known as the principal stresses.
Using this same coordinate system where the axes are parallel to the principal stresses $\sigma_1$ and $\sigma_2$, we can look at the stress vectors $p_x$ and $p_y$. They become

$$p_x = \sigma_1 \cos \theta \quad \text{and} \quad p_y = \sigma_2 \sin \theta.$$  

Substituting into the equation $\sin^2 \theta + \cos^2 \theta = 1$, we can generate the equation for an ellipse where the $p_x$ and $p_y$ are on the ellipse called the stress ellipse. The semi axes of the ellipse are $\sigma_1$ and $\sigma_2$.

The Mohr's circle is a graphical method of representing the state of stress of a rock in two dimensions. The equations used for the Mohr's circle representation are those derived above for the coordinate system with axes parallel to the principal stresses. The Mohr's circle may be used to derive the normal $\sigma_n$ and shear $\tau$ stresses on any plane whose normal is oriented at $\theta$ from $\sigma_1$. The coordinate system for the Mohr's circle representation is $\sigma_n$ along the horizontal axes with increasing compression to the right and $\tau$ along the vertical axes. Critical points along the $\sigma_n$-axes are $OP = \sigma_1$, $OQ = \sigma_2$, and $C = 1/2(\sigma_1 + \sigma_2)$. Point C is the average stress. The angle measured as PCA counter clockwise from OP is $2\theta$. Now we have

$$\sigma_n = OB = OC + CB = \frac{1}{2}(\sigma_1 + \sigma_2) + \frac{1}{2}(\sigma_1 - \sigma_2) \cos 2\theta$$  

$$\tau = AB = \frac{1}{2}(\sigma_1 - \sigma_2) \sin 2\theta.$$  

Diameter of the Mohr’s circle = Differential stress ($\sigma_d$)  
Radius of the Mohr’s circle = Maximum shear stress ($\tau_{\max}$)
1.1.5 – Stress in the Earth

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Dynamics = Stress: The heart of structural geology

The mechanical traffic light

- Stress  Strain
- Compression  Shortening
- Shear  Shear
- Tension  Elongation
SURFACE TRACTIONS

Blue = superposition of shear and normal tractions

Normal traction
\[ \vec{t} = \vec{t}_i = [ t_1, t_2, t_3 ] \]

Shear traction
\[ \vec{s} = \vec{s}_i = [ s_1, s_2, s_3 ] \]

Force on surface
\[ \vec{f} = f_i = [ f_1, f_2, f_3 ] \]

The mechanical traffic light

1. Compression
2. Shear
3. Tension
A change in shape (something visible) \rightarrow A change in stress (something invisible)

Reaction in the rock

**SURFACE TRACTIONS**
(reaction within the rock)

coordinate system

\[ \mathbf{n} = n_j = [n_1, n_2, n_3] \]

force on surface

\[ \mathbf{f} = f_i = [f_1, f_2, f_3] \]

surface orientation

stress within the solid

coordinate system defines reference cube within the solid

volume change

Stress within the solid will cause deformation of the solid of two fundamental types later called **pure shear** and **simple shear**.
Stress: That invisible reaction within a body to a surface traction
Acts on three orthogonal planes specified by a coordinate system \((1,2,3)\) or \((x,y,z)\)

- \(\sigma_i\) = principal stresses \((i = 1 \text{ to } 3 \text{ – Einstein’s notation})\)
  - \(\sigma_1 > \sigma_2 > \sigma_3\) \((\text{note: one subscript signifies a principal stress})\)
    - In a solid there are three orthogonal planes on which shear stress does not act. Principal stresses act on planes devoid of shear stresses. Shear stresses act on all other planes in the solid body.

- \(\sigma_{ii}\) = normal stresses but not necessarily principal stresses \((i = 1 \text{ to } 3 \text{ – Einstein’s notation})\)
  - \(\sigma_{11} > \sigma_{22} > \sigma_{33}\) \((\text{note: two like subscripts signify a normal stress})\)

- \(\sigma_n\) = normal stress

- \(\sigma_{ij}\) = shear stresses \((i,j = 1 \text{ to } 3 \text{ – Einstein’s notation})\)
  - \(\sigma_{12} = \sigma_{21}; \sigma_{13} = \sigma_{31}; \sigma_{23} = \sigma_{32}\) \((\text{note: two different subscripts signify a shear stress})\)
    - No orientation implied relative to earth coordinates

- \(\tau_{ij}\) = shear stresses \((i,j = 1 \text{ to } 3 \text{ – Einstein’s notation})\)
  - \(\tau_{12} = \tau_{21}; \tau_{13} = \tau_{31}; \tau_{23} = \tau_{32}\)
  - \(\tau_{ij}\) and \(\sigma_{ij}\) are the same parameters. The symbols represent the same values.

- \(S_v\) = vertical stress
- \(S_{H_{\text{max}}}\) = maximum horizontal stress
- \(S_{h_{\text{min}}}\) = minimum horizontal stress
  - No relative magnitude implied
  - These are generally (but not always) principal stresses.

Stress is specified as a 3 by 3 matrix
\[
\begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix} =
\begin{bmatrix}
\tau_{11} & \tau_{12} & \tau_{13} \\
\tau_{21} & \tau_{22} & \tau_{23} \\
\tau_{31} & \tau_{32} & \tau_{33}
\end{bmatrix} =
\begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3
\end{bmatrix}
\]

These matrices are three versions of the same stress.
Density of Important Earth Materials
(mass per unit volume)

- Quartz: 2.65 gm/cm$^3$
- Calcite: 2.71 gm/cm$^3$
- Salt (NaCl): 2.165 gm/cm$^3$
- Common Clays
  - Chlorite: 2.6 to 3.3 gm/cm$^3$
  - Kaolinite: 2.65 gm/cm$^3$
  - Illite: 2.77 gm/cm$^3$
- Siltstone: 2.6 – 2.63 gm/cm$^3$
- Sandstone: < 2.6 gm/cm$^3$
  - Depends on porosity
- Carbonate: ≈ 2.69 gm/cm$^3$
- Salt (NaCl): 2.165 gm/cm$^3$
- Shale: 2.68 to 2.71 gm/cm$^3$
- Pure Water: 1.0 gm/cm$^3$
- Salt Water: > 1.02 gm/cm$^3$
  - Depends on salinity
- Petroleum: ≈ 0.84 gm/cm$^3$
- Compressed methane: 0.06 gm/cm$^3$

The Earth is a self-gravitating body with its primary stress as consequence of the 'weight' (i.e., force) of overburden. That force is proportional to the (integrated) density of the overburden.
Stress-Depth Diagram

• Overburden Stress ($S_v$)
  \[ S_v = \rho_{rock}gz \]

• Pore pressure ($P_p$)
  \[ P_p = \rho_{water}gz \]

• Rock (solid) → Stress
• Pore water (fluid) → Pressure
• Natural gas (fluid) → Pressure

\( z = \text{depth} \)
\( \rho = \text{density} \)
\( g = \text{gravitational acceleration} \)

Note: In rock deformation experiments, fluids are used to simulate \( \sigma_2 \) and \( \sigma_3 \) and these stresses are misnamed confining pressure (but only in the laboratory).

Fluids can not sustain a stress because they exhibit to shear strength under static (slow) loads, hence, they are assigned a pressure as a proxy for stress.
Andersonian states of stress plotted on a stress-depth diagram (Anderson, 1951)

- Thrust Faulting: \( S_{Hmax} > S_{hmin} > S_v \)
- Strike Slip Faulting: \( S_{Hmax} > S_v > S_{hmin} \)
- Normal Faulting: \( S_v > S_{Hmax} > S_{hmin} \)

Here, the variables for stress (\( \sigma \) and \( S \)) are used interchangeably only because the coordinate axes for laboratory experiments on fault generation have been rotated to their field positions.
<table>
<thead>
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<th>Name</th>
<th>Definition</th>
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<td>A Component of Principal Stress</td>
<td>$\sigma_{ii}$ or $\sigma_i$</td>
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<tr>
<td>Any Component of Stress</td>
<td>$\sigma_{ij}$</td>
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<tr>
<td>Differential Stress</td>
<td>$\sigma_d = \sigma_1 - \sigma_3$</td>
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<td>Maximum Shear Stress</td>
<td>$\tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2}$</td>
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<td>Lithostatic Stress</td>
<td>$S_H = S_h = S_v$</td>
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<td>$P_p = P_p = P_p$</td>
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<td>$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$</td>
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<td>$\sigma_1 - \sigma_m$, $\sigma_2 - \sigma_m$, $\sigma_3 - \sigma_m$</td>
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<tr>
<td>Effective Stress</td>
<td>$\sigma_i - P_p$</td>
</tr>
</tbody>
</table>
Strain:
Visible Reaction to Surface Traction

- Volume Change – caused by **normal stresses** ($-\sigma_n$) or **normal stresses** ($\sigma_n$)
- Distortion – caused by **shear stresses** ($\tau$)
- Mohr Diagram – simple graph illustrates relative tendency for volume change vs. distortion

Note: compression (+) here
Mohr’s Circle

- Unconfined Body
  - One force (normal stress) can generate stresses at right angles in a confined body
  - Normal stresses produce a volume change

- Confined Compression
  - Note: Confined compression = negative $\Delta V$
  - $0 < S_h < S_v$

Uniaxial Strain

Terry Engelder
Initial stress state during a burial cycle

SEABED STRESSES

overburden
weight ($S_v$)

stress = solid
pressure = fluid

fluid

$S_v = P_p$

mud with > 70% porosity: acts as a liquid with no shear strength
Shear Stress

- Unconfined Body
Coordinate System (i.e., interior cube) oriented so that axes (i.e., cube faces) are parallel and normal to principal stresses!

**STRESS (the principal orientation)**

Within the solid there is a coordinate system parallel to the principal stresses.

\[ \sigma = \sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \]

\[ \bar{n} = n_j = [n_1, n_2, n_3] \]

\[ \bar{f} = f_i = [f_1, f_2, f_3] \]
Note: Same surface traction as before (i.e., same magnitude, same orientation on same surface) means stress inside cube is the same as before. Just a different coordinate system.
Rotate Orientation of Reference Body

• Mohr Circle: Graphic means of representing a tensor quantity

\[
\begin{vmatrix}
\sigma_{11} & \sigma_{13} \\
\sigma_{31} & \sigma_{33}
\end{vmatrix}
= 
\begin{vmatrix}
\sigma_{11} & \tau_{13} \\
\tau_{31} & \sigma_{33}
\end{vmatrix}
= 
\begin{vmatrix}
\sigma_1 & 0 \\
0 & \sigma_3
\end{vmatrix}
\]
**STRESS** (a matrix which relates the force on the surface and orientation of the surface)

\[
\bar{n} = n_j = [n_1, n_2, n_3]
\]

\[
\bar{f} = f_i = [f_1, f_2, f_3]
\]

\[
[f_1, f_2, f_3] = \begin{bmatrix}
\sigma_{11}, & \sigma_{12}, & \sigma_{13} \\
\sigma_{21}, & \sigma_{22}, & \sigma_{23} \\
\sigma_{31}, & \sigma_{32}, & \sigma_{33}
\end{bmatrix} n_j
\]

coordinate system
State of stress during burial

Using arbitrary coordinate system

STRESS (normal traction)

$\mathbf{n} = n_j = [n_1, n_2, n_3]$  

Surface orientation

$\mathbf{f} = f_i = [f_1, f_2, f_3]$  

Force on surface

$\mathbf{t} = t_i = [t_1, t_2, t_3]$  

Normal traction

$\sigma = \sigma_{ij} = \begin{bmatrix} \sigma_{11}, \sigma_{12}, \sigma_{13} \\ \sigma_{21}, \sigma_{22}, \sigma_{23} \\ \sigma_{31}, \sigma_{32}, \sigma_{33} \end{bmatrix}$

Stress within the solid
Rotate the coordinate system to parallel the principal stresses

**STRESS (the principal orientation)**

- \( \sigma = \sigma_{ij} = \begin{bmatrix} \sigma_1, & 0, & 0 \\ 0, & \sigma_2, & 0 \\ 0, & 0, & \sigma_3 \end{bmatrix} \)
- \( \bar{n} = n_j = [ n_1, n_2, n_3 ] \)
- \( \bar{f} = f_i = [ f_1, 0, 0 ] \)

State of stress during burial
The Cauchy tetrahedron

Front face of the tetrahedron represents a free surface on which a surface traction (force \( f \) = a vector) acts. The orientation of the surface is given by direction cosines relative to a coordinate system \((x_1, x_2, x_3)\).

Back three faces represent orthogonal planes in the interior of the stressed body.
**Reaction of the solid to a stress vector** -- Now we examine a solid piece of the rock in the shape of a tetrahedron with corners ABCO (Fig. 10-5). Because the rock is in equilibrium no surface of the tetrahedron is moving relative to any other surface. Let the surface defined by ABC be the surface element \( \delta S \) discussed above. The force \( \sigma(f) \) transmitted across \( \delta S \) is \( \sigma(f) \) times (area ABC). The forces on the three faces at right angles may be each denoted by three components \( \sigma_{ij} \) so that we have nine components in all. Each face has two components parallel to the surface and one component normal to the surface. Note that these components are parallel to the three components \( l_1, l_2, \) and \( l_3 \) of the unit vector \( l \). Remember that a stress multiplied by an area is a force.

Resolving forces parallel to Ox we have

\[
f_1 (ABC) = \sigma_{11}(BOC) + \sigma_{12}(AOC) + \sigma_{13}(AOB) \quad (10-2)
\]

Dividing the area of each side (e.g. BOC) by the area of the face (ABC) gives the component of the unit vector \( l \) in the direction normal to the side. Thus:

\[
f_1 = \sigma_{11}l_1 + \sigma_{12}l_2 + \sigma_{13}l_3
\]

\[
f_2 = \sigma_{21}l_1 + \sigma_{22}l_2 + \sigma_{23}l_3
\]

\[
f_3 = \sigma_{31}l_1 + \sigma_{32}l_2 + \sigma_{33}l_3. \quad (10-3)
\]

Hence,

\[
f_i = \sigma_{ij}l_j \quad (10-4)
\]

where both \( f_i \) and \( l_j \) are three components of the force vector \( f \) and the unit directional vector \( l \).
Mohr circle construction - We start with the principal axes of a tensor

\[
\sigma_{ij} = \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3 \\
\end{bmatrix}
\]  

and consider the transformation of the components of stress by a clockwise rotation about the principal axis \(\sigma_3\). For this rotation the direction cosine matrix is the following

\[
a_{ij} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33} \\
\end{bmatrix} = \begin{bmatrix}
cos \theta & -sin \theta & 0 \\
sin \theta & cos \theta & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]  

We now transform \(\sigma_{ij}\) using the direction cosines to obtain

\[
\sigma'_{ij} = \begin{bmatrix}
\sigma'_{11} & \sigma'_{12} & 0 \\
\sigma'_{12} & \sigma'_{22} & 0 \\
0 & 0 & \sigma_3 \\
\end{bmatrix}
\]  

According to the transformation we generate four components. For example,

\[
\sigma'_{11} = a_{1k} a_{1l} \sigma_{kl} = a_{11} a_{11} \sigma_1 + a_{11} a_{12} \sigma_2
\]

where \(\sigma_{11} = \sigma_1\) and \(\sigma_{22} = \sigma_2\) so that

\[
\sigma'_{11} = \sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta \\
\sigma'_{22} = \sigma_1 \sin^2 \theta + \sigma_2 \cos^2 \theta \\
\sigma'_{12} = + \sigma_1 \sin \theta \cos \theta - \sigma_2 \sin \theta \cos \theta
\]  

(10-8)
**Field tensors verses matter tensors** - Stress and strain are examples of field tensors whereas the tensors which measure crystal properties such as magnetic susceptibility are matter tensors. Matter tensors must conform to crystal symmetry whereas stress is not a crystal property but is akin to a force impressed on the crystal. Both field and matter tensors have similar special forms.

<table>
<thead>
<tr>
<th>Field Tensor</th>
<th>Symmetry of Matter Tensor</th>
</tr>
</thead>
</table>
| \[
P_p \quad 0 \quad 0 \\
0 \quad P_p \quad 0 \\
0 \quad 0 \quad P_p
\] | Hydrostatic Stress        |
| \[
\sigma_1 \quad 0 \quad 0 \\
0 \quad \sigma_2 \quad 0 \\
0 \quad 0 \quad \sigma_3
\] | Uniaxial Stress           |
| \[
\sigma_1 \quad 0 \quad 0 \\
0 \quad \sigma_2 \quad 0 \\
0 \quad 0 \quad \sigma_3
\] | Biaxial Stress            |
| \[
\sigma_1 \quad 0 \quad 0 \\
0 \quad -\sigma_2 \quad 0 \\
0 \quad 0 \quad 0
\] | Triaxial Stress           |
| \[
\sigma_1 \quad 0 \quad 0 \\
0 \quad -\sigma_2 \quad 0 \\
0 \quad 0 \quad 0
\] | Pure Shear                |
| \[
M_1 \quad 0 \quad 0 \\
0 \quad M_1 \quad 0 \\
0 \quad 0 \quad M_1
\] | Isometric                 |
| \[
M_1 \quad 0 \quad 0 \\
0 \quad M_2 \quad 0 \\
0 \quad 0 \quad M_3
\] | Tetragonal                |
| \[
M_{11} \quad M_{12} \quad M_{13} \\
M_{21} \quad M_{22} \quad M_{23} \\
M_{31} \quad M_{32} \quad M_{33}
\] | Monoclinic                |
| \[
M_{11} \quad M_{12} \quad M_{13} \\
M_{21} \quad M_{22} \quad M_{23} \\
M_{31} \quad M_{32} \quad M_{33}
\] | Triclinic                 |
1.2.1 - Soft Sediment Structures

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Given the large range of topics in the 12 books on the geometry, kinematics, and dynamics of structural geology (see 1.1.2), there is very little attention given to role of water. It most commonly appears as the medium responsible for pore pressure in rocks. Its role is one of the most underestimated in the formation of the structures mentioned in all 12 books largely because it is such an effective friction reducer. More to the point, it does not reduce friction (i.e., the rock property) but rather it reduces the normal stress so that slip between surfaces can take place at a lower shear stress. An approach that makes this set of short course notes different from the large collection of such notes, is the larger focus on the role of water in structural geology relative to other treatments of this ilk.

One subject that is common in structure texts from a half century ago is the chapter on the deformation of soft sediments (Maltman, 1994). Causes of sediment deformation include gravitational mass movement (slumps, slides, creep), fluid-sediment movement, glacial shoving, and earthquake shaking. Specific structures include density inversions, pore water escape structures, sediment shrinkage, sediment wetting, deformation related to compaction and deformation related to early chemical precipitation. Many of these structures are enabled by a decrease in effective stress with the most common being the liquefaction from earthquake shaking. Examples of most of these structures are abundant in the Devonian section of the Appalachian Basin the Acadian tectonics taking place just to the east. Regardless of which basin examples come from, these soft sediment structures are usually a manifestation of tectonics.
A plate-tectonics context for Middle Devonian Marcellus and Upper Devonian sediments of the Catskill Delta complex which carry abundant soft sediment structures.

Blakey, 1994
Density inversion: Ball & Pillow Sags in Upper Devonian Sandstone
Appalachian Basin, Sugar Run, Pennsylvania, USA

Underside of Bedding
Liquefaction features that are incipient ball & pillow structures
Devonian Rhinestreet Formation, Dansville, New York, USA

Look at the underside of bedding
Balls of silt pulling apart to slump down into shale
Devonian Ithaca Formation, Marathon, New York, USA
Balls of silt falling more than a meter down into shale
Devonian Ithaca Formation, Marathon, New York, USA
Pencil cleavage forming around the siltstone ball as it falls
Devonian Ithaca Formation, Marathon, New York, USA
Large ball deforming while dropping into shale
Devonian Ithaca Formation, Marathon, New York, USA
Small-scale slumps near the shelf-slope break of a delta front
Mahantango Fm, Appalachian Valley & Ridge, Pennsylvania, USA
Bentonite deposited rapidly on sea-floor mud
Appalachian Mountains, Pennsylvania, USA

Tioga bentonite: water-layed volcanic ash bed
Marcellus Shale

soft sediment structure *(ball and pillow)*
Clastic dike dewatering & slaty cleavage
Martinsburg Formation, Delaware Water Gap, Pennsylvania, USA
Buckled folding of chert beds as a consequence of peptization of porcellanite
Miocene Monterey Formation, Santa Barbara

When colloidal particles bear a same sign electric charge, they mutually repel each other and cannot aggregate together. Flocculation occurs by adding a concentrated solution of salt to the system. These was the condition for deposition of the Monterey porcellanite. The electrical charges present at the surface of the particles are so "neutralised" and disappear. When the water chemistry of the pore space changed by the invasion of fresh water (groundwater), the electrical double layers present before flocculation in sea water expands again and the electrical repulsion reappears: the precipitate peptizes. The expansion of the porcellanite caused the buckling of the chert.
1.2.2 - Elasticity

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Hooke’s Law

When a body changes shape we say that the body is strained. Strain, a deformation, is measured as the ratio of the change in shape of a body to its initial shape. During compaction, for example, individual beds on the sea floor were separated by a certain distance (L). When the beds were compacted under overburden load, the separation decreased (i.e., displacement took place) by an amount ΔL. Strain (ε) is ΔL divided by L. If the beds had been compacted elastically, would have returned to their initial separation, L, upon exhumation. Of course, we know that compacted beds are not elastic so they don’t ‘rebound’ upon exhumation.

Linear elasticity has a predictable relationship between the amount of force and the amount of strain. We note, for example, that when a spring is loaded with one, two, and three weights, respectively, it gets longer in equal increments with the addition of comparable increments of weight. In this example, weight is a force acting at a point but if the force was spread over an area we would say that a stress causes the springs to lengthen. Constant and predictable relationship between change in force (or stress) and change in length (or strain) is characteristic of an elastic material.

Atomic forces which govern the atomic distances in a lattice act just like springs which return to their natural shape regardless of whether they were subject to a push or a pull. The law that governs the action of a spring is called Hooke’s Law. We can write the equation for the Hooke's Law of a spring as follows:

\[ \text{force} = \text{spring constant} \times \text{displacement} \]

The constant which specifies the relationship between the force on the spring and its displacement is called the spring constant. Often rock deformation is given in units of stress (force per unit area) and strain (change in length divided by initial length). In this case Hooke's Law is written as:

\[ \text{stress} = \text{Young's modulus} \times \text{strain} \]
Hooke’s law is illustrated in a plot of strain (displacement) versus stress (force)

\[
\text{Stress (s)} = \frac{\text{force}}{\text{unit area}}
\]

\[
\text{Young’s Modulus} = \frac{\text{stress}}{\text{strain}}
\]

\[
\text{Spring Constant} = \frac{\text{force}}{\text{displacement}}
\]

Above, stress (force) is plotted as the independent variable as it should be. However, geologists (and engineers) are an odd bunch who by convention always plot stress (force) on the vertical axis as if it were the dependent variable. Don’t be fooled by this convention in the other plots that appear during this short course.

Measurements of strain of rocks usually involve very small displacements. This is because most rocks are very stiff relative to a rubber band. In fact, rocks are so stiff that strain is measured using the unit, microstrain. A strain of $10^{-1}$ is a 10% strain which means that a body of one meter in length will shorten by 10 centimeters. A strain of $10^{-3}$ is a 0.1% strain which means that a body of one meter in length will shorten by 1 millimeter. A microstrain is $10^{-6}$ or a 0.0001% strain which means that a body of one meter in length will shorten by one micron (one micrometer = $10^{-3}$ millimeter).
If we wish to measure very small strains we must use strain gauges which are small wires wound like an accordion. The strain gauge wires are glued to the surface of specimen so that when the specimen stretches, the strain gauge wires also stretch. If the wires are stretched the cross sectional area of the wires gets smaller. The electrical resistance of the wire increases if the cross sectional area decreases. The strain gauge operates on the principle that the change in resistance of the accordion-like wire is equivalent to the strain of the sample. A strain indicator measures strain in the strain gauge wires by measuring the change in resistance of the wire as the specimen stretches.

Quartzite cylinder subject to a tensile stress sufficient to cause brittle failure. A strain gauge was used to monitor strain before failure.
Elementary Elasticity (1-D problems)

- Stress: \( \sigma = \frac{\text{force}}{\text{area}} \)
- Strain: \( \varepsilon = \frac{\Delta l}{l} \)
- Hooke’s Law: \( \sigma = E \varepsilon \)
- Poisson’s Ratio: \( \nu = \frac{\varepsilon_x}{\varepsilon_z} \)
  - If \( \Delta V \) (volume) = 0, \( \nu_{\text{H}_2\text{O}} = 0.5 \) (incompressible)
  - \( \nu_{\text{rock}} < 0.5 \) (compressible) due to pore space
- Poroelasticity: \( S_{h\text{min}} = f(\Delta P_p) \)
The Earth is elastic. This can be said for many geological phenomena from the short-term loading from earthquake waves to the long-term loading by seamounts that cause lithospheric flexure. These longer-term loads may lead to a dual rheological behavior such as viscoelasticity but, never-the-less, the elastic load always present even under these long-term conditions controls the behavior of fracture propagation. Short-term loads are those acting at time scales less than $10^5$ seconds and at these time scales the mechanical behavior of rock is described reasonably well by the theory of elasticity. Under many circumstances crack propagation in the crust appears to occur during short enough time intervals that the stresses driving the propagation are short-term loads. For this reason, it is appropriate to start our discussion of the physical process controlling fracture distribution in the crust with a discussion of elementary elasticity followed by a discussion of the more common states of stress in the crust.

Stress in the Earth can be easily considered using either of two standard reference states (Engelder, 1993). A reference state is taken to by the hypothetical stress condition in the absence of tectonic stresses that are a manifestation of global plate tectonics. Without tectonic stress a planetary body would be left with gravitational and thermal stresses, as presumably is the case for the Moon. Rock near its melting temperature under long-term loads will flow to relieve differential stress, $\sigma_d$. In such a state the three principal stresses will approach each other to generate a state of stress that we call the **lithostatic reference state**

$$\sigma_1 = \sigma_2 = \sigma_3 = S_H = S_h = S_v$$

where the Greek letters refer to principal stresses with reference to Earth coordinates and the Arabic letters refer to the Earth coordinates of vertical and horizontal. In general, geologists refer to compressive stresses as positive. However, there are times in the analysis of fracture propagation when it is more convenient to assign a positive sign to tensile stresses.

The **uniaxial strain reference state** is based on the postulated boundary condition that strain is constrained at zero across all fixed vertical planes (Savage et al., 1985). Such a boundary condition leads to the stress state which approximates newly deposited sediment in a sedimentary basin: the state of stress arising from uniaxial strain. The intrusion, solidification and cooling of a diabase sill demonstrates both reference states, as does the generation of mud cracks, a structure that may later be used as a geological strain marker.
Thermoelastic contraction generates the tensile stress that drives the propagation of these columnal joints.
Mudcracks:
Eastern Desert of Egypt (near Gulf of Suez)
Initial state of stress in a granite based on thermoelastic behavior

The stress-depth diagram

- Lithostatic Stress – Granite pluton going through its solidus:
  - $S_{Hmax} = S_{hmin} = S_v$

- Cooling
  - Cooling is a negative temperature change which causes a decrease in horizontal stress and increase in differential stress.

\[
\Delta S_{hmin} = \frac{\alpha E}{1 - \nu} \Delta T
\]

- Magma cools by isobaric contraction causing $S_{hmin}$ to decrease

Here are three elastic constants (material properties) that we would like to understand!
The elementary response of rock to a state of lithostatic stress is to deform (i.e., strain, $\varepsilon$) in a manner characterized by Hooke’s law,

$$\varepsilon = \frac{\sigma}{M}$$

where $M$ is a property of the rock called an elastic modulus. Under lithostatic stress, $\varepsilon_{kk} = \frac{\sigma_{kk}}{3K}$

where $M = 3K$ and $K$ is the bulk modulus of the rock.

The second reference state in the crust of the Earth arises as a consequence of burial of sediment where the lateral constrain of neighboring rock prevents lateral stretching. This is a state called uniaxial strain where additional overburden will cause a vertical contraction but $\varepsilon_H = \varepsilon_h = 0$. Under such conditions an elastic body will support a differential stress, $\sigma_d$. Under a state of $\sigma_d$, the body is subject to a shear deformation characterized by a different form of Hooke’s law,

$$\varepsilon_{ij} = \frac{\sigma_{ij}}{2\mu}$$

where $M = 2\mu$ and $\mu$ is the shear modulus or modulus of rigidity.

For the purpose of understanding the relationship between the various elastic constants a third reference state, the uniaxial stress state where $\sigma_1 \neq 0$ and $\sigma_2 = \sigma_3 = 0$, is useful but not common in the Earth. The general statement of Hooke’s law relates each principal stress the three principal strains as follows:

$$\sigma_1 = (\lambda + 2\mu)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3$$
$$\sigma_2 = \lambda\varepsilon_1 + (\lambda + 2\mu)\varepsilon_2 + \lambda\varepsilon_3$$
$$\sigma_3 = \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2\mu)\varepsilon_3$$

where $\lambda$ is a Lame constant and $\mu$ is the shear modulus. The most general statements of Hooke’s law involves two elastic constants. Under a state of uniaxial stress equations of elasticity are written:

$$\sigma_1 = (\lambda + 2\mu)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3$$
$$0 = \lambda\varepsilon_1 + (\lambda + 2\mu)\varepsilon_2 + \lambda\varepsilon_3$$
$$0 = \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2\mu)\varepsilon_3$$
rearranging \[ \varepsilon_2 = \varepsilon_3 = -\frac{\lambda}{2(\lambda + \mu)} \varepsilon_1 \]

Substituting back into the stress (\(\sigma_1\)) equation

\[ \sigma_1 = \frac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)} \varepsilon_1 \]

Now we see that Young’s modulus is a function of two other elastic constants:

\[ E = \frac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)} \]

We may also define Poisson’s ratio, \(\nu\), as a function of two other elastic constants:

\[ \nu = \frac{\lambda}{2(\lambda + \mu)} \]

Hooke’s law for uniaxial compression can also be applied to write expressions for \(\mu\) and \(K\) in terms of \(E\) and \(\nu\):

\[ 2\mu = \frac{E}{1+\nu} \quad 3K = \frac{E}{1+2\nu} \]

Typical elastic properties of a well-lithified sandstone might be on the order of \(E = 40\) GPa, \(\mu \approx 20\) GPa, \(K = 15\) GPa and \(\nu \approx 0.2\).

The evolution of the state of stress leading to the generation fractures can be quite complex. During the history of burial, lithification, tectonic deformation, and exhumation there may be several instances when stress conditions are conductive for fracture propagation. One can assume that these favorable states of stress are achieved during inelastic deformation. In most cases, a thermoelastic excursion is imposed on the sequence of inelastic events. The first of these inelastic events takes place during burial and lithification.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>density</td>
<td>ML$^{-3}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>stress</td>
<td>ML$^{-1}$T$^{-2}$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>shear stress</td>
<td>ML$^{-1}$T$^{-2}$</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>normal stress</td>
<td>ML$^{-1}$T$^{-2}$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>strain</td>
<td>dimensionless [LL$^{-1}$]</td>
</tr>
<tr>
<td>$E$</td>
<td>Young's Modulus</td>
<td>ML$^{-1}$T$^{-2}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson's ratio</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Engineering shear strain</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$P_p$</td>
<td>pore pressure</td>
<td>ML$^{-1}$T$^{-2}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>porosity</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
<td>C$^\circ$</td>
</tr>
<tr>
<td>$q$</td>
<td>heat flow</td>
<td>JL$^{-2}$T$^{-1}$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>thermal conductivity</td>
<td>JLT$^{-1}$C$^\circ$-1</td>
</tr>
<tr>
<td>$z$</td>
<td>depth</td>
<td>L</td>
</tr>
</tbody>
</table>
1.2.3 Effective Stress

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Karl Terzaghi was a pioneer in the field of soil mechanics. Much of what he proposed for the behavior of water saturated soils can be applied to the mechanics of rocks with fluid-filled pores. His most important concept is known as the effective stress principle. The effective stress within a soil or rock is equal to the total stress minus the pore pressure. The effective stress principle is as follows. Across any open surface around a pore within a rock, there act a total stress $\sigma$ and a pore water pressure $P_p$. Total stress ($\sigma$) can be visualized as the weight of a water-saturated column rock. Two components of that weight are the rock with empty pores and the weight of the water that fills the pores. We define effective stress as

$$\bar{\sigma} = \sigma - P_p$$

Distinguishing the total vertical stress $\sigma_v$ and the total horizontal stress $\sigma_{h_{\text{min}}}$, we have

$$\bar{\sigma}_v = \sigma_v - P_p \quad \bar{\sigma}_h = \sigma_h - P_p$$

The coefficient of lateral stress ($\Lambda$) is based on effective stresses

$$\Lambda = \frac{\bar{\sigma}_h}{\bar{\sigma}_v}$$

Note: Here $\sigma_v$ and $\sigma_{h_{\text{min}}}$ provide information on orientation but not magnitude.
Coupled Behavior

• “All measureable effects of a change in stress, such as compression, distortion and a change in shearing resistance are due exclusively to changes of effective stress.”

• Deformation is then the product of the combined effect of total stress $\sigma$ and pore pressure $P_p$ through effective stress, $\bar{\sigma}$

\[
\bar{\sigma} = \sigma - P_p
\]

Terzaghi, K., 1936, The shearing resistance of saturated soil and the angle between the planes of shear, Proceedings of 1st International SMFE Conference, Harvard, Mass. V. 1, p. 54-56
Friction between grains holds grains in place

**Dry**

\[ S_v = \rho_{dry} g z \]

\[ S_v > S_h \]

**Wet**

\[ S_v = \rho_{wet} g z \]

\[ S_v > S_h \]
Dry – Friction between sand grains prevents lateral loading
Wet – Water adds to load against the side of the container

Mohr’s Circle

\[ \sigma_1 \text{ – vertical} \]
\[ \sigma_3 \text{ – horizontal} \]

Dry – Total Stress
Hydrostatic Pore Pressure – Total Stress

Terry Engelder
Mohr Circle (Terzaghi Behavior)

- Effective stress acts only on normal stresses.
- Starting condition (sample saturated = wet)

\[ \sigma_1 - P_p = \bar{\sigma}_1 \]

\[ \sigma_3 - P_p = \bar{\sigma}_3 \]
Mohr’s Circle: Effective Stress

- $\sigma_1$ – vertical
- $\sigma_3$ – horizontal

- Dry Sand – Total Stress
- Saturated Sand Hydrostatic $P_p$ – Total Stress
- Saturated Sand – Effective Stress

$\bar{\sigma}_1$, $\bar{\sigma}_3$, $\sigma_1$, $\sigma_3$
Sedimentary Basin: Basic Mechanical Unit:

Dry Clastic Material

Porosity: \[ \phi = \frac{A - a}{A} \]
vertical forces \[ \sigma_v A = \sigma_s a \]

vertical stress \[ \sigma_s > \sigma_v \]
Sedimentary Basin: Compaction

vertical forces \( \sigma_v A = \sigma_s a \)

vertical stress \( \sigma_s > \sigma_v \)

contact area

total surface area
• A coupling of changes in total stress to changes in pore pressure
• Because of free boundary at the Earth’s surface, total stress and pore pressure are NOT coupled in the **vertical** direction (to a first approximation).
Consider the effect of pores on strength of rocks. A rock without pores is considerably stronger. This stands to reason because pore space is unsupported within the rock. With the presence of unsupported space the intact rock between the pores has to take larger loads to sustain an overall applied stress. This important effect also has a direct bearing on the effective stress law which is more properly written

$$\bar{\sigma} = \sigma - \alpha P_p$$

Terzaghi suggested that $\alpha = 1$ based on experiments in soils. His interpretation was that grain boundaries and grain contacts both have an effective porosity of one, which is to say, water at a pressure $P_p$ fills all space between grain contacts. Others including Geertsma have suggested that

$$\alpha = 1 - \left( \frac{K}{K_0} \right)$$

where $K$ and $K_0$ are the effective and grain bulk moduli. Nur and Byerlee have a very interesting derivation of $\alpha$. They assume an isotropic aggregate of solid material with connected pores. The outside of the solid is subject to confining pressure $P_c$ and the pores are subject to pressure $P_p$. Both of these pressures have an effect on the volumetric strain of the aggregate $\Theta$. But what we will do is consider the effect of these pressures by dividing the application of the confining pressure into two steps. The first application of pressure involves balancing $pp$ and $pc$, so that $P_p' = P_c'$. The second application of pressure involves the remaining confining pressure $P_c'' = P_c - P_p$. Volumetric strain $\Theta$ of a dry aggregate caused by the addition of a confining pressure is

$$\Theta = \beta \cdot P_c$$

where $\beta = 1/K$ is the effective compressibility of the dry aggregate. Now the volumetric strain of the aggregate $\Theta''$ due solely to the confining pressure $P_c''$ is

$$\Theta'' = \beta \cdot P_c'' = \beta \cdot P_c - \beta \cdot P_p$$

The volumetric strain $\Theta'$ caused by the equal addition of pressure around the rock body and inside the pores is of interest. The solution is to assume a solid in which the pores are filled with a solid. Then an external confining pressure $P_c'$ causes an equal hydrostatic compression $P_c'$ in the solid. The volumetric strain of the solid $\Theta_s$ is then
where $\beta_s$ is the compressibility of the solid devoid of any cavities. In this analysis we can remove the solid from the pore and replace it with a fluid at $P_p' = P_c'$. In doing so the deformation of the solid remains unchanged. Consequently

$$\Theta' = \Theta_S = \beta_s \cdot p_c'$$

Adding the equations to find the total strain

$$\Theta = \Theta' + \Theta'' = \beta \cdot p_c - (\beta - \beta_s) p_p.$$

This equation can be written in terms of the bulk moduli where $K$ is the bulk modulus of the dry aggregate and $K_s$ is the intrinsic bulk modulus of the solid itself

$$\Theta = \frac{1}{K} p_c - \left[ \frac{1}{K} - \frac{1}{K_s} \right] p_p.$$

Because the volumetric strain can be written

$$\Theta = \frac{1}{K} \cdot p_c''$$

the expression for the effective pressure can now be written

$$p_{c''} = p_c - \left[ 1 - \frac{K}{K_s} \right] p_p$$

and

$$\alpha = 1 - \frac{K}{K_s}.$$
Sedimentary Basin: Vertical Effective Stress

Biot Effective Stress

\[ \bar{\sigma}_v = \sigma_v - P_p \left( \frac{A - a}{A} \right) \]

Terzaghi Effective Stress

\[ \bar{\sigma}_v = \sigma_v - P_p \]

only surface film along contact

free water along contact

contact area \( A \)
total surface area

contact area \( a \)
effective porosity \( 0 \rightarrow 1 \)
total surface area

Terry Engelder
Mohr Circle

- Terzaghi vs. Biot effective stress.
  - Start experiment saturated then increase pore pressure ($P_p$)

![Diagram of Mohr Circle showing Terzaghi and Biot effective stresses with labels for vertical ($\sigma_1$) and horizontal ($\sigma_3$) directions.](image)
Mohr’s Circle:

1. **Effective Stress**
2. **Abnormal Pore Pressure**

- Hydrostatic Pore Pressure – Total Stress ($S_h$)
- Hydrostatic Pore Pressure – Effective Stress
- Abnormal Pore Pressure – Effective Stress
- Abnormal Pore Pressure – Total Stress ($S_h$)

$\sigma_1$ – vertical
$\sigma_3$ – horizontal
Evidence of Poroelastic Deformation (Biot Behavior):
Production of gas reduces pore pressure!
Horizontal stress does not reduce as much as pore pressure!

Figure 3. Plot of $\Delta S_h/\Delta P_p$ vs. $\Delta P_p$ in McAllen Ranch field, south Texas. Depth of each datum is indicated in metres. Data taken from Salz (1977).
Stress increases when pore pressure increases
North Sea, Norway Sector

Evidence of Poroelastic Deformation:
Generation of gas increases pore pressure!
Horizontal stress does not increase as much as pore pressure increases!

Leakoff data from the North Sea, Central Graben

Gaarenstroom et al., 1993
• $\Delta P_p$ is fully effective in an unceментed sand (i.e., soil)
• $\Delta P_p$ works against a cement in a rock and thus is not fully effective
1.2.4 – Consolidation, Compaction, and Compaction Disequilibrium

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
• Compaction is the process in which a stress applied to a soil causes instantaneous densification
  • Air is displaced from the pores between the soil grains.
• After stress is applied, there is a densification due to water (or other liquid) being displaced from between the soil grains. This slow densification is called consolidation, not compaction.
• Compaction is the term commonly used when strain associated with densification is visible in rock.
• Seafloor mud converts to rock through porosity decline.
• Porosity loss occurs through the combined effects of compaction (if water loss is instantaneous) consolidation and cementation,
  • Consolidation often dominates in the upper 2 km of burial
  • cementation dominates porosity evolution at greater depths.
Mechanical Rotation -- mechanism for mechanical compaction (called compaction because this is no sense of time in the observation)

- March Model
  - Expresses the relationship between mechanical rotation and flattening strain

- Detection of mechanical rotation during flattening strain
  - Anisotropy of magnetic susceptibility from paramagnetic grains (i.e., mica)
  - Anisotropy of intensity of X-ray diffraction from chlorite grains

Seal-bed clay

Compacted Shale
Flattening of grains
Devonian Rhinestreet gas shale, New York, USA

 kerogen particles have been flattened to conform to the shape of inorganic grains (scale = 0.1 mm)

Lash and Engelder, 2005
compacted clay grains wrapping a pyrite framboid in a clay laminae sample

Lash and Engelder, 2005

Flattening of grains
Devonian Rhinestreet gas shale, New York, USA
Pressure Solution
(mechanism for chemical compaction)

Solution occurring preferentially at the contact surfaces of grains (crystals) where the normal stress at the grain contacts exceeds the fluid pressure in the pore space. Preferential solution is a consequence of the higher chemical potential in the water film trapped between grain contacts.

Chemical potential is commonly used in predicting the influence of stress on the tendency of a solid to dissolve or grow in the presence of a solution (Kamb, 1959). For equilibrium of a solid under all possible states of strain

$$\mu_L = F_s + P_w V_s$$

where $\mu_L$ is the chemical potential of the solute in solution, $F_s$ is the molar Helmholtz free energy of the stressed solid, $P_w$ is the pressure of the solution in contact with the solid, and $V_s$ is the molar volume of the solid in the stressed state. Differentiating at a constant temperature, a change in $\mu_L$ is represented by three terms:

$$d\mu_L = dF_s + P_w dV_s + V_s dP_w$$

In this equation, $V_s dP_w$ represents a change in normal pressure on the surface of a solid suspended in the fluid undergoing a pressure change. $dF_s$ is the change in elastic strain energy and $P_w dV_s$ is the change in specific volume as the result of stress under grain-grain contacts. For rocks where grains are in contact, stresses within the grains arise from a combination of pore water pressure and grain to grain contacts. To treat this situation, several including Paterson (1973) and Robin (1978) deal with a variation in stress ($\sigma_n$) along the surface of the grains arising from grain to grain contacts. Unlike the hydrostatic case, chemical potential varies along the surface of the grain because of the pressure variation from grain to grain contacts.

Pressure solution removing the edge of this crinoid columnal
Sandstone compaction: Note the role of pressure solution in developing closer packing!

Weedman et al., 1992
• Bedding will flatten and fold around a rigid body in a compacting matrix:
  • Pebbles in a tillite
    • Difficult to sort out sub-glacial compaction from overburden compaction
  • Concretions
  • Dropstones
Devonian Rhinestreet Formation, Lake Erie

Shale matrix compacts around concretions

Concretions are carbonate deposits within seafloor mud
Consolidation

The dissipation of excess pore pressure, accompanied by volume change is called **consolidation**.

Usually (but not always) the total stress remains constant and the pore pressure and volume slowly change. The rate of consolidation (volume change with seepage) is dependent on the permeability of the rock and the size of the consolidating layer.
Specific Volume of Rock

- Specific volume \( (v) \) decreases during consolidation
  \[ V - \text{volume of sample} \]
  \[ V_o - \text{volume of grains} \]
  \[ V_p - \text{volume of pores} \]

- Porosity
  \[ \phi = \frac{V_p}{V} \]

- Void Ratio
  \[ e = \frac{V_p}{V_o} \]

Terry Engelder
• Consolidation is a form of compaction but driven by changes in fluid volume in the pore space of rocks.
• In this context pore water is incompressible relative to a compressible rock.
• Consolidation reflects a coupled behavior expressed as:

$$\Delta S_h = f \left( \Delta P_p \right)$$
Effective Stress Parameter (k)

- \( \sigma_h = S_h - P_p \) and \( \sigma_v = S_v - P_p \) are effective horizontal and vertical stresses.
- Effective stress parameter is also known as the coefficient of Earth stress at rest.

\[
k = \tilde{R} = \frac{\overline{\sigma_h}}{\overline{\sigma_v}} = \frac{S_h - P_p}{S_v - P_p}\]

\[k < 1\]
Uniaxial Consolidation

• During the early stages of burial in a sedimentary basin, especially where tectonic deformation is slow, stress is commonly governed by uniaxial strain conditions where lateral boundaries are fixed.
Sedimentary Basin: Drained Conditions
Hydrostatic Pore Pressure

- Earth consolidates only under drained conditions
Experimentally consolidated silty clay including unloading-reloading cycles

Karig & Hou, 1992

Some will call this a compaction curve
• $k = K_o =$ stress ratio for no lateral strain.

Karig & Hou, 1992
• Some call this a consolidation coefficient

• The mechanism responsible is friction at grain/grain contacts

• It is a mistake to consider that k is equivalent to Poisson’s ratio (v)
  • Poisson’s ratio is an expression of elastic behavior
  • Consolidation is not elastic
Mechanisms for controlling horizontal stress early during burial

The consolidation coefficient (k) is just one of three mechanisms that might control the state of stress during early basin development. The fourth, pore pressure, is already incorporated in the effective stress ratio that defines k. The first process that must be followed in burial is lithification. Before lithification the grain-grain contacts carry the overburden weight and the rock column is maintained by the uniaxial mechanism where part of the vertical stress is transferred to the lateral boundaries. In the case of sand this might be 53% of the overburden. More of the vertical load is transferred in the case of shale (62%). When the grain framework carries the overburden weight, lithification can take place without changing the lateral stress (Voight and St. Pierre, 1974). If nothing else happened, this would leave a layered sandstone-shale with sandstone carrying a lower least principal stress ($S_{\text{hmin}}$). As will be seen, fractures in sand-shale sequences follow this rule.

The other two mechanisms for controlling horizontal stress early in the history of a basin are viscous relaxation and tectonic strain. A viscoelastic material will strain when a new load is applied and when strain is held constant the material will relax. Experiments by Hagin and Zoback (2004) show that creep will allow the relaxation of as much as 75% of a differential stress on an unconsolidated dry sand in as little as 10 hours. The implications of this behavior is that the $\sigma_d$ that Karig and Hou (1992) observe might not last in a basin until lithification can ‘cement’ in this stress difference in place. However, the evidence is that a bed by bed stress difference does survive for a long time (millions of years) in a sedimentary basin.

The Nolte-Smith (1981) model for hydraulic fracture in sedimentary basins
Aside from consolidation and viscoelastic relaxation, the third mechanism for controlling the least stress ($S_{hmin}$) in a basin is tectonic strain. Following lithification, the rock will not creep with the short time constant as a soil or sand aggregate, never-the-less the rock will creep. However, as well be shown, basins are subject to a differential stress that is sufficient to cause slip on favorable oriented faults if the stress field is disturbed only slightly (Townend and Zoback, 2000). This means that rocks of the basin are subject to a significant elastic tectonic strain above that which is derived from overburden acting on a constrained earth. If tectonic strain is important in setting the horizontal stress profile, the rock with the higher Young’s modulus (E) will exhibit extreme stresses, high or low, depending on whether the basin is stretching or shortening.

If a basin is being stretched, sandstones would have a lower $S_{hmin}$ relative to shale because of the sandstone’s higher E. This is the type of stress profile seen in the Piceance Basin (Warpinski, 1989). Modern offshore basins such as the Gulf of Mexico are presently being stretched and might exhibit this familiar stress profile in sandstones versus shale. However, the Piceance Basin sits in a back arc where the original sedimentary deposits were subjected to shortening tectonics. Under such an environment we might expect that the sandstone would have a higher $S_{hmin}$. 

![Diagram showing minimum horizontal stress vs. depth in the Mesa Verde Group, Colorado.](image-url)
Mechanisms for controlling horizontal stress early during burial

The Appalachian Basin is a place where the stiff rocks, both sandstone limestone are subject to higher stresses (Evans et al., 1989). In some cases the stiff layers have a stress state with $S_{H\text{max}} > S_v$ as indicated hydraulic fracture stress tests in the stress-depth diagram shown below. The Appalachian Plateau is a test case for the three mechanisms for early stress in basins.

Today the propagation of vertical fractures should favor shales rather than the stiff beds. However, natural fractures propagated during the Alleghanian Orogeny which was a layer-parallel shortening event about 270 Ma. Even then, like today, the stiff rocks should have been least likely to fracture under effective tensile stress. Rather, the ‘stiff’ rocks fractured in a compressive environment, the same situation observed in the Piceance Basin. One explanation is that, despite the possibility of both stress relaxation by creep and a tectonic stress that favored shortening, the fracture pattern indicates the least stress was in the stiff rocks, as would have been the case if the overprint of the original consolidation stress was still significant. The other possibility is that the stretch of the oroclinal bend was active (Srivastava & Engelder, 1990).
1-D strain

Undeformed State

Deformed State

\[ \Delta z \]

\[ \Delta u \]
Uncompacted bedding preserved within concretion

Taughannock Falls
\( \varepsilon \) (compaction) = 42%

Geneseo Black Shale
Mudrocks display higher depositional porosities than sandstones and also experience more porosity loss with burial. At a depth of 1.5 km, many mudrocks have porosity lower than the 26% intergranular volume of a sandstone.


Red Dot: Intergranular volume of sand at deposition.

Green Dot: Intergranular volume of sand after compactional stabilization.
• the mechanism stopping compaction acted at a relatively shallow depth

Dickinson, 1953

42% concretions

700 m
(a) Porosity (%) vs. Depth (m)

- Normal compaction curve
- Onset of disequilibrium compaction

(b) Pressure (MPa) vs. Depth (m)

- Hydrostatic pressure
- Lithostatic pressure
- Pore pressure
- Fluid retention depth
- Constant vertical effective stress
• When pore fluid can not leak in response to additional overburden the rock is **undrained**.

• Water in a subsiding basin in incompressible.

• This means that under undrained conditions, water supports the total increment of overburden.
The top of abnormal pressure and the top of the zone undercompaction as indicated by shale density, sonic, and conductivity logs usually do not occur at the same depth at a given location. Rather, these tops in most wells are separated by hundreds of feet (tens of meters) and in some wells by vertical distances of over 2000 ft (600 m). Furthermore, the top of abnormal pressure is usually above or coincident with the zone of undercompaction, and therefore the zone of undercompaction in many cases is abnormally pressured. Since the top of abnormal pressure and the top of the zone of undercompaction are usually not coincident the use of electropressure methods to determine geopressures quantitatively yields results that are in many cases inaccurate and unreliable. The Tertiary section of the Gulf of Mexico Basin is characterized by two geopressure profiles consisting of three or four linear segments. The Monte Christo field shows a profile with four segments.

Geopressure profile for the Monte Christo field, Hidalgo County, Texas. (100 bottom hole pressure measurements from 87 wells with 11 data points excluded.)
1.2.5 – Stress in the Schizosphere

An AAPG Short Course by
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Extension fractures (i.e., joints) are by far more common in the crust of the earth than shear fractures. The explanation is simple: the state of stress within the crust of the earth is such that the differential stress \( \sigma_d = \sigma_1 - \sigma_3 \) is rarely large enough to fracture intact rock. In many instances smallish appearing faults are nothing more then reactivated extension fractures. Reactivated means that slip occurs parallel to the extension fracture after it propagated normal to the least principal stress \( \sigma_3 \). Slip in shear requires that the stress field be reoriented so that the extension fracture is no longer sitting in a principal stress plane.

If differential stress within the crust of the earth is rarely large enough to cause shear fractures, how are most fractures formed? This question is particularly interesting because tensile stresses are required for the generation of extension fractures. Yet, how are tensile stresses generated when lithostatic stress which is compressive become increasingly large with depth in the crust. Remember that the vertical stress \( \sigma_v \) is proportional to the density \( \rho \) of the rock and the depth of burial \( z \):

\[
\sigma_v = \rho \cdot g \cdot z
\]

where \( g \) is the acceleration due to gravity. In the absence of tectonic stresses the horizontal stress \( \sigma_h \) increases with depth as a fraction of the vertical stress.

\[
\sigma_h = \frac{\nu}{1 - \nu} \cdot \sigma_v = \frac{\nu}{1 - \nu} \cdot \rho \cdot g \cdot z
\]

Using the foregoing equation, the stress-depth diagrams on the top of the next slide is a thought experiment showing two examples of the development of \( \sigma_h \) with depth of burial in a delta environment, however unlikely. One case assumes a dry sandstone which formed with its present elastic properties at the surface of the earth (the black line on the left diagram). The other extreme assumes a dense clay matrix supported by water so it can’t consolidate (the black line on the right diagram). The clay has such a low permeability that its leak rate is too slow to keep up with burial. Therefore we call a clay "undrained". A fluid like water with no shear strength has a Poisson’s ratio \( \nu = 0.5 \). In this state, the clay has \( \sigma_h = \sigma_v \). There is some difference in the overburden induced \( \sigma_h \) between these two situations with \( \sigma_h \) being less in the sandstone. A dry sandstone might have a \( \nu = 0.25 \) and so the sandstone would have a horizontal stress gradient \( (\Delta \sigma_h/\Delta z) = 0.33 \). The undrained clay would have a \( (\Delta \sigma_h/\Delta z) = 1.00 \).
These cartoons are stress-depth diagrams that help understand the evolution of stress in basins subject to both tectonic and pore pressure change. These extremes are not realistic for several reasons. First, the consolidation of mud and sand set the original stress gradient (for sand $\Delta \sigma_h/\Delta z = 0.53$ psi/ft and for clay $\Delta \sigma_h/\Delta z = 0.62$ psi/ft). Second, water is nearly universal at depth in basins. If the sandstone is saturated and drained, the hydrostatic $P_p$ in would have a pressure gradient of about 0.45 psi/ft (blue line) depending on the salinity of the pore water (fresh water $\Delta P_p/\Delta z = 0.43$ psi/ft). Note that the horizontal stress gradient ($\Delta \sigma_h/\Delta z =$) 0.33 psi/ft (black line) is less than the pressure gradient for water, an impossible situation. In fact, through consolidation of an unlithified sand aggregate ($\Delta \sigma_h/\Delta z =$) 0.53 psi/ft. Lithification does not change this stress state but does transform sand (i.e., buried soil) to a sandstone (i.e., an elastic rock). Through coupled behavior of water acting one an elastic rock, least horizontal stress ($\sigma_h$) would exceed pore pressure (red line). Third, mud is not very dense on the sea floor. A water supported mud would have a stress gradient that was a function of the density of the mud which is not much denser than the water column (orange line).
Natural hydraulic fractures

Propagation direction is right to left

Ithaca (Devonian siltstone-gray shale)

Watkins Glen, N.Y.

Terry Engelder
Sedimentary Basin: Undrained Conditions
Compaction Disequilibrium

Change in Total Stress
No Change in Effective Stress
Change in Pore Pressure

Terry Engelder
Sedimentary Basin: Undrained Conditions
Compaction Disequilibrium

\[ \Delta \sigma \]

\[ \sigma_v \]

\[ \sigma_s \]

\[ P_p < S_h \]

S\text{-}h \quad \text{Integrity Good}

Terry Engelder
Sedimentary Basin: Undrained Conditions
Migration of Hydrocarbons

Natural Hydraulic Fracturing

Pp > S_h
Integrity Lost

Terry Engelder
Mechanisms for increasing pore pressure

- Compaction disequilibrium
- Hydrocarbon maturation
- Pore-space filling by chemical diagenesis
- Development of a gas column
- Tectonic compaction
Mechanisms for increasing pore pressure

- Compaction disequilibrium
- Hydrocarbon maturation

Hydrocarbon maturation

Stress / Pore Pressure (MPa)

Depth below sea level (km)

- ○ Pore Pressure
- △ Leak-off Pressure
- --- $S_v$
- --- $S_h$

Bell, 1990

SCOTIAN SHELF, CANADA
Mechanisms for increasing pore pressure:

• Compaction disequilibrium
• Pore-space filling by chemical diagenesis

Stress-insensitive compaction

Pore-space filling by chemical diagenesis

Bell, 1990
• Poroelastic deformation causes a decrease in $\sigma_d$ (differential stress) because total $\sigma_3 (S_h)$ increases with increasing $P_p$. 
• Jointing is only permitted in the crust at $\sigma_3 = -T$ if $\sigma_1 < 3T$

Secor, 1965
Mechanisms for increasing pore pressure

- Compaction disequilibrium
- Development of a gas column

Bell, 1990

Gas column above oil or water

Compaction Dis-equilibrium

Development of a gas column
Mechanisms for increasing pore pressure:

- Compaction disequilibrium
- Tectonic compaction

Bell, 1990

Graph: Stress / Pore Pressure (MPa) vs. Depth below sea level (km)

- ○ Pore Pressure
- ▲ Leak-off Pressure
- --- $S_v$
- --- $S_h$

Legend:

- Blue line: Hydrostatic Pressure
- Orange line: Compaction Disequilibrium
- Red line: Tectonic Compaction

Bell, 1990
General stress-depth diagrams in the schizosphere

a. Normal faulting
   - $S_V \geq S_{H_{\text{max}}} \geq S_{\text{hmin}}$
   - Hydrostatic
   - $P_p \approx -23 \text{ MPa/km}$
   - $P_p \approx -0.44 \text{ psi/ft}$

b. Strike-slip faulting
   - $S_{H_{\text{max}}} \geq S_V \geq S_{\text{hmin}}$
   - Hydrostatic
   - $P_p$

(c) Reverse faulting
   - $S_{H_{\text{max}}} \geq S_{\text{hmin}} \geq S_V$
   - Hydrostatic
   - $P_p$

(d) Normal faulting
   - Overpressure at depth
   - $S_V \geq S_{H_{\text{max}}} \geq S_{\text{hmin}}$
   - $P_p$

(e) Strike-slip faulting
   - Overpressure at depth
   - $S_{H_{\text{max}}} \geq S_V \geq S_{\text{hmin}}$
   - $P_p$

(f) Reverse faulting
   - Overpressure at depth
   - $S_{H_{\text{max}}} \geq S_{\text{hmin}} \geq S_V$
   - $P_p$
Stress measurement in the KTB scientific research well indicate a strong crust, in a state of failure equilibrium as predicted by Coulomb theory and laboratory-derived coefficients of friction of between 0.6 and 0.7 (after Zoback and Harjes, 1997)
2.1.1 – Fracture Mechanics

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Boundary forces must balance on any internal plane.

Boundary forces are not supported at the edge of a hole!

Redistribution of forces produces a tensile stress.
Kirsch’s Two Contributions:

- Far-field stress magnified **three times** (3x) at a circular hole
- Circular hole turns a far-field compressive stress into a **tensile stress**

\[
\sigma_{\theta\theta} = \frac{S_H}{2} \left( 1 + \frac{R^2}{r^2} \right) - \frac{S_H}{2} \left( 1 + 3 \frac{R^4}{r^4} \right) \cos 2\theta
\]
If rock containing a vertical cylindrical hole is subject to a uniform uniaxial horizontal stress at infinity, \( S_H \), then the stresses near the hole, as expressed in polar coordinates at a point \( \theta \), and \( r \), are given by

\[
\begin{align*}
\sigma_{rr} &= \frac{S_H}{2} \left(1 - \frac{R^2}{r^2}\right) + \frac{S_H}{2} \left(1 + 3 \frac{R^4}{r^4} - 4 \frac{R^2}{r^2}\right) \cos 2\theta \\
\sigma_{\theta\theta} &= \frac{S_H}{2} \left(1 + \frac{R^2}{r^2}\right) - \frac{S_H}{2} \left(1 + 3 \frac{R^4}{r^4}\right) \cos 2\theta \\
\tau_{r\theta} &= \frac{S_H}{2} \left(1 - 3 \frac{R^4}{r^4} + 2 \frac{R^2}{r^2}\right) \sin 2\theta
\end{align*}
\]
• Elliptical cracks magnify (i.e., concentrate) stress even more than circular holes.

\[ S_{\text{tip}} = S_h (1 + 2c/b) \]
Stress near an elliptical hole can become really large.

\[ S_{\text{tip}} = S_h \left(1 + \frac{2c}{b}\right) \]
Cracks are NOT elliptical and best modeled as a mathematical singularity.

- Sharp crack tips lead to an infinitely large stress magnification (an impossible condition).
Irwin’s contribution (1957)

- identified Westergaard’s variable as containing the well known stress intensity factor which incorporated the length of the crack, 2c, the geometry of the crack and loading system, Y, and the remote stress, $\sigma_{yy}$.

$$K_I = Y \sigma_{yy}^r \sqrt{\pi c}$$

Stress Intensity ($K_I$)

- $K_I$ is the stress in the vicinity of a sharp crack tip
- **Stress intensity** depends on three variables and NOT just crack-normal stress
  - Length
  - Shape
  - Effective normal stress
Stress Intensity (K_i)

\[ \text{Stress Intensity} = \text{Stress} \times \frac{\text{Length}}{\text{Shape}} \]
Critical Stress Intensity \((K_{lc})\)

- \(K_{lc}\) is the stress intensity at the crack tip necessary to cause crack propagation.
- When \(K_{lc}\) is measured in the laboratory this is a rock property known as **fracture toughness**
Fracture toughness ($K_{lc}$) is stress intensity at the initiation of crack propagation.

\[ K_{lc} = K_I = YS_h^r \sqrt{\pi c} \]

Stress Intensity:

\[ K_I (\text{stress intensity}) = Y (\text{shape}) S_h^r (\text{stress}) \sqrt{\pi c} (\text{length}) \]
Crack driving stress ($\Delta \sigma$)

- One component of stress intensity ($K_I$)
- $\Delta \sigma$ comes from more than one source
- In the petroleum environment, the driving stress is the amount that $P_p$ exceeds $\sigma_3$

$$\Delta \sigma = P_p - \sigma_3$$
CRACK DRIVING STRESS

\[ \Delta \sigma = \frac{K_{lc}}{Y \sqrt{\pi c}} \]

CRACK DRIVING STRESS IS ALWAYS A NET TENSILE STRESS GOVERNED BY \( K_{lc} \).
A CRACK DRIVING PRESSURE \( (P_i) \) THE FLUID PRESSURE \( (P_P) \) WITHIN A CRACK IN EXCESS OF COMPRESSIVE ROCK STRESS.

\[ P_i = \Delta \sigma = P_P - S_h^r \]

tension = positive number

\[ \Delta \sigma = S_h^r \]
Stress field in vicinity of crack tip
- Takes a shape often describes as ‘rabbit ears’
Stress in the vicinity of a crack tip (rectangular coordinates)

\[
\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \left\{ \cos\left(\frac{\theta}{2}\right) \left[ 1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] \right\}
\]

\[
\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \left\{ \cos\left(\frac{\theta}{2}\right) \left[ 1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] \right\}
\]

\[
\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \left\{ \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right) \right\}
\]

\[
\sigma_{zz} = v(\sigma_{xx} + \sigma_{yy}) \quad \text{Plain strain}
\]

\[
\sigma_{zz} = 0 \quad \text{Plain stress}
\]

\[
\tau_{xz} = \tau_{yz} = 0
\]
Linear Elastic Fracture Mechanics (LEFM)

The study of the stress concentration in an elastic medium caused by a crack tip.

\[ S_H = \text{maximum horizontal compressive stress} \]
\[ S_h = \text{minimum horizontal compressive stress} \]
2.1.2 - Microcracks

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
The Brazilian Test

The Earth is in a state of compression because it is a self-gravitating body. Yet, ‘tensile’ fractures are the most pervasive of structures at the outcrop scale. So the question arises, “How does the Earth allow for the initiation of tensile cracks in rocks below the surface subject to three principal stresses, each in compression?” In fact, as a rock is buried more deeply it is subject for greater compressive stresses. One solution is found by considering a rock mechanics experiment called a Brazilian Test which is accomplished by loading a disc of rock between parallel plates. A contact load is developed along a line at the top and bottom of the disc.

If we imagine grains loaded at the depth of 3.5 km and if the overburden is supported by a line load as is found in a Brazilian Test, a tensile stress of 60 MPa is developed along the vertical diameter of the grain. This is the primary mechanism by which microcracks will develop from even smaller flaws in the crystalline lattice of the grain (Jaeger and Hoskins, 1966).
Aside from grain-grain contacts, there are a number of mechanisms, many depending on elastic contrasts between rock volumes, that allow for the generation of absolute tension. Examples include inclusions which depend on the relation between the modulus of the inclusion and the modulus of the matrix (Kemeny and Cook, 1987).
Concretion with internal cracks
Pennsylvanian Llewellyn Formation, Bear Valley, Pennsylvania, USA
Absolute Tension Required in Initiate the Propagation of Flaws

Crack-tip stress Intensity
mode I loading

\[ s = \text{stress} \]
\[ Y = \text{shape} \]
\[ c = \text{half length} \]

Solving for driving stress

\[ \Delta\sigma = \frac{K_{lc}}{Y\sqrt{\pi c}} \]
Various rocks had different tensile strengths. However, when fracture toughness $K_{ic}$ was measured for each of these rocks, it was discovered they all had roughly the same $K_{ic}$. The difference in tensile strength tests is that the different lithologies had different flaw size ranging from a very porous sandstone to a very fine-grained diabase.
Vertical fluid inclusion tracks representing healed microcracks
Milford Granite, New Hampshire, USA

Vertical microcracks in a quartz grain within granite. These are now tracks of fluid inclusions. The initial flaws in granite that allowed the growth of microcracks under the weight of overburden were on the order of 20-50 μm (see slide on Brazilian test).

Once microcracks are present in a rock, then tensile stresses less than, say, 10 MPa will be sufficient to allow the growth of larger joints and other fractures (white dashed box).
Fractography of Glass & Ceramics

- Four general classes of crack surface morphology
  - Initiation point
  - Mirror
  - Mist
  - Hackle

Glass rod fractured in tension
Fractography of Glass & Ceramics

- Initiation point
- Mirror
- Mist
- Hackle

In-plane propagation
Out-of-plane propagation
Subcritical Propagation Velocity

- $K_I = \text{stress intensity (a measure of stress at the crack tip)}$

- Subcritical propagation at initiation
  - $K_I < K_{ic}$

- Critical propagation
  - $K_I = K_{ic}$
Post Critical Behavior

Branching = Crack leaves its plane in irregular manner

Terry Engelder
2.1.3 – Joint Surface Morphology

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Twist Hackle is NOT Crack

Branching: These fringe cracks are NOT indicative of high-speed propagation

By analogy the surface of joints and the surface of fractured ceramics are the same. Inferences about joint initiation, propagation and arrest are based on surface morphology

Kulander, et al., 1979
Fracture Plume = Mirror and/or Mist

Hodgson, 1961
Blast Fracture (*high velocity*)

- Initiation
- Branching
- Hackle
- Mirror
- Ordovician Dolomite

no stress guide at crack tip
Miocene Chalk, Israel
Joint initiation at a vertical burrow
initiation point = borrow (Devonian, Appalachian Plateau)
initiation point = small concretion (Devonian Appalachian Plateau)
Hesitation line (Devonian siltstone, Appalachian Valley and Ridge)
The Joint

Progressive rupture with multiple arrests: Appalachian Plateau
multiple hesitation lines (mudcrack, Gulf of Suez, Egypt)
Rib marks = hesitation lines (siltstone, Appalachian Valley and Ridge)
rib marks = termination of large joint (Appalachian Plateau)
blast fracture (Ordovician dolomite)
Crack branching = critical propagation (Appalachian Plateau)
Crack branching = critical propagation (Appalachian Plateau)
2.1.4 - Initiation of Joints

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
From the previous lesson we have learned that an unreasonably large crack driving stress is necessary to trigger and then sustain joint propagation from the pervasive and open but very small in situ flaws (i.e., grain-boundary cracks and pores) in clastic rocks. A larger structure is required to magnify the remote stress and, thereby, initiate crack propagation from these open flaws that are otherwise too small for crack growth under geologically reasonable crack-driving stresses. We know that larger structures are required from outcrops on the Appalachian Plateau, New York, where inhomogeneities such as concretions, fossil fragments, ripples and flute casts are the loci of many initiation points (IP). Their clear association with joint initiation is ample evidence that sedimentary structures act to magnify the remote stress at the onset of joint propagation.

Pores and other flaws in sandstone and particularly shale are, generally, too small to allow a stress concentration sufficient for

\[ K_I \geq K_{IC} \]

How does tensile stress from macroscopic concentration points superimpose on microscopic flaws and pores in clastic rocks?

Joint initiation requires

- microscopic flaws
- primarily pore space
- macroscopic structures
- stress concentration points

McConaughy & Engelder, 2001
Shape factors ($Y$) for some microscopic flaws in sedimentary rocks

**SOME IDEALIZED PORE GEOMETRIES**

- **Pore around Kerogen Flake**
  - *Penny-shaped crack*
  - $Y = 1.13$

- **Pore between Shale Clay**
  - *Flat tunnel crack*
  - $Y = 1.77$

- **Pore within Sandstone**
  - *Sphere with edge crack*
  - $Y = 2.98$

- **Pore Throat**
  - *Cylinder with edge crack*
  - $Y = 3.98$

McConaughy & Engelder, 2001
Because the microscopic stress concentration points require an unrealistically high tensile stress, joint initiation in sedimentary rock must operate from a larger stress concentration structure. Indeed, joints don’t initiate spontaneously with matrix but rather grow from larger structures around which a tension of a few MPa can be magnified to trigger propagation from microscopic flaws usually associated with pore space.
initiation at a vertical borrow
initiation at an internal structure = small concretion
initiation at an internal structure = fossil
initiation at an internal structure = shale lamination
initiation at a bedding boundary = flute cast
initiation at a bedding boundary = flute cast
initiation at a bedding boundary = gutter cast
Initiation at ball and pillow

Joint Development in Ball-and-pillow Structure

Concretion

Propagation of crack front

McConaughy & Engelder, 2001
initiation at a bedding boundary = soft sediment structure
initiation at a bedding boundary = soft sediment structure
initiation at a bedding boundary = flat boundary
initiation at a bedding boundary = irregular boundary
joint initiation across bed boundaries
Asymmetrical Groove Cast with Frictional Interface

Model Geometry and Boundary Conditions
Models using a frictional interface between the two beds generate stress concentrations near vertically oriented bedding surfaces, locations that field evidence shows are associated with joint initiation. Pore fluids at pressures in excess of the least principal stress, acting against non-horizontal surfaces of a bed form, may provide the additional drive for joint initiation. In mechanically coupled siltstone beds, a modest joint-tip stress concentration across a shale layer (e.g., $3 \times 10^6$ for a 1 cm thick bed) permits a competition between sedimentary structures and pre-existing joint tips to initiate new jointing.
Three categories of sedimentary structures that concentrate stress necessary for primary joint initiation include: bedforms; trace fossils; and other IP including inclusions and soft-sediment deformation structures. Initiation at irregular boundaries, such as bedforms or ball-and-pillow structures, commonly occurred at a region of sharp curvature on the boundary. A sharp transition from a horizontal bed boundary to a vertical surface on a bedform is the favored spot for an initiation point. In instances in which a linear bedform intersects the plane of a joint at an angle (i.e., non-orthogonal) the local fracture surface at the IP is typically misaligned with the overall orientation of the joint. This indicates a strong geometric component to the mechanism responsible for the stress concentration.

Fracture Initiation Points

- bed forms:
  - groove casts, flute casts, gutter casts
- trace fossils:
  - worm burrows
- inclusions:
  - concretions, pyrite nodules, fossil shells
- soft sediment deformation structures:
  - ball and pillow structure

Fracture Initiation Points: Location within Bed

- bed boundaries:
  - top of bed
  - bottom of bed
- middle of bed
- end of bed

McConaughy & Engelder, 2001
2.1.5 – Propagation of Joints

An AAPG Short Course by
Terry Engelder
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The Pennsylvania State University
CRACK-WALL DISPLACEMENT (PENNY-SHAPED CRACK)

Tearing

Simple-Shear Loading

Sliding

Opening Mode

Tensile Loading

Shearing Mode

MixeD-Mode Loading
Crack growth: Driven by gas generation from coaly plant material
Middle Devonian Geneseo/Burket gas shale, Howard, Pennsylvania, USA
Growth of a Penny-shaped crack
Ordovician Bellefonte Dolomite, State College, Pennsylvania, USA
Growth of a Penny-shaped crack
Martinsburg Slate, Slatedale, Pennsylvania, USA
The principal stresses

- tension
- compression

Stress on planes NOT parallel with principal stresses

Stress Around a Crack

absolute magnitude of 2 principal stresses equal

45°
Mixed Mode Crack Path

Maximum Circumferential Stress

Mode I crack-tip displacement

Mixed-mode crack-tip displacement

Mode II crack-tip displacement

The principal tensile stress

local propagation direction always normal to mode I displacement
Mixed Mode Crack Path

Crack-Tip Strain

Mode I Crack

non-linear zone

θ₀

Sᵥ < Sᵰ

Sᵥ > Sᵰ

Sₘᵣₐₓ

Distortion

Volume Change

Sₘᵢₙ
Angular Variation in Strain Energy Density Near a Crack Tip

Pure Mode I

Combined Mode I–II

$K_1 = K_{II}$

Pure Mode II
Mode II Loading

\[ \sigma_{xx} = \frac{K_{II}}{\sqrt{2\pi r}} \left\{ -\sin\left(\frac{\theta}{2}\right) \left[ 2 + \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right) \right] \right\} \]

\[ \sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \left\{ \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right) \right\} \]

\[ \tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \left\{ \cos\left(\frac{\theta}{2}\right) \left[ 1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] \right\} \]

\[ \sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \]

\[ \sigma_{zz} = 0 \]

\[ K_{II} = Y \tau_{xy}^r \sqrt{\pi c} \]

\[ \tau_{xz} = \tau_{yz} = 0 \]
Kinks

wing crack

low stress

high stress
wing crack

low stress
MODES OF CRACK DISPLACEMENT

- Mode I Opening
- Mode II Sliding
- Mode III Tearing
Mode III Loading

\[ \sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0 \]

\[ \tau_{xy} = 0 \]

\[ \tau_{xz} = \frac{K_{III}}{\sqrt{2\pi r}} \left\{ -\sin\left(\frac{\theta}{2}\right) \right\} \]

\[ \tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \left\{ \cos\left(\frac{\theta}{2}\right) \right\} \]

\[ K_{III} = Y\tau_{xz}^r \sqrt{\pi c} \]
Tilt versus Twist

Mixed Modes I&II
- Tilted fringe crack (A kink)
- Parent joint
- Surface morphology
- Parent joint tip lines

Mixed Modes I&III
- Segmented fringe crack
Mixed-Mode Crack-Tip Stress Field

\[
\sigma_{\theta\theta} = \frac{\cos^2\left(\frac{\theta}{2}\right)}{\sqrt{2\pi r}} \left[ K_I \cos\left(\frac{\theta}{2}\right) - \frac{3}{2} K_{II} \sin \theta \right]
\]

\[
\sigma_{rr} = \frac{\cos\left(\frac{\theta}{2}\right)}{\sqrt{2\pi r}} \left\{ K_I \left[ 1 + \sin^2\left(\frac{\theta}{2}\right) \right] + \frac{3}{2} K_{II} \sin \theta - 2 K_{II} \tan\left(\frac{\theta}{2}\right) \right\}
\]

\[
\tau_{r\theta} = \frac{\cos\left(\frac{\theta}{2}\right)}{\sqrt{2\pi r}} \left[ K_I \sin \theta + K_{II} (3 \cos \theta - 1) \right]
\]
2.1.6 – Stress Trajectories in an Elastic Earth

An AAPG Short Course by Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Vertical cracks propagate parallel to trajectory of $S_H$ (Irwin, 1957).

$S_H =$ maximum horizontal compressive stress
$S_h =$ minimum horizontal compressive stress

Stress field orientation varies in space = local stress

Ship Rock, New Mexico
Dikes (i.e., cracks) follow tectonic stress field
Spanish Peaks, Colorado, USA

The radial dike pattern around the Spanish Peaks (adapted from Johnson, 1961). This pattern is a cross section in plan view of the volcano. The architecture of a volcano showing the orientation of dikes and eruptive fissures in a stress field where $S_H > S_h$ (adapted from Nakamura et al., 1977). Note the tendency of the dikes to rotate into the direction of $S_H$ after propagating from the main feeder of the volcano.
Dikes (i.e., cracks) follow tectonic stress field
Aleutian Arc, Alaska, USA

Map of Alaska showing the orientation of eruptive fissures (black lines) on volcanoes of the Aleutian Arc (adapted from Nakamura et al., 1977). The direction of convergence of the Pacific Plate relative to the North American Plate is shown with open arrows. Other features include the Aleutian Trench (dashed line), the 500 m below sea level depth contour (dotted line), and lines of longitude (straight lines). Detailed maps of three volcanoes show the orientation of cinder cones (dots) associated with eruptive fissures.
At Robin Hood's Bay, a model based on the perturbation of the stress field due to stress concentration at points of contact along fault planes can help explain the observed joint patterns. In the proposed model, the local direction of the maximum horizontal stress is orientated towards each point. Joints follow the stress trajectories, and so converge at the points of contact. In places where there are closely spaced faults with adjacent points of convergence, the final joint pattern is the result of a complex superposition of the joints converging towards each point.

Stress point concentration model for the perturbation of the stress field during the formation of set J2, Robin Hood's Bay. The regional direction of the horizontal stress is 145°.

Movement on the fault produces a second point of contact to the north. The stress trajectories converge at the second point.
Fault #5

Stress concentration point on fault #5 (red arrow) showing a counter-clockwise rotation in the regional stress field

Joint sequence near rollover folds
Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom
Joint sequence near rollover folds
Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom

Engelder & Peacock, 2001
Joint sequence near rollover folds
Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom

Stress concentration point on fault #5 (red arrow) showing a counter-clockwise rotation in the regional stress field

Bed 3883

Engelder & Peacock, 2001
A stress field rotation of about 120°

Veins can record a complex stress history:

**Fractures** create elastic discontinuities

**Veins** return the rock to an elastic continuum

Devonian New South Wales, Australia
Later joints abut earlier joints
Entrada Sandstone, Arches National Park, Moab, Utah, USA
In a study of the joints in the Entrada Sandstone in Arches National Park, Utah, Dyer (1988) observed that, on approaching older joints, younger joints follow one of two distinct propagation paths. Younger joints rotate to become either parallel or perpendicular to the older joint. The remote stress field changed orientation after propagation of a primary joint set. Upon realignment of the remote stress field, a second set of joints propagated from the mid region between the primary joints. In the mid region, the younger joints propagated normal to the remote $S_h$ in its new orientation. The change in orientation of the younger joints near the primary joints indicates a local rotation of the principal stresses in the vicinity of the wall of the preexisting joints.
Cross joints are late-formed nonsystematic fractures that extend across intervals between systematic joints. Traces of such cross joints are seen on bedding-plane surfaces of Devonian Catskill elastic sedimentary rocks of the Appalachian Plateau of western New York State, where the maximum horizontal principal stress ($S_h$) is oriented $-N65^\circ E$, as indicated by in situ stress measurements. Between pairs of closely spaced systematic joints, traces of cross joints are commonly planar and orthogonal to the preexisting joints. However, in the mid-region between some widely spaced systematic joints in western New York, cross joints strike parallel to the Su of the present lithospheric stress field.

Engelder & Gross, 1993
Fossil Crinoids
Stress concentration

Crinoid columnal:
single crystal of calcite

Mechanical twins in calcite

Direction of LPS = Alleghanian Compression

Engelder, 1982
Borehole Breakouts
Four common borehole geometries

BOREHOLE SHAPES

Washout

Key Seat

STRESS CONCENTRATION

BREAKOUT

Caliper Increase
Bit Size

In Gauge Hole

Stress Concentration

Depth

Cal 1-3
Cal 2-4

Plumb & Cox, 1987
Maximum Horizontal Stress Directions

$S_h$ Direction Inferred From
- Borehole Elongation
- Hydraulic Fracture
- Overcoring

Atlantic Coast Stress Province Boundary (Zoback & Zoback, 1980)

- Appalachian Fold Belt
- Grenville & Kenoran Fold Belts
- Coastal Plain
- Water

100 Miles
100 Kilometers

Plumb & Cox, 1987
Stress orientation as measured by hydraulic fracture
Appalachian Basin, USA

Borehole formation microscanner image

Induced fractures after Mini-fracture oriented ENE-WSW.

Hydraulic Fracture Orientations

Engelder, 2016, RPSEA experiment
2.1.7 - Natural Hydraulic Fracturing

An AAPG Short Course by Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Fracture development
Devonian black shales, Appalachian Basin, USA

Early studies of joints in the Devonian section of the Catskill Delta Complex showed that black shales carried an unusually high density of joints in two orientations (Sheldon, 1912, Engelder, 1982, Engelder et al., 2009). This association led eventually to the conclusion that high pore pressure from hydrocarbon maturation was capable to driving natural hydraulic fractures.

Plot of joint density (number of joints per unit length of core) within various Devonian formations of the Appalachian Basin (black shale units are shaded). Data are from shale core drilled as part of the Eastern Gas Shales Project (EGSP) and published in a report from Cliffs Minerals, Inc. (1982).

Box-and-whisker diagrams representing the joint-spacing distribution for two joint sets (i.e. one crossfold set and the 070° set) across the transition from the black shale to gray (light blue arrow) shale. Small spacing equals high joint density.
Cyclic propagation of joints
Upper Devonian Ithaca Fm., Appalachian Basin, USA

Bahat & Engelder, 1984
Secor (1965, 1969) observed that the plumose morphology on the surface of joints within the Devonian Catskill Delta Complex indicated a cyclical propagation cycle. Each cycle was marked by a fan-shaped pattern which increased in surface roughness until abruptly ending in a semi-circular tip. The length of the propagation cycles had approximately the same dimension as the bed thickness. In this example that siltstone bed is about 10 cm thick and the propagation cycles are each about 10 cm.
Cyclic propagation of joints
Upper Devonian Ithaca Fm., Appalachian Basin, USA

Propagation direction is right to left

Watkins Glen, NY: This is an example of a 44 cm thick bed with propagation increments concomitantly larger.

Lacazette & Engelder, 1992
Catskill Fm., Pine Creek, PA: Here is a bed > 2 m in thickness with propagation increments arresting after > 2 m of propagation. Again, each increment starts as a relatively smooth surface and eventually become rougher before the propagation cycle is arrested. This smooth to rough surface is an important element in the interpretation of propagation velocity (Savalli and Engelder, 2005).
Catskill Fm., Pine Creek, PA: Another observation that became important was that the joints always propagated in sandstone layers which were contained between shale beds. These rocks were channel sands in a fluvial system that was punctuated by overbank mud deposits. Initially the beds were red but those rock layers with joints of cyclic propagation were green relative to the red overbank muds (now shale).
Catskill Fm., Pine Creek, PA: The plume morphology on joint surfaces is subtle and requires that the sun illuminate the joint surface at a low angle to accent the morphology (Kulander and Dean, 1995).
Catskill Fm., Buttonwood, PA: when joint propagation starts a relatively smooth surface is produced.

Tan et al., 2014
Catskill Fm., Buttonwood, PA: While this sandstone appears red, the color is from mud washing across the joint surface. A fresh piece of this rock has a green color.
Catskill Fm., Buttonwood, PA: This joint surface gives an idea of the evolution in surface roughness just before the propagation cycle is arrest. This is the same surface as in the previous slide.

Tan et al., 2014
Cyclic propagation of joints
Upper Devonian Ithaca Fm., Appalachian Basin, USA

Propagation cycles for one joint cutting a 44 cm thick bed. 68 cycles appear over a propagation length of 28 m.

Lacazette & Engelder, 1992
Cyclic propagation of joints
Upper Devonian Ithaca Fm., Appalachian Basin, USA

Lacazette & Engelder, 1992
Cyclic propagation of joints
Upper Devonian Ithaca Fm., Appalachian Basin, USA

Lacazette & Engelder, 1992
Cyclic propagation of joints
Upper Devonian Ithaca Fm., Appalachian Basin, USA

Cyclic propagation increments increase in length with the last of 68 cycles being one of the longest.

Termination of the Joint
Lacazette & Engelder, 1992

INCREMENTAL PROPAGATION (ITHACA SILTSTONE)
Why does a coke fizz when opened?

Gas Law: $PV = nRT = \text{constant}$
PSU lab experiment:
plexiglass cylinder hydraulically
fractured from internal hole
discrete propagation events
What is the cause of incremental fracturing?
What is the cause of incremental fracturing?

PSU lab experiment:
plexiglass cylinder hydraulically fractured from internal hole

discrete propagation events

Gas Law:
\[ PV = nRT = \text{constant} \]
The regularity and rhythmic nature of several types of fractures suggest that cyclic propagation also arises from dynamic instability of the fracture-fluid-rock system. A 40-90-m-long cross-fold joint that propagated within a single bed of the Devonian Ithaca Siltstone near Watkins Glen, New York has a plumose surface morphology with multiple arrest lines indicating that cracking occurred in increments rather than in one smooth rupture. The crack increments increase in overall length in the propagation direction over the final 28-m portion of the exposed end of the study joint with the largest increments increasing in length from 0.6 m to 1.0 m.
Mechanisms for natural hydraulic fracture

Infiltration-limited propagation is described elsewhere (i.e., Segall and Pollard, 1983). The infiltration-limited model assumes that the driving fluid is incompressible. The rate of infiltration of fluid into the joint from adjacent pore space in the host rock maintains fluid pressure to counterbalance total stress and fracture toughness. Throughout propagation $K_1$ remains at a level where crack growth can occur and join propagation is stable.

Expansion of the fluid within the joint can maintain the crack driving stress even though propagation causes an increase in crack volume. For our compressibility-limited model, propagation distance during a rupture event is a function of the compressibility of the fluid within the joint. Rates of fluid flow into the newly created portion of the joint are equal to or greater than the speed of joint propagation. Thus fluid pressure changes at an equal rate within all portions of the joint at all times during the fracturing process. Arrest occurs because the increase in joint volume will eventually reduce the fluid pressure until the fluid drive is counterbalanced by fracture toughness and the crack stress.

If the crack tip is restrained and is suddenly released, the crack, at least momentarily, becomes unstable. In this case, the crack may outrun the fluid within the crack (Sammis and Julian, 1987). In the most extreme case of a flow-limited model the rate of fluid flow into the propagating joint tip is infinitely slow relative to the rate of joint propagation. A case similar to this is described by Secor and Pollard (1975).

Lacazette & Engelder, 1992
The mechanism for natural hydraulic fractures: The shrinkage fracture model.

Engelder & Lacazette’s Poroelastic Contraction Model for NHF

Poroelastic contraction model for a rock with an initial flaw and constrained by rigid boundaries on all sides. The model consists of grains, elastic grain-grain contacts, an interconnected pore space. Vectors are shown to represent the balance of forces along the interface between the initial flaw and the joint-rock surface.

Miller (1995) expanded on the Engelder & Lacazette (1990) model for poroelastic contraction by using a more complete set of poroelastic equations. Miller points out that vertical (shrinkage) fractures are favored and should occur at lower pore pressures in less compressible rocks buried in basins with lower geothermal gradients.
After Lacazette and Engelder (1992) used the incremental propagation of joints to argue that this particular surface morphology was a manifestation of natural hydraulic fracturing, the research team at Penn State sought independent evidence supporting the NHF interpretation. That evidence came in the form of joints interacting with concretions in black shale. Here we see a concretion completely suspended in space when a joint face was exposed by erosion, evidence that the joint did not penetrate the concretion as it propagated around the carbonate body.
Joint cutting around but not through a carbonate concretion
Devonian Rhinestreet black shale, Eighteen Mile Creek, New York, USA

Black shale of the Appalachian Plateau carries more than one joint set. $J_1$ joints predate the major layer-parallel shortening event and $J_2$ joint propagation on the Appalachian Plateau.
The oldest joint set in Devonian rocks of western New York state has an atypical NS strike and predates regionally more abundant NW-striking and ENE-striking joints driven by hydrocarbon-related fluid decompression. The NS joints originated in higher modulus diagenetic carbonate and were driven initially by a different mechanism, either joint-normal stretching and/or thermoelastic contraction. The origin of these joints in higher modulus carbonate concretions indicates the presence of a tensile stress produced by uniform regional extensional strain. Upper Devonian shale hosting the NS joints crops out in that area of the Appalachian Basin where a Morrowan erosional unconformity marks the region of maximum upward lithospheric flexure of an early Alleghanian forebulge. The NS strike of these early joints points to a forebulge stretching axis oriented approximately east-west and associated in time and space with crustal loading that drove both the Northfieldian Orogeny and the underplating of the Bronson Hill Anticlinorium in New England. Ultimately, subsidence of the Morrowan forebulge buried the Upper Devonian shale succession to the oil window during the latter part of the Alleghanian tectonic cycle resulting in the propagation of fluid driven NW- and ENE-trending joints in black shale.
Joint cutting around but not through a carbonate concretion
Devonian Geneseo black shale, Watkins Glen, New York, USA

The problem of the non-penetrating joints was solved by McConaughy & Engelder (1999) using numerical modeling.
Crack propagation path (solid line) and normalized Mode I stress intensity for a joint originating 2 m from the concretion and driven by fluid loading (a natural hydraulic fracture). The stress perturbations associated with the concretion cause the crack to be deflected towards the x-axis. This is accompanied by an increase in normalized stress intensity as the crack-tip moves into a region of reduced compressive stress (caused by the presence of a stiff concretion). Normalized $K_I$ decreases sharply as the crack-tip approaches to within a few millimeters of the concretion, an effect caused by the presence of the interface. This is the explanation for natural hydraulic fractures failing to penetrate concretions where as tensile cracks penetrate into concretions.

McConaughy and Engelder, 1999
Closely spaced joints in a gas shale:
Middle Devonian Marcellus Fm, East Waterford, Pennsylvania, USA

High pressure gas splits gas shale by natural hydraulic fracturing!
These are the high permeability tunnels necessary for rapid production!

Engelder, et al., 2009
Closely spaced natural hydraulic fractures: the mechanism by which hydrocarbons often migrate out of a source rock and toward a reservoir rock. Honda SUV for scale.

**Direction of Reservoir Rocks:** Upper Devonian Sandstones

**Source Rock:** Genesee Black Shale

Engelder, et al., 2009
Cracks driven by elevated fluid pressures are closely spaced as a consequence of the crack-driving stress (i.e., the difference between the crack-normal compression and the fluid pressure in the crack). Various parameters influence the driving-stress distribution around pressurized cracks in layered media, and influence the spacing of natural hydraulic fractures. Numerical experiments show that: (1) crack-driving stress is reduced in the vicinity of pressurized joints, and that the extent of the stress reduction depends on the contrast in elastic properties between the layers; and (2) crack-driving stress distribution depends on the ambient pore pressure during jointing. Both factors lead to a small spacing relative to bed height.

Fischer et al., 1995
Industry hydraulic fractures

Industry hydraulic fracture simulation of conventional reservoirs indicate that stress ($S_{hmin}$) in sandstone reservoirs is often less than $S_{hmin}$ in the enveloping shale.

- Pressure-time curve for driving a hydraulic fracture into a pay zone from wellbore
- Pressure-fracture height curves

Nolte & Smith, 1981
Low stress siltstone fractures before high stress shale just like the industrial hydraulic fracture!

Engelder, 1985
Upper Devonian Sandstones near Dimock, PA:
Vertical fractures in reservoir rocks grow to great heights but widely spaced.
2.1.8 – Hydrocarbon Fluid Migration

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Horizontal bitumen-filled microcracks are common within clay laminae of the finely laminated organic carbon-rich shale in the lower half of a heavily jointed black shale. Horizontal microcracks in a hydrocarbon source rock that carries regional vertical joints indicates that a horizontal least principal stress owes its presence to material properties of the fractured shale and the magnitude and orientation of the crack-driving stress during kerogen maturation. Three material properties favored the horizontal initiation of microcracks: (1) the abundance of flat kerogen grains oriented parallel to layering; (2) a marked strength anisotropy in large part caused by the laminated nature of the rock; and (3) the tight, strongly oriented planar clay-grain fabric produced by gravitational compaction of flocculated clay at shallow-burial depth. The latter was especially important to sustaining elevated pore pressure, the crack-driving stress, which was generated by the conversion of kerogen to bitumen. In addition, poroelastic deformation of the low-permeability laminated shale pressurized by catagenesis elevated the in-situ horizontal stress in excess of the vertical stress, which remained constant during pore-pressure buildup, thereby favoring the propagation of microcracks in the horizontal plane.
Oil migration from initiation microcracks accumulates into larger volumes as shown in these bitumen veins.
Immissibility of bitumen and water in veins and faults
Jurassic Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom
Intergrowth of bitumen and calcite fibers in bedding-parallel veins
Jurassic Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom
Maturation of hydrocarbons is an endothermic reaction (+ ΔV) that maintains fluids in their reduced state. Fe is maintained in solution in a reducing fluid. One of the chief signals that hydrocarbon fluids have migrated through a reservoir rock is that the fluids reduce iron which is carried out of the system in solution (e.g., Fe in the form of red hematite is removed).
Recycling Fracture Flowback!
Green = Archean Ocean
Red = Proterozoic Ocean

One interpretation of this air photograph is that flowback water (small impoundment) is being recycled. When the flowback (the green with gray surfactant floating to the surface) is oxidized by mixing with fresh water, the dissolved Fe immediately precipitates, turning the larger impoundment a rust red color. This is a very modest fracture stimulation for which there may be just one pump truck and one small sand mixer (left of drill).
In outcrop, rock colors that really matter!
(black, gray, red, green)
Redox front moves through Catskill Fm. Sandstone:
Indicative of hydrocarbon (with gas) flow
The low-permeability Mercia Mudstones of the Bristol Channel Basin, southwest England, however, contain bodies of sand that, during the opening of the basin, were injected along some of the hydraulic fractures in the mudstones, preserving them as sedimentary dikes and sills.

Cosgrove, 2001
Field observations indicate that fluid pressures within the Mercia Mudstones were very high during basin inversion and that hydraulic fracturing provided a transient permeability that relieved this excess pressure. The fractures are not visible in most of the mudstones but have been preserved within evaporite-rich horizons as a network of satin spar veins. Thus, the chance preservation of the sedimentary dikes and satin spar veins shows that at different times during the evolution of the basin, fluids migrated through low-permeability units along transient networks of hydraulic fractures.

Cosgrove, 2001
Gypsum vein cut by a normal fault: Migration reducing fluids
Triassic Mercia mudstones, Watchet Beach, Bristol Channel, United Kingdom

Vertical fault allows the invasion of oxidizing fluids which then stain the satin spar veins with a red iron oxide.

Terry Engelder
Hydrocarbon fluids reduce iron oxides

Chevron employee for scale

迁移的烃类流体沿裂隙迁移
Paradox盆地，墨西哥帽，美国犹他州

Hydrocarbon fluids reduce iron oxides

Terry Engelder
Reducing hydrocarbon fluids migrate along a channel sandstone
Catskill Fm., Bottonwood, Pennsylvania, USA

Outcrop example of a 2-meter thick channel sandstone that was the host of hydrocarbon fluids at some point before exhumation. Gamma ray, neutron porosity and density porosity logs through an equivalent sandstone bed in the subsurface which is gas charged (the yellow neutron/density porosity cross over) today. Depth intervals are two feet.
Lock Haven Formation: Pre-exhumation hydrocarbon-fluid charged sandstone layers. Logs same as in the previous slide.
Natural hydraulic fractures are restricted to the green rocks. Presumably these were gas charged and probably of pore pressures well above hydrostatic. Same logs as above.
Hydraulic fractures are contained (stopped) by a higher stress mudstone.
Reducing fluids penetrate one meter of red mudstone
In the absence of $J_2$ joints there is very little (10 cm) penetration of reducing fluids into the red mudstone.
2.1.9 – Characteristics of a Single Joint Set

An AAPG Short Course by Terry Engelder
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parallel joints growing in a rectilinear stress field
growth limited by neighboring joints
Spacing limited by bed thickness
Fracture Spacing Ratio (FSR)

FSR << 1

bed thickness to joint spacing
uniform: more joints where aperture is smaller
wider spacing = larger aperture
Characteristic of cracks in an elastic material
Effect on Bulk Modulus

reduces bulk rock modulus
Interconnectivity

little to none
Effect on Bulk Rock Permeability

little to none
2.2.1 Veins

An AAPG Short Course by
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Vein Growth

In recent years fibrous syntectonic veins have been recognized to be an important fabric element in rocks deformed particularly at low metamorphic grades. Durney and Ramsay (1973) demonstrated that fibres in such veins have grown by an accretionary process, and that in many cases fibre shapes can be related to the progressive displacement history during vein formation. Durney and Ramsay (1973) and Ramsay (1980) made a major contribution to the understanding of syntectonic vein formation by demonstrating that some fibrous vein infillings develop by repeated increments of microcrack opening followed by sealing of the microcrack by deposition of material from solution. This process, termed “crack-seal” deformation (Ramsay, 1980), occurs where accumulation of elastic strain is followed by brittle failure, release of elastic strain, solution transfer of material to the microfracture site, and deposition therein. Once sealing of the microfracture is accomplished, stresses can once again be transmitted across the region of the initial microcrack, and elastic strains again build up until there is renewed microfracture.

Diagram illustrating the sequence of events involved in fibre growth by the crack-seal deformation mechanism. A. Accumulation of elastic strain ($\sigma = \text{max. principal stress}$). B. Elastic strain release following brittle failure. Solution transfer of material into microcrack, and nucleation of phases on microcrack walls. C. Sealing of microcrack and accumulation of further elastic strain. D. Second microcrack and elastic strain release increment. E. Syntaxial overgrowth of material on microcrack walks to seal microcrack. F. Repeated crack-seal increments build up a fibrous microstructure.
Oriented or “syntaxial” overgrowth (Bathurst, 1975) of phases in microcrack walls may occur if the depositing species is the same as the substrate.

Nucleation and initial growth mechanisms in fluid-filled microcracks. A. Syntaxial overgrowth of grains in the microcrack walls. B. Oriented initial growth due to competition between neighbouring grains having anisotropic growth kinetics. C. Random nucleation and initial growth.

A. Fiber growth sequence. B. Overall vein morphology developed during a displacement history in which principal strain increment directions are coaxial. Arrows indicate fiber younging directions. C. Overall vein morphology developed with changing principal strain directions.

Development of lattice preferred orientation during crack-seal fiber growth by preferential rejoining at each crack-seal growth increment of those grains having fast growth directions parallel to the incremental displacement vector. Bars indicate orientations of fast growth directions.

Cox and Etheridge, 1983
Vein filling is usually derived from local dissolution. Here pressure solution at the edge of a calcite crinoid columnal feeds calcite to local veins the cut normal to the shortening enabled by pressure solution.
Antitaxial vein growth takes place by crack propagation at the interface between the vein of the country rock. Slivers of country rock are incorporated in the vein where fiber growth is not seeded by crystals in the country rock.
Vein opening in tearing mode (mode III) as indicated by tilted fibers
Jurassic Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom
Cross cutting veins allow for analysis of growth sequence
Ordovician Bellefonte Dolomite, Nittany Mountain Syncline, Pennsylvania, USA

Note: When this material was examined in the middle 1990s, it did not exhibit fluorescence and was interpreted as insoluble residue associated with cleavage development. However, bitumen does not exhibit a strong fluorescence, leaving the possibility the dark material is bitumen.
Chevron Folds at Loughskinny, Ireland
Crossing micro-veins
Loughskinny, Ireland
Conjugate shear fracturing – Beach type I veins
Church Floor in Croatia
Conjugate shear fracturing – Beach (1977) type I conjugate veins
Jurassic Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom
Conjugate shear fracturing – Beach (1977) type II conjugate veins
Shear fracturing with minor shear displacement
Jurassic Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom

Veins < 45° to shear fracture

Peacock & Sanderson, 1995
Shear fracturing with minor shear displacement
Jurassic Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom

veins ≈ 45° to shear fracture
Shear fracturing with minor shear displacement
Jurassic Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom

veins > 45° to shear fracture

Peacock & Sanderson, 1995
Bitumen in cross-fold vein
Limestone layer in lowest Marcellus Fm, Canoga, New York, USA
2.2.2 - Transitional-Tensile Fracture

An AAPG Short Course by
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A common perception promulgated in many recent structural geology textbooks is that joints and shear fractures are the end members of a spectrum of brittle fracture types, and that some fractures are a hybrid of both end members (cf. Hancock, 1985). Such fractures presumably open as tensile cracks and simultaneously propagate in the same plane while subject to a shear traction. As an example, Price and Cosgrove (1990) show a vein for which "it is clear that the planar fracture developed in hybrid extension and shear failure".

One model for the development of hybrid shear fractures is transitional-tensile fracture propagation, a process described as the in-plane propagation of a crack subject to a shear traction while held open by a tensile normal stress. Presumably, such propagation leads to a brittle structure that is the hybrid of a joint and a shear fracture. Crack-seal veins with oblique fibers are possible candidates. While these veins clearly show shear offset, this is not conclusive evidence that a shear traction was present at the time of initial crack propagation. Many recent structural geology textbooks use a parabolic Coulomb-Mohr failure envelope to explain the mechanics of transitional-tensile fracturing. However, the laboratory experiments cited as demonstrating transitional-tensile behavior fail to produce the fracture orientation predicted by a parabolic failure envelope. Additional attempts at verification include field examples of conjugate joint sets with small acute angles, but these conjugate joints form neither simultaneously nor in the stress field required by the transitional ± tensile model. Finally, linear elastic fracture mechanics provides strong theoretical grounds for rejecting the notion that individual cracks propagate in their own plane when subject to a shear traction. These observations suggest that transitional-tensile fracture propagation is unlikely to occur in homogeneous, isotropic rock, and that it is not explained by a parabolic Coulomb-Mohr failure envelope as several recent structural geology textbooks have suggested.

A sketch of a vein with oblique fibers that Price and Cosgrove (1990) offer as an example of hybrid extension and shear fracture, running laterally into a shear zone. The eraser head of a pencil is shown for scale.
Transition tensile fracture
Jurassic Limestone, Brijuni, Croatia
Strain during development of en echelon veins
Blue Lias, Bristol Channel, Lilstock, United Kingdom
Veins in a Transitional-tension mode of failure
South Coast of New South Wales, Australia
Transitional-tensile failure by en echelon crack growth
Conway Granite, New Hampshire, USA
Brittle shear failure by linking en echelon cracks
Tensleep Sandstone, Wyoming, USA
The stress concentration at the tip of a crack showing contours of stress intensity near the tip (adapted from Reches and Lockner, 1994).
Fault inclination $\theta$ (solid line) predicted by the mutual enhancement cracking model and the intensity of the induced tensile stress (dashed line) along the M curve (adapted from Reches and Lockner, 1994). Data from the Mount Desert Island granite (Engelder, 1989) are plotted as a test of the stepped-crack theory for the development of hybrid shear fractures.
Predicted Fault Inclination
(degrees from $\sigma_1$)

Induced Tension
(% of internal pressure)

Crack Spacing/Crack Length (S/L)

Engelder, 1989
Three examples of Coulomb-Mohr failure envelopes as redrawn from Handin and Hager (1957) for the Green River Shale, and from Price (1958) for the Pennant and Darley Dale sandstones.
The Coulomb-Mohr failure envelope for diabase drawn from seven experiments at various confining pressures (adapted from Brace, 1964). Experiments 6 and 7 were subjected to stresses that should have produced transitional-tensile fracture propagation. Fracture angles of 18° and 13° were predicted whereas the samples failed at angles of 5° and 0°, respectively.
• Laboratory configurations for brittle fractures

Ramsay & Chester, 2004
Fracture angle as a function of confining pressure

Ramsay & Chester, 2004
2.2.3 Shear Failure

An AAPG Short Course by
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Borehole failure was a significant problem as industry drilled deeper following the second world war. It was presumed that failure was in compression. Years before a criterion for brittle failure in compression was proposed by the French engineer, Coulomb (1773). The Coulomb failure criterion is empirical requiring several rock deformation experiments at different \( \sigma_d = (\sigma_1 - \sigma_3) \) conditions. From these experiments the failure of a certain type of rock can then be predicted largely because \( \sigma_d = f(\sigma_3) = f(\text{confining pressure}) \). The criterion contains no information on the mechanism or the microscopic process involved in rock fracture. Interestingly, the Coulomb criterion was formulated about 1770 and more than 150 years passed before the actual process of rock failure was understood. About 1920 an engineer named Griffith first developed a mechanistic failure criterion which could be applied to rocks. Griffith's theory was originally formulated to explain fracture in glass.

Griffith's (1924) criterion was based on the observation that glass is not internally homogeneous but rather contain pores. The same can be said for rocks. Pores in rocks can be original spaces between grains that did not completely fill with cement or they can be microcracks that develop when the rock is stressed. These voids play an important role in rock failure directly through weakening the rock and indirectly through hosting pore water which under pressure can also affect the rock strength. The effect of voids on the elastic properties of a rock can include: 1.) The Young's modulus (E) of a rock containing voids and cracks is less than the intrinsic E of a solid rock (an intrinsic property refers to that property in a body without flaws or cracks); 2.) The Poisson's ratio (v) of a rock containing cracks is less than its intrinsic u; 3.) cracks give rocks a different modulus during loading and unloading (this difference in behavior is known as hysteresis).

The distinction between failure in tension and failure in compression is significant. Griffith's criterion gave a convenient physical explanation for crack propagation when the crack (2c) is subject to tensile stresses. The crack propagates in the plane normal to the tensile stress (T) or least principal stress (F/a = T = -\( \sigma_3 \)). However, we have learned that whole-rock stresses including \( \sigma_3 \) increase with depth in the crust of the earth. Yet, rock fracture in shear depends on microcrack propagation where the propagation is caused by tensile stresses. Where are the tensile stresses generated when rock is deep within the crust of the earth? The answer is that on a microscopic scale grains press against each other at sharp contacts. These contacts are called "stress risers". Under these microscopic contacts very large tensile stresses can be generated even though the whole-rock stress in highly compressive.
Microcrack Growth
Three gains in a rock

Two discs pressing into each other is an example of a stress riser. The point contacts of each disk serve to concentrate the stress which was distributed over the end plattens. The increase in stress is shown by the circular lines originating from the point contact. These lines represent lines of equal stress difference. The numbers indicate that the stress difference increases as the point contacts are approached. Another characteristic of point contacts is that a portion of the volume under the point contact is in a state of tension and this tension produces Hertzian cracks under the point contact. In addition, loading discs in this fashion, called a Brazilian test, will cause a tensile failure and microcrack growth (i.e., an axial crack near the central portion of the disc.

When the rock is compressed under high $\sigma_d (= \sigma_1 - \sigma_3)$ conditions, many of these microscopic contacts are activated and microscopic crack propagation is common. As the population of microcracks increases a zone of weakness develops near but not along a plane of maximum shear stress ($\tau_{\text{max}}$). This zone of microcrack intersection eventually hosts the through-going shear fracture.

Shear fractures do not form at 45° to $\sigma_1$ and in the plane of $\tau_{\text{max}}$ but rather in a plane whose normal is closer to $\sigma_3$ than $\sigma_1$. The reason for this behavior is found in a closer examination of the Coulomb criterion where

$$\tau = S_0 + \mu_0 \sigma_n$$

where $\sigma_n$ and $\tau$ the normal and shear stress on the plane of failure respectively and $S_0$ is the cohesive strength of the rock. $\mu_0$ is the angle of internal friction

$$\mu_0 = \tan \phi$$
Tan $\phi$ can not be measured directly but, rather, is derived from the slope of the Coulomb failure envelop. $\mu_0$ should be distinguished from the coefficient of sliding friction ($\mu$) which relates $\tau$ and $\sigma_n$ during slip of a fault.

$$\mu = \frac{\tau}{\sigma_n}.$$  

$\mu_0$ predicts the angle of shear failure

$$\mu_0 = \frac{(\tau - S_0)}{\sigma_n}.$$  

A graphical plot of $\tau$, $\sigma_n$, and $(\tau - S_0)/\sigma_n$ shows that shear failure occurs on the plane where $(\tau - S_0)/\sigma_n$ is maximized.
Shear fracturing requires a high differential stress. Such stresses are not common within the crust. Places where $\sigma_1 - \sigma_3$ may be high include:

**Fault zones** -- Along fault zones asperities which interlock can act as local stress concentrators. In fact, studies of earthquakes show that many faults lock and then slip violently. Asperities cause the locking and more often than not the earthquake is a manifestation of the asperity being sheared off.

**Folds** -- Rocks can act like elastic plates which bend. It is known from the theory of elasticity that part of the bent plate is compressed. In this zone of compression within folded rocks compressive stresses can become so large that the shear strength of the rock is exceeded.

**Man-made structures** -- Pillars in mines often have to bear loads many times that of neighboring rock left undisturbed by mining. Mine shaft openings also act as stress concentrators in much the same manner as microscopic point contacts within rock. The stresses around holes in rock can be many times as large as was found in undisturbed rock.

There are several rock structures associated with shear fracturing. Cataclastic rock is the product of concentrated microfracturing within a shear zone. Cataclasis is the process of mechanical reduction of grain size by brittle fracture and rigid grain rotation. Microfractures develop across the shear fracture in the direction of $s_1$. Continued slip on the shear fracture cause more microfracturing which leads to a wider zone of cataclasis. Comminution is another term applied to the mechanical reduction of grain size. As the pressure on the rock increases, tension gashes form the the shear zone becomes semiductile. Tension gashes are filled macrocracks or extension fractures. The origin of extension fractures is the subject of a future lecture. Repeated shear fracturing causes deformation bands. These are zones of cataclastic material most commonly observed in sandstones. Deformation bands occur because each successive shear fracture or zone of cataclasis is permeable to cements which make the zone stronger than surrounding sandstone. A successive shear fracture must then rupture intact rock or cemented fracture and is not focused by a previous plane of weakness.
Geological structures range across the spectrum from brittle fracture to ductile flow. The crust of the earth may also be divided in the same fashion with the **schizosphere** being that part of the outer crust where confining stress governs crustal behavior (Scholz, 2002). Below is the **plastosphere** where temperature governs behavior virtually independent of confining stress. Throughout this range of behavior, but particularly in the schizosphere, crack propagation has a universal presence. Crack propagation is governed by the elasticity properties of the rock.

A laboratory experiment in plexiglass where a ductile shear zone is about to develop with en echelon cracks guiding the evolution of this shear zone. In cross section the crack opens most in the center and narrows to a sharp tip. In plan view, the crack grows as a circular, ‘penny’, hence the name, penny-shaped crack.
Growth of a fault within an intact rock

Reches & Lockner, 1994
Growth of a fault within an intact rock

σ₁
fracture
larger flaw

σ₃

σ₁
failure plane

σ₃
σ₃

σ₁
failure plane

σ₃
Shear failure of poorly lithified sandstone is one of the unambiguous examples of shear failure in the crust, probably because of the inherent weakness of such ‘rocks’. First named, ‘braided shear fractures’ by Engelder (1974), they were renamed ‘deformation bands’ by Adyin and Johnson (1978) and that is the name that stuck in the literature.
Deformation Bands with Reidel Shears
Devonian, New South Wales, Australia

Schematic of reidel shears relative to major slip surface (after Logan et al., 1979)
Concentrated slip zones bound subsidiary shears at three scales within the Cerro Brass Fault, which cuts Cambrian dolomites of the Nittany Anticlinorium, Pennsylvania. On the outcrop scale thrust faults along both Cerro Brass fault zone boundaries acted as concentrated slip zones bounding subsidiary shears developed along original bedding planes. The outcrop scale subsidiary shears are zones of concentrated slip bounding hand specimen-scale subsidiary shears. In turn, the hand-specimen-scale subsidiary shears are concentrated slip zones bounding even smaller-scale subsidiary shears. Subsidiary shears in Cerro Brass fault zone are analogous to subsidiary shears (i.e. R_1 and R_2) commonly found in laboratory gouge-friction experiments. The orientations of subsidiary shears at the outcrop and hand-specimen scales define a ‘Riedel within Riedel’ geometry in which the original bedding played the role of R, shears at the outcrop scale and, at the same time, operated as boundary faults at a smaller scale. The presence of subsidiary shears on more than one scale suggests that the Coulomb failure theory is not sufficient to explain their origin.
Many studies have shown that displacements in brittle fault zones are primarily accommodated within well-defined, narrow layers of cataclasite or gouge, which are bounded by zones of damaged host rock pervaded by fractures and breccia (e.g., Sibson, 1979). The cataclasite or gouge layers represent the fault surface, the site where the majority of displacements are accommodated. The damaged zones record long-continued deformation from movement on those faults; that is, the damaged zones are a consequence of progressive fracturing from stress cycling caused by continuous or repeated movements on non-parallel or nonplanar faults (Chester and Logan, 1986).
Growth of a fault within an intact rock: A field example

(a) Characteristic fracture patterns

(b) Breccia and damage zone

Damage zone
Marginal part
Fault core
Damage zone
Marginal part
2.2.4 Friction: Critically Stressed Earth

An AAPG Short Course by
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Rock friction is of interest because it controls many processes in the earth's crust including: flexural slip folding; earthquakes; landslides; subduction of continental crust; strike-slip motion of the earth's major plate as they slide past each other. Ultimately it is the frictional strength of the crust that controls the magnitude of differential stress within the earth's crust. Friction is an active mechanism of deformation above the brittle-ductile transition within the earth's crust. It determines the level of shear stress ($t$) required to induce slip along any existing discontinuity.

So far, we made the point that the earth's crust is pervaded by discontinuities, most of which were mode I cracks (extension fractures, joints, tension cracks, etc.). Because of these discontinuities a buildup in earth stress is controlled by friction rather than the intact strength of rocks. In a sense, the generation of fractures, particularly by shear failure, is a local phenomena occurring on relatively small scales. Measurement of in situ stress (i.e. earth stress) by techniques such as hydraulic fracturing shows that the differential stress throughout much of the crust is less than or equal to the frictional strength of local rocks.

In the laboratory friction where normal stress $\sigma_n$ and the shear stress $\tau$ are measured during fault slip, $\mu$, for a single experiment is a the ratio $\tau/\sigma_n$.

\[ \mu = \frac{\tau}{\sigma_n}. \]

If $\sigma_n$ and $\tau$ for several experiments are plotted the line connecting the data points may converge at the origin. In this case $\mu$ is the slope of the line and friction has the same meaning as for the single experiment. But if the line does not converge to the origin, it will cut the $t$ axis at the value $S_0$ which represents the cohesive strength of the fault zone. Here, $\mu$ is the slope of the line but

\[ \mu = \frac{(\tau - S_0)}{\sigma_n}. \]
Laboratory Friction Tests

The frictional strength of rocks comes from laboratory tests. Samples A and B are cross sections of cylinders cut for triaxial testing. A is a sample sliding on a saw cut in a triaxial test. B is a sample sliding on a previously induced shear fracture in a triaxial test. C is a conventional shear test without confining pressure but with a normal stress indicated by the vertical arrow. D is a double shear test with one plate sliding between two other plates. E is a torsion test. In tests C and D the normal stress $\sigma_n$ and the shear stress $\tau$ is measured directly. In the triaxial tests $\sigma_n$ and $\tau$ must be calculated from the principal stresses $\sigma_1$ and $\sigma_3$. For the torsion test the $\tau$ must be integrated over the surface of the cylinder. The advantage of the torsion test over the other four is that infinite (very large) displacement may be achieved during a test.

Friction tests on fault gouge: Cylinder cut in half (l) and jacketed cylinder (r). Engelder Ph.D. samples (Engelder et al., 1975)
Coulomb Failure vs. Frictional Slip

There is a distinction between sliding friction and internal friction that is best illustrated in Mohr Space. Shear fractures do not form at 45° to $\sigma_1$ and in the plane of $\tau_{\text{max}}$ but rather in a plane whose normal is closer to $\sigma_3$ than $\sigma_1$. The reason for this behavior is found in a closer examination of the Coulomb criterion where

$$\tau = S_0 + \mu_0 \sigma_n$$

where $\sigma_n$ and $\tau$ the normal and shear stress on the plane of failure respectively and $S_0$ is the cohesive strength of the rock. $\mu_0$ is the angle of internal friction

$$\mu_0 = \tan \varphi$$

Tan $\varphi$ can not be measured directly but, rather, is derived from the slope of the Coulomb failure envelop. $\mu_0$ should be distinguished from the coefficient of sliding friction ($\mu$) which relates $\tau$ and $\sigma_n$ during slip of a fault

$$\mu = \tau/\sigma_n.$$ 

$\mu_0$ predicts the angle of shear failure whereas the $\mu$ predicts the angle of frictional slip.

Given this stress state (the Mohr circle) and friction envelope any fault with a $2\theta$ angle between 50° and 170° is capable of slipping. However, this is true for only those faults with a strike parallel to the $\sigma_2$ direction. To predict which faults will slip in a 3-D stress state we must employ three Mohr circles (see below).
During laboratory experiments force-displacement curves are recorded as illustrated below. A shows the ideal curve for a sliding friction test. Here the force increases until the rock slips. Once slip is initiated the rock will continue to slip without further increase in the force necessary to maintain slip. B illustrates a more realistic force-displacement curve where there is a slight increase in force necessary to maintain slip. Curves A and B are examples of a behavior called stable sliding. C illustrates a situation where force builds and then suddenly drops. This same cycle is repeated many times throughout the experiment. D is the same behavior measured at the sliding surface. This contrasts with C where displacement was measured at some distance from the sliding surface. The displacement shown in C is a reflection of the elastic distortion of the load frame during force buildup. The rock slips instantaneously with a major drop in force as shown in C. If displacement is measured at the rock sample, it becomes obvious that the force drop accompanies the slip of the rock sample. This behavior is known as stick-slip and is believed to be similar to earthquakes along a major fault zone within the crust.
Coulomb Failure

Stick-slip occurs because unloading of the elastic system which was responsible for the force buildup can not follow the force drop of the rock. The figure below illustrated this behavior. The straight line AB is the uncontrolled unloading of the elastic load frame holding the sample (the earth on either side of a fault zone). Here the earthquake (stick-slip event) starts at point A where the shearing force along the fault zone drops faster than the unloading of the load frame. This creates excess potential energy which must be absorbed by the loading system. Slip on the fault will stop when the available excess potential energy represented by the shaded area below the line AB is absorbed by the extra work represented by the shaded area above the line AB. At point B the load frame must reloaded (shearing stress increased) to the point C before slip starts anew. Stick-slip starts again at point D.
The calculation of the shear stress and normal stress on an arbitrarily oriented fault plane or joint is accomplished using a 3-D Mohr diagram. This diagram includes three Mohr circles which are drawn with the diameters equal to the differences between the three effective principal stresses ($\sigma'_1$, $\sigma'_2$, $\sigma'_3$) which define the ambient stress field. The orientations of all faults and joints are represented by points falling between the largest Mohr circle ($\sigma'_1 - \sigma'_3$) and the two smaller Mohr circles ($\sigma'_1 - \sigma'_2$) and ($\sigma'_2 - \sigma'_3$). The position of an arbitrary geological plane in 3-D Mohr space defines the normal stress and shear stress on that planar feature, a crack, a joint, a shear fracture, a fault, and more. If we know frictional properties of the rock hosting the planar geological structure, then we can plot the frictional strength of that rock as a line predicting frictional slip in Mohr space. Of course, a lower coefficient of friction means that the rock discontinuity will sustain a smaller shear stress at a given normal stress.

To calculate the effective normal stress ($\sigma'_n$) and shear stress ($\tau$), we need to know the angles $\phi_1$ and $\phi_3$ between the normal to the fault and coordinate axes denoting the direction of the principal stresses, $\sigma_1$ and $\sigma_3$, respectively. The point representing $\sigma'_n$ and $\tau$ on the plane in question is found by using the angles $2\phi_1$ (counterclockwise from the center of $\sigma'_1 - \sigma'_2$) and $2\phi_3$ (clockwise from the center of $\sigma'_2 - \sigma'_3$) to define two arcs with their centers of the smaller Mohr circles ($\sigma'_1 - \sigma'_2$) and ($\sigma'_2 - \sigma'_3$). The intersection of the arcs defines the stress acting on the plane.
Critical stress induces frictional slip on faulted joints
Chinle Formation and Glen Canyon Sandstone at Split Mountain, Utah, USA

Geometries and kinematics of brittle deformation features found within sandstone reveal that early subvertical joints confined to jointed beds were subsequently reactivated in shear. Reactivation of the joints with normal dip-slip produced “faulted joints”, which are interpreted to have formed in conjunction with a second set of joints and clusters of pinnate joints. Faulted joints in layered rocks may be identified by their geometric similarities to non-faulted, bed-confined joints: their positively skewed spacing distributions, their lack of conjugate pairs, their consistent terminations at or near discrete bed boundaries, and the absence of significant amounts of fault breccia and/or gouge. Because faulted joints attain considerable length prior to slip, their D/L ratios are initially much smaller than those for primary faults. Furthermore, magnitude of slip is not a consequence of fault growth, and consequently displacement across faulted joints in certain cases may be independent of length.
Critical Stress Controlled by Rock Friction

The relationship between in-situ stress and fluid flow in fractured and faulted rock is dependent on critical stress theory. Barton et al., (1995) show that data obtained from three boreholes that penetrate highly fractured and faulted crystalline rocks indicate that potentially active faults appear to be the most important hydraulic conduits in situ. The data indicate that the permeability of critically stressed faults is much higher than that of faults that are not optimally oriented for failure in the current stress field.

Barton et al., 1995

Normalized shear vs. effective normal stress for hydraulically conductive (left column) and nonhydraulically conductive (center column) fractures based on precision temperature logs (refer to Jaeger and Cook, 1979, p. 28, for details of construction of these diagrams). Open squares in upper left plot show stress state calculated for fractures in Cajon Pass borehole where flow was indicated by direct flow tests. Right column shows lower-hemisphere stereographic projections of poles to fracture planes for hydraulically conductive (solid circles) and nonhydraulically conductive (plus signs) planes.
The Three Elements of Structural Geology

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2.2.5 – The Tapered Wedge

An AAPG Short Course by
Terry Engelder
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A common feature of foreland thrust belts is the presence of a basal surface of detachment or décollement which dips toward the interior of the mountain belt, and below which there is relatively little deformation. The horizon along which the décollement is located is often a relatively weak one. The overall cross-sectional taper of the thrust belt above this horizon has been demonstrated to be related to the relative strengths of the thrust belt and its basal detachment (e.g., Chapple, 1978; Davis et al., 1983; Stockmal, 1983; Dahlen et al., 1984). In essence, such “bulldozer” models state that a relatively weak basal detachment zone can be overthrust by a body of nearly rectangular geometry. But greater basal strength requires that the overthrusting body attain a certain cross-sectional wedge taper, the critical taper ($\alpha + \beta$).

The original idea for a tapered wedge goes to Chapple (1978). His analysis assumed a perfectly plastic material slipping on a frictional base. A plastic model might seem appropriate because the material is simultaneously at failure throughout. In fact, this is the case for the plastosphere where temperature, not confining pressure, controls rock strength. However, the mechanism for failure of earth materials in the schizosphere is not so dependent on temperature as it is on confining pressure. We have established that such materials obey a Coulomb failure criterion and so the tapered wedges of the schizosphere are Coulomb wedges.
When a Coulomb material is at failure, its strength is controlled by friction and resistance to frictional slip increases in proportion to confining pressure. A stack of rocks on an incline can be pushed upward within limits before the push causes the back side to collapse and thicken. The process of thickening adds an additional strength to the backend so the wedge can grow longer until collapse repeats itself at the backend and the section thickens again. This process repeats itself as long as there is space to push the wedge uphill.

Pushing the tapered wedge on a flat or up a slope becomes a force-balance problem just like that used to solve for stresses in a triangular wedge. The push uphill \( (\sigma_x) \) against an element within the wedge is resisted by gravity \( \rho_{\text{rock}} g H \, dx \, \sin \beta \), the weight of water (as given in the diagram), and frictional resistance of the décollement \( \mu_f (1 - \lambda_f) \rho g H \, dx \) where \( \lambda \) is the pore pressure ratio. The orientation of the principal stresses within the wedge is defined by its Coulomb strength. From this force-balance problem the equation for the critical taper is determined to be

\[
(\alpha + \beta) = \frac{(1 - \lambda_f) \mu_f + \beta}{(1 - \lambda) k + 1}
\]

where \( k \) is a measure of Coulomb strength in the wedge closely related to the consolidation coefficient determined earlier and also a function of the coefficient of friction of the décollement, \( \mu_f \).

Davis et al. (1983) tested their critical wedge theory by adding the dip of the basal decollement and the topographic slope for a number of thrust belts around the globe. They came to the conclusion that when surface slopes were measured, they were in good agreement with wedges predicted from actual rock properties. They also found that for pore pressures above ($l > 0.7$), the surface slopes were relatively low (i.e., $< 4^\circ$).

Surface slopes predicted for subaerial and submarine accretionary wedges, assuming $\lambda = \lambda_f$ compared with observed wedge shapes. Davis et al. (1983) note that the wedges for which fluid-pressure data are available are shown with heavy boxes, which are in good agreement with predictions.
3.1.1 Quantifying Strain: Examples of Shortening

An AAPG Short Course by
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Deformation is conveniently separated into three components, of which two are displayed in a one-dimensional analysis. In one dimension there can only be a rigid-body translation (burial) and a stretch (compaction around a concretion is negative stretch). The third component shows up in two and three dimensions where there is an additional deformation known as a rigid-body rotation where the body spins about an axis. Even though this is a 2-D image, the concretion does not ordinarily spin during burial but the shale in the vicinity of the concretion does. For this example we consider the shale at a remote location relative to the concretion.
Strain: Change in length over original length --- \( \frac{\Delta z - \Delta u}{\Delta z} \)

The change in length is commonly referred to as a stretch which can either be positive (longer) or negative (shorter).

\( u = \) displacement in the z direction (unknown)
Strain ($\varepsilon$) is a dimensionless quantity defined as a change in length ($\Delta l$) per unit length ($l$).

$$\varepsilon = \frac{\Delta l}{l}$$

Strain is actually defined near a point by the limiting process of differential calculus.

$$\varepsilon = \lim_{\Delta l \to 0} \left( \frac{\Delta l}{l} \right)$$

Now differential quantities can be introduced by letting $dx = l$ and $du = \Delta l$.

$$\varepsilon = \lim_{\delta x \to 0} \frac{\delta u}{\delta x} = \frac{\delta u}{\delta x}$$

$\varepsilon = \Delta u/\Delta x$

$u = \text{displacement}$
Tectonic Shortening = like compaction it is a negative stretch!

\[ \varepsilon < 0; \quad S < 1 \]

for strain analysis we must keep track of the relative position of two points within a deforming body.
There two techniques for keeping track of what happened to two points within an undeformed body (i.e., the center and edge of a crinoid columnal). First, we can develop a series of equations relating the final resting spot of the point relative to their initial points. Or, we can develop a series of equations describing a vector (red arrow) pointing to the final resting spot from the initial position of a point like center of the crinoid columnal.

The material point in its undeformed position is specified by a vector \( \mathbf{X} \) with components \( X_1, X_2, X_3 \) or \( X_i \). In the new or deformed position the vector \( \mathbf{x} \) has components \( x_1, x_2, x_3 \) or \( x_i \).

### 2-D Deformation Equations:

\[
\begin{align*}
x_1 &= f(X_1, X_2) \\
x_2 &= f(X_1, X_2)
\end{align*}
\]

### A complete deformation includes rigid-translation:

\[
\begin{align*}
x_1 &= a_0 + a_1X_1 + a_2X_2 + a_3X_3 \\
x_2 &= b_0 + b_1X_1 + b_2X_2 + a_3X_3 \\
x_3 &= c_0 + c_1X_1 + c_2X_2 + c_3X_3
\end{align*}
\]

### 3-D Deformation Equations:

\[
\begin{align*}
x_2 &= f(X_1, X_2, X_3) \\
x_3 &= f(X_1, X_2, X_3)
\end{align*}
\]
The constants, $a_0$, $b_0$, and $c_0$ specify the rigid-body translation. If there is no rigid-body translation, then the deformation equations for a **homogeneous deformation** may be written in matrix form:

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

The reason that this is called a homogeneous deformation is that all points $x'_i$ are linearly related to points $x_i$. This is an example of deformation in plane strain where all motion is parallel to the plane normal to the $x_3$ axis.

In contrast a **non-homogeneous** deformation the points $x'_i$ are related to $x_i$ in a nonlinear manner.

$$x_1 = a_0 + a_1\cdot X_1 + a_2\cdot X_2$$

$$x_2 = b_0 + b_1\cdot X_1 + b_2\cdot X_2$$

$$x_3 = X_3$$

Exercise 3.1.1: During the Alpine orogeny a shark’s tooth was deformed in the Swiss Alps according to the following deformation equation. Map the deformed shape of the shark’s tooth using the following equation!
2-D Displacement Equations

The motion of the material point from $X_i$ to $x_i$ is also described by a displacement $u_0$ or $(u_0)_i$. $u_0$, which is a displacement vector, is nothing more than the final position, $x$, minus the initial position, $X$.

\[
(u_0)_1 = x_1 - X_1 \\
(u_0)_2 = x_2 - X_2 \\
(u_0)_i = x_i - X_i \\
\]

$u_0 = x - X$

The displacement $(u_0)_i$ of the point $X_i$ represents a major part of the motion of all points within a rock body. The motion $(u_0)_i$ is called rigid-body translation because it does not describe the motion of particles or rock relative to each other but rather specifies that all particles follow the same path.

In this brief introduction we have only specified how individual points move during deformation. We have not yet considered the relative motion of the points.
In geology kinematics is the description of the path that rocks took during deformation. It is also the mathematical description of the relative position of two infinitesimal points during the deformation of rocks. Two points can change by translating together, rotating around each other, or changing in distance relative to one another. We shall call such a mathematical description deformation mapping.

If during rigid-body translation the particles of rock move relative to each other we must devise other equations to account for their relative motion. To the undeformed state we can attach a line segment $d\mathbf{X}$ whose components are $dX_i$. The study of deformation is concerned with the change in orientation and length of $d\mathbf{X}$ as the point at $\mathbf{X}$ is moved by deformation to the point at $\mathbf{x}$. We say that the vector $dX_i$ is both stretched and rotated to become the new vector $dx_i$. To account for the relative motion of particles within the rock, we consider how the motion $u_i$ of any vector $d\mathbf{X}$ differs from the motion $(u_o)_i$ of the vector $\mathbf{X}$.

If $x_i = f(X_j)$ then $x_i + \Delta x_i = f(X_j + \Delta X_j)$ And by a Taylor’s expansion $\Delta x_i = \left(\frac{\partial x_i}{\partial X_j}\right) \Delta X_j$
Deformation and Displacement Gradient

\( \frac{\partial x_i}{\partial X_j} \) are coefficients called the deformation gradient and are a function of the location within the rock \((X_i)\). The coefficients for the deformation of the shark’s tooth are:

\[
\begin{vmatrix}
\frac{\partial x_1}{\partial X_1} \\
\frac{\partial x_2}{\partial X_2} \\
\frac{\partial x_3}{\partial X_3}
\end{vmatrix} = \begin{pmatrix}
1 & -1 & 0 \\
2 & 2 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

The displacement gradient, \( E_{ij} \), is a function of position within the rock:

\[
E_{ij} = \frac{\partial u_i}{\partial x_j}
\]

These scalar quantities, \( E_{ij} \), are the components of the displacement equations which take the following form:

\[
\begin{align*}
\frac{\partial u_1}{\partial X_1} X_1 + \frac{\partial u_1}{\partial X_2} X_2 + \frac{\partial u_1}{\partial X_3} X_3 &= u_1 = (u_0)_1 + E_{11} dX_1 + E_{12} dX_2 + E_{13} dX_3 \\
\frac{\partial u_2}{\partial X_1} X_1 + \frac{\partial u_2}{\partial X_2} X_2 + \frac{\partial u_2}{\partial X_3} X_3 &= u_2 = (u_0)_2 + E_{21} dX_1 + E_{22} dX_2 + E_{23} dX_3 \\
\frac{\partial u_3}{\partial X_1} X_1 + \frac{\partial u_3}{\partial X_2} X_2 + \frac{\partial u_3}{\partial X_3} X_3 &= u_3 = (u_0)_3 + E_{31} dX_1 + E_{32} dX_2 + E_{33} dX_3
\end{align*}
\]

Deformation is separated into three components: rigid-body translation, stretch and rigid-body rotation.

\[
\text{Stretch} = \epsilon_{ij} = \frac{1}{2} (E_{ij} + E_{ji}) \quad \text{Rigid-body rotation} = \omega_{ij} = \frac{1}{2} (E_{ij} - E_{ji}).
\]
Exercise #2 The first deformation to occur in the history of a sedimentary rock is overburden compaction. This is represented by flattening in the vertical direction with no deformation in the horizontal directions. Compaction around a concretion was used to illustrate this deformation. The following equations represent overburden compaction. Where does the underformed point $X_i = [1,1,1]$ end up after compaction (we say that the initial point is mapped to the final point using the displacement equations)? What is the vector connecting the undeformed point $X_i = [1,1,1]$ to its deformed counterpart? Hint: See figure for deformation!

Displacement equations:

\[
\begin{align*}
u_1 &= 0X_1 + 0X_2 + 0X_3 \\
u_2 &= 0X_1 + 0X_2 + 0X_3 \\
u_3 &= 0X_1 + 0X_2 - 0.5X_3
\end{align*}
\]

Deformation equations:

\[
\begin{align*}
x_1 &= 1X_1 + 0X_2 + 0X_3 \\
x_2 &= 0X_1 + 1X_2 + 0X_3 \\
x_3 &= 0X_1 + 0X_2 + 0.5X_3
\end{align*}
\]
Exercise #3: If the compacted shale from Exercise #2 were turned on its side this deformation could represent a tectonic compaction. The Martinsburg shales near Harrisburg, Pennsylvania, have been isoclinally folded. Beds which were once flat-lying are now standing on end with a dip of 90°. Assuming that the shales were deformed with no internal strain we can use the following equations represent the rigid rotation of a block of shale by 90°. Where does the underformed point \( X_i = [1,1,1] \) end up after rigid-body rotation (we say that the initial point is mapped to the final point using the displacement equations)? What is the vector connecting the shale at point \( X_i = [1,1,1] \) to its rotated counterpart? Hint: See figure for this deformation!

Displacement equations (A vector from the old point to its new location):

\[
\begin{align*}
u_1 &= 0X_1 + 0X_2 + 0X_3 \\
u_2 &= 0X_1 - 1X_2 - 1X_3 \\
u_3 &= 0X_1 + 1X_2 - 1X_3
\end{align*}
\]

Deformation equations (A map of the new point given the coordinates of its old position):

\[
\begin{align*}
x_1 &= 1X_1 + 0X_2 + 0X_3 \\
x_2 &= 0X_1 + 0X_2 - 1X_3 \\
x_3 &= 0X_1 + 1X_2 + 0X_3
\end{align*}
\]
Exercise #4 As a final review of deformation and displacement equations, we shall take a look at the behavior of a fault zone subject to simple shear. In higher grade terrains these fault zones are simply called shear zones. The deformation and displacement equations are given below. Where does the point $X_i = [1,1,1]$ end up after simple shear in a fault zone (we say that the initial point is mapped to the final point using the displacement equations)? What is the vector connecting the fault at point $X_i = [1,1,1]$ to its sheared counterpart? Hint: See figure for this deformation!

Displacement equations (A vector from the old point to its new location):

$$
u_1 = 0X_1 + 0X_2 + 0X_3$$
$$u_2 = 0X_1 + 0X_2 + 0.5X_3$$
$$u_3 = 0X_1 + 0X_2 + 0X_3$$

Deformation equations (A map of the new point given the coordinates of its old position):

$$x_1 = 1X_1 + 0X_2 + 0X_3$$
$$x_2 = 0X_1 + 1X_2 + 0.5X_3$$
$$x_3 = 0X_1 + 0X_2 + 1X_3$$
A formal definition of shear strain ($\gamma$) is the change in angle ($\psi$) between two initially perpendicular lines.

A second measure of shear strain is the tensor shear strain which is half the tangent of the change in angle between initially perpendicular lines.

\[
\text{Tensor shear strain} = \frac{\gamma}{2}.
\]

Shear strain ($\gamma$) is also called engineering shear strain.

The following are useful definitions:

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon = \frac{\Delta l}{l_0}$</td>
<td>(elongation)</td>
</tr>
<tr>
<td>$S = \frac{l_1}{l_0} = (1 + \varepsilon)$</td>
<td>(stretch)</td>
</tr>
<tr>
<td>$\lambda = (\frac{l_1}{l_0})^2 = (1 + \varepsilon)^2$</td>
<td>(quadratic elongation)</td>
</tr>
<tr>
<td>$\varepsilon = \frac{dl}{l_0}$</td>
<td>(infinitesimal strain)</td>
</tr>
<tr>
<td>$\varepsilon = \frac{\Delta l}{l_0}$</td>
<td>(small increment of strain)</td>
</tr>
<tr>
<td>$\varepsilon = \int \frac{l_1}{l_0} \frac{dl}{l_0} = \ln (\frac{l_1}{l_0}) = \ln(1 + \varepsilon) = \frac{1}{2} \ln \lambda$</td>
<td>(natural strain)</td>
</tr>
</tbody>
</table>
Irrotational vs. Rotational Strain

The rotational strain tensor, $e_{ij}$, applies to infinitesimal strains and is a general (or asymmetric) second rank tensor and can be expressed as the sum to a symmetric and an antisymmetric tensor:

$$e_{ij} = \varepsilon_{ij} + \omega_{ij}$$

where

$$\varepsilon_{ij} = \frac{1}{2} (e_{ij} + e_{ji})$$

and

$$\omega_{ij} = \frac{1}{2} (e_{ij} - e_{ji})$$

These simple definitions help us understand the distinctions between simple shear and pure shear. Overburden compaction is a case of pure shear with the deformation gradient matrix:

$$\frac{\partial u_i}{\partial x_j} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.005 \end{bmatrix}$$

The rotational strain tensor, $e_{ij}$, is

$$e_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.005 \end{bmatrix}$$

The irrotational strain tensor, $\varepsilon_{ij}$, is

$$\varepsilon_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.005 \end{bmatrix}$$

The rotational component of strain is

$$\omega_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

For pure shear there is a remarkable similarity between the rotational strain tensor and the irrotational strain tensor. This similarity disappears for the case of simple shear.
Irrotational vs. Rotational Strain

The deformation gradient matrix and the rotational strain tensor for simple shear in a fault zone is

\[
\frac{\partial u_i}{\partial x_j} = e_{ij} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0.05 \\
0 & 0 & 0 
\end{bmatrix}
\]

The **irrotational** strain tensor, \( \varepsilon_{ij} \), is

\[
\varepsilon_{ij} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0.025 \\
0 & 0.025 & 0 
\end{bmatrix}
\]

The rotational component of strain is

\[
\omega_{ij} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0.025 \\
0 & -0.025 & 0 
\end{bmatrix}
\]

The distinction between pure shear and simple shear is further clarified by considering the principal strain axes. If the directions of the principal axes of strain do not change as a result of displacement, then that deformation is termed irrotational strain. In the case of simple shear the principal axes of strain always differ depending on the amount of shear. The difference defines the rotational component of strain which is known as rotational strain. It is important to note that although simple shear is a rotational deformation, there has been no actual rotation in space within a fault zone. The development of rotational strain does not necessarily imply that the body has to spin physically around some axis. Because of this lack of real rotation of within a fault zone, some like to refer to the rotation as an **internal rotation**. This type of rotation is in contrast to our example of **external rotation** where bedded sediments being turned on end as given previously by the displacement equations.
15% Layer-Parallel Shortening
Layer parallel shortening
Appalachian Plateau, New York, USA

Numerical Values = Axial Ratios

Principal Axis Orientation
Calcite Twin Data

Cleavage and Pencil Orientation

Principal Axis Orientation
Crinoid Data

FINITE STRAIN DATA SET
N.Y. PLATEAU

Engelder & Engelder, 1977
Engelder & Geiser, 1979
Geiser, 1988
Disjunctive Cleavage
Onondaga Limestone, Appalachian Plateau, Geneva, New York, USA
Disjunctive Cleavage on edge of Crinoid Columnal
Canadaway Formation, Appalachian Plateau, Wellsville, New York, USA
Layer parallel shortening
Appalachian Plateau, New York, USA

PRINCIPAL FINITE STRAIN TRAJECTORIES
New York Plateau

Rochester

0 30 Km

N Y
Pa.
Iso-strain map prepared from strain data
Appalachian Plateau, New York, USA

LPS Strain - Silurian Salt Isopach Relations
NY Plateau

Geiser, 1988
Deformed Grid
Appalachian Plateau, New York, USA

DEFORMED STATE GRID
NEW YORK PLATEAU

Iso-strain Contour
Deformed State

Iso-strain Contour
Undefomed State

Finite Strain Trajectory \( (1+e_3) \)

Geiser, 1988
Anthracite Coal District

Bear Valley Strip Mine
Bear Valley Strip Mine
(Appalachian Valley & Ridge)

Llewellyn Formation
(Pennsylvanian)

$S_1 < 1.0$

$S_2 = 0.51$ to $0.66$

$S_2 < 0.5$
Tectonic Compaction

- **Tectonic Compaction** is a process that brings about a *layer-parallel shortening*, with a further *decrease in pore volume*. During tectonic compaction there is also a *net change in water content*. The degree of compaction depends on structural position within the orogenic belt.
Concretions on a bedding plane

$S_2 = 0.51 \text{ to } 0.66$
lycopsid branch captured by concretion ($S_2 = 0.78$)
3.1.2 – Strain Markers: Examples of Extension

An AAPG Short Course by
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Deformation and then strain were introduced in the context of mechanical and then chemical compaction during burial. This leads to a stretch which is less than one. Furthermore, in a foreland like the Appalachian Plateau a tectonic compaction takes place which means there is a stretch of less than one in the direction of layer-parallel shortening. In other tectonic settings there is an extension which means that stretch is greater than one. In considering more strain markers outside those found on the Appalachian Plateau, a positive stretch is common. We start with an example of regional stretch by normal faulting, the Basin and Range, USA, where the unstrained upper crust was about half its present length along a line from Carson City Nevada to Salt Lake City.
In studying deformation and strain for extension of the Basin and Range and layer parallel shortening of the Appalachian Plateau detachment sheet, all we could do was compare the final shape (line length) with its initial shape (line). There was no information about intermediate deformation yet we know from the propagation of multiple joint sets that the orientation of principal stress change during tectonic deformation and in this case it is unlikely that the orientation of incremental strain was the same for each step. This is in contrast with invariant stress orientation during compaction accompanying burial where strain is described as **coaxial** or pure shear. When stress orientation changes over time, the geometry of deformation changes in a progressive way and we say that the strain is **non coaxial**. We call this progressive deformation which is most easily seen in the way that fibers grow in extension veins and stress shadows.

The fiber growth in extension has have four geometries. In syntaxial veins the fiber material is very close or identical to the composition of grains in walls. Often the fibers grow in crystallographic continuity with the wall rock grains and growth of the veins takes place at the medial line. The common sedimentary rocks find that syntaxial fibers in carbonates are calcite and in sandstone they are quartz. Antitaxial fibers commonly entrain wall rock along a central band and have no crystallographic relationship with the wall. Growth of these fibers is toward the wall and commonly the wall is fractured a number of times with the incorporation of a number of fragment trains in a process called **crack seal** (Ramsay, 1980). Sometime a vein may display a combination of both antitaxial and syntaxial growth (a composite vein). In some instances the fibers are straight but at an angle to the wall which means the opening direction did not change with time.

Black and white crystals represent different orientations of the crystalline lattice. Arrows indicate the direction of fiber growth. Drawings from Ramsay and Huber, 1983
Some sedimentary rocks contain inclusions that resist deformation relative to their matrix. We have already seen that carbonate concretions fail to compact as overburden is added, layer by layer. Pyrite frambooids are also strong relative to a clay of a shale matrix. This relative strength is preserved through lithification as indicated by matrix detaching from concretions during tectonic compaction. Some inclusions are more susceptible to deformation along with the rock matrix as is the case with crinoid columnals during layer-parallel shortening.

Rigid inclusions respond much like the walls of a vein during extension in that there can be either syntaxial or antitaxial growth in pressure shadows. In the case of rigid inclusions the pyrite pressure shadows grow in crystallographic continuity with the wall rock which means the fiber growth takes place at the fiber-framboid contact (i.e., syntaxial growth). The fiber growth for crinoid type pressure shadows takes place at the contact between the pressure shadow and the wall rock. In the drawing on this page, the crinoid is a single crystal that has been mechanically twinned during tectonic deformation. Pressure shadows indicate either coaxial strain (straight fibers) or non-coaxial strain (curving fibers).

Arrows give an indication of the direction of fiber growth. Drawing taken from Ramsay and Huber, 1983.
When the growth of the fiber pressure shadow is at the rigid inclusion, such as against pyrite, this is said to be an example of **displacement controlled fibers**. For example, in the figure to the right, fibers grew on the left wall at time a. But because extension rotated clockwise and shifted away from the left wall as fiber growth continued growth, the initial growth was displaced away from the wall. Eventually growth continued on the left wall but in a different direction.

Two fiber growth models are possible during coaxial strain (straight fibers). A rigid fiber growth model presumes that the fibers are not deformable so each increment is identical (model B). If the fibers are deformable then the outline of the fibers (model C) takes the shape of the strain ellipse (outline A).

Two fiber growth models are possible during non coaxial strain (curved fibers). A rigid fiber growth model presumes that the fibers are not deformable so each increment is identical (model B). If the fibers are deformable then the outline of the fibers (model C) takes the shape of the strain ellipse (outline A).

Drawings after Huber and Ramsay (1983)
Folds after Pure Shear Deformation

**Pure Shear**

After 50% horizontal elongation under pure shear.

Original

After 50% add’tl horizontal shortening.
(Pure shear)

**Volume Constant Strain**

From Marli Miller, U of Washington
3.1.3 - Rheology

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
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Ductile Deformation

Ductile deformation occurs if the rock under stress does not lose its strength by means of a brittle failure. This behavior is illustrated using stress-strain curves from rock deformation experiments. Each test is run at constant strain rate which means that in a triaxial test the piston is advanced into the cylindrical rock sample at a constant rate. The initial behavior of the rock is elastic for which a linear stress-strain curve is shown. Brittle failure causes a complete lose of strength. Ductile flow shows that the strength is maintained during continuous straining of the sample.

Percent ductility is a measure of the amount of strain that a rock undergoes before losing strength. Ductility varies with lithology. The strongest and most brittle of the rocks is a quartzite or silica cemented sandstone. In contrast, halite is very weak and will undergo large amounts of ductile flow without brittle failure. There is a variety of rocks and their relative ductilities as a function of depth of burial within the earth. Starting with the most brittle there is silica cemented sandstone, dolomite, calcite-cemented sandstone, shale, limestone, and halite.

Initially constant strain-rate tests were most convenient for laboratory experiments. However, conditions within the crust of the earth closely resemble constant stress tests. This is so because the differential stress within the crust does not change rapidly with time. The most interesting characteristic of constant stress tests is that steady state creep is achieved. This is a state where the rock exhibits no change of strain rate with time.
Various mechanisms of ductile flow were introduced during the previous lecture. Each of these mechanisms can dominate during the creep of rocks. The dominate mechanism depends on the temperature and differential stress affecting the rock. For plotting the temperature of deformation verses stress, the temperature $T$ is normalized to the melting temperature ($T_m$) by the ratio $T/T_m$. The stress of deformation is normalized by the shear modulus of the rock ($\mu$).

Various creep mechanisms include the following:

- **Nabarro-Herring Creep** - bulk diffusion of point vacancies down a stress gradient. Recall that a point vacancy is a single missing atom.
- **Anelastic Creep** - below a critical shearing stress for large dislocation movement mechanisms as Coble Creep take place.
- **Low-Temperature Creep** - includes multiplication and glide of dislocations. Stresses have to be reasonably high to cause this type of creep.
- **High-Temperature Creep** - at higher temperatures edge dislocations can climb and screw dislocations can cross slip.

An equation for Nabarro-Herring Creep gives the strain rate ($\dot{\varepsilon}$) in terms of stress $\sigma$

$$\dot{\varepsilon} = \frac{(\alpha D V_a \sigma)}{k T L^2}.$$  

$\alpha$ is a geometric factor; $L$ is the diameter of the grain; $D$ is the diffusion coefficient; $V_a$ is the atomic volume; $T$ is the temperature; $\sigma$ is stress; and $k$ is the Boltzman number. Here strain rate is proportional to stress which is the behavior of a Newtonian viscosity.
Steady State Creep

Steady creep flow of rock materials can also be modeled using the Weertman Equation

\[ \dot{\varepsilon} = A \exp\left(-\frac{Q_c}{RT}\right) f(\sigma) \]

where \( T \) = temperature, \( Q_c \) = creep activation energy, and \( R \) = gas constant. This equation can be evaluated using a plot of \( \log \sigma \) versus \( -\log(\dot{\varepsilon}) \) by rearranging the above equation

\[ \log(\dot{\varepsilon}/A) = \frac{Q_c}{RT} + \phi \log \sigma \]

where \( \phi \) is the slope of the lines in the plot to the left and \( Q_c \) is determined as the slope of the plot of \( \log \dot{\varepsilon} \) versus \( 1/T \) at constant stress. Experiments show that creep rate at high temperature is a strong function of stress.

\[ \dot{\varepsilon} = \alpha \sigma^n \]

The plot to the right is a deformation mechanisms map for calcite (limestone). Given a stress and temperature, the deformation mechanism diagram shows which of six mechanisms are favored. These mechanisms include cataclasis, pressure solution, dislocation glide, dislocation climb, Coble creep, and Nabarro-Herring creep.
During a systematic investigation of the strength of rocks, focusing largely on ductile behavior, a critical step occurred when F. J. Turner (1948) hypothesized that twin lamellae in calcite or deformation lamellae in quartz originated by plastic flow and formed when the slip systems were in orientations of high shear stress. The study of rock strength narrowed to focus on the experimental deformation of marble largely because calcite is ductile under a wide range of experimental conditions.

Two distinct intracrystalline slip mechanisms are evident in petrographic thin sections of deformed rocks. These mechanisms, translation gliding and twin gliding, are controlled by crystallographic structures and represent the motion of dislocations (missing atoms in the atomic lattice). Both mechanisms obey Schmid's law which states that an intracrystalline slip mechanism will operate only when the shear stress resolved along the slip direction in the slip plane has reached a certain critical value ($\tau_c$). Although $\tau_c$ is a function of temperature and strain rate, it is independent of the stress normal to the active slip plane, a very important observation because slip is an indication of a specific $\sigma_d$.

Twin gliding in calcite along with pressure solution of both quartz and calcite are the primary ductile deformation mechanisms in the brittle portion of the crust, the schizosphere.

Schematic block diagrams of an edge dislocation and a screw dislocation in cubic lattices. The Burger's vector, $\mathbf{b}$, is defined as the closure vector (the direction of motion) for the loop counted around the dislocation (dark line). The tangent vector, $\mathbf{t}$, is defined to lie parallel to the dislocation line. The Burger's and tangent vectors are perpendicular for an edge dislocation; in contrast, they are parallel for a screw dislocation.
Translation gliding is the macroscopic manifestation of edge dislocation motion along a slip plane. The large-scale effect of translation gliding is the deformation of the host grain in simple shear with each atomic plane displaced an integral atomic distance relative to the plane. Displacement along any slip plane is limited by crystallographic symmetry to one or a few slip directions with the slip plane and slip direction comprising a slip system. Crystals may have more than one slip system, each with a unique \( \tau_c \). The slip system with the lowest \( \tau_c \) is the most active system. The resolved shear stress \( (\tau_r) \) along a given slip direction is given by

\[
\tau_r = (\sigma_1 - \sigma_3)\Omega_o \quad \text{and} \quad \Omega_o = \cos \lambda \cos \phi
\]

where \( \lambda \) is the angle between \( \sigma_1 \) and the normal to the slip plane and \( \phi \) is the angle between \( \sigma_1 \) and the slip direction. The maximum \( \tau_c \) occurs on a slip system where \( \lambda = \phi = 45^\circ \) so that \( \Omega_o = 0.5 \). Slip will occur only if \( \tau_r \geq \tau_c \). However, for homogeneous deformation of an aggregate, five slip systems must operate, an observation called the von Mises criterion.

Although twin gliding is also regarded as a simple shear of the crystalline lattice along the slip plane, it differs from translation gliding in two respects. First, it is homogeneous which means that each lattice plane is displaced the same amount relative to the plane below. There are no such constraints on the motion of each lattice plane during translation gliding. Secondly, the twinned portion of a crystal is deformed into a mirror image of the undeformed crystalline lattice across the twin plane.

Diagram showing the nature of the gliding system for calcite e-twins relative to principal stress axes and angles given here. (b) Contoured values of \( \Omega_0 \) for a horizontal twin plane (after Jamison and Spang, 1976)
Calcite twinning is one of the key deformation mechanisms used in the study of stress during ductile flow in the schizosphere. For twinning on the e-planes, the atomic layers above the twin plane move toward the c-axis in the host crystal with respect to the atomic layers below the twin plane. Here is a micrograph of calcite twin lamellae in a deformed marble.

Twin-gliding in calcite. Diagrammatic projection of the twinned calcite structure parallel to the plane containing the c-axis and the normal to $\sigma_1$, showing the twinning elements (adapted from Friedman, 1964). Equal-area projection showing the graphic method for constructing principal stress axes best oriented to produce twin gliding in calcite for two e-planes (adapted from Carter and Raleigh, 1969).

Position of compression and extension axes that would most favor development of calcite twin lamellae where plane of diagram is normal to e-twin plane and contains glide direction [e1:r2]. (b) Shear strain in a partially twinned calcite grain (adapted from Groshong, 1972). The values $t_1$ and $t_2$ are the widths of twins and $t$ is the width of the host grain perpendicular to the twin planes.

Engelder, 1993
Intergranular Mechanisms: Pressure solution
Pebble Conglomerate, Beavertail Point, Rhode Island, USA

Ductile deformation mechanisms alone or combined with other mechanisms act to facilitate the changes in the shape of rocks at various levels in the crust of the earth. The higher the temperature (i.e. the deeper within the crust), the more likely that ductile mechanisms dominate over brittle mechanisms in controlling the style of deformation. Ductile mechanisms can be divided into intergranular and intragranular processes. Higher temperatures favor intragranular processes.

Near the surface of the crust pressure solution is the most common intergranular deformation mechanism. It is manifested by cleavage in sedimentary rocks. The basic process involves a dissolution at contact points between grains of rock. Characteristically one grain will dissolve faster than the other. The product of this differential dissolution is a stylolite in fine-grained rocks or pits and beards in pebbles of a conglomerate. The dissolution process is believed to be activated by the high normal stress between grain contacts. The rock, dissolved from the high stress contacts, then diffuses into the pore spaces of the rock where pore fluid exerts the only normal stress at pressures much less than found along grain-to-grain contacts.

In two or more component systems one material will dissolve faster than another. For example, in a pelagic limestone the calcite dissolves and clay forms as an insoluble residue. In a one component system, the single component dissolves uniformly. In a shale (clay) the composition of the shale after 50% shortening is the same as the initial shale. In quartzites the grain boundaries are irregular as the grains penetrate each other. The normal to the irregular surface that is the stylolite points in the direction of $\sigma_1$ acting on the rock. If the interpenetration is large enough, large protrusions of one material penetrates another material to point in the direction of $\sigma_1$. 

Terry Engelder
The orientation of stylolites and pressure solution cleavage is always consistent relative to $\sigma_1$. The maximum rate of dissolution is in the direction along a surface normal to $\sigma_1$. This favors the formation of a cleavage with its "planar" direction normal to $\sigma_1$ and $\sigma_1$ may be either vertical or horizontal.
Different flow laws describe the behavior of rocks when different deformation mechanisms are rate controlling. Various flow regimes are identified with subdivisions of a temperature-c(rate)-σ_d plot known as a deformation mechanism map (Stocker and Ashby, 1973). In the \textbf{upper tier} of the ductile-flow regime, grain-size-sensitive creep mechanisms are important. Three grain-size mechanisms include diffusional flow, pressure solution, and superplastic flow (e.g., Durney, 1972; Schmid, 1983). The \textbf{middle tier} of the ductile-flow regime is characterized by low-temperature creep for which flow stress is relatively insensitive to temperature and strain-rate changes (Kirby, 1980). Here dislocation glide dominates and dislocation pile-up leads to failure which may be brittle. In the high temperature creep (\textit{power-law creep}) regime (\textbf{lower tier}), relatively small temperature changes have a marked effect on the steady-state flow stress, cr; (Kirby, 1983).
3.1.4 – Plastosphere v. Schizosphere

An AAPG Short Course by Terry Engelder
   Professor of Geosciences
   The Pennsylvania State University
Constraints on lithospheric stress are based on ductile as well as brittle rock strength. Temperature and confining pressure have fundamentally different effects on brittle and ductile rock strength. Ductile strength is almost unaffected by confining pressure, whereas the brittle strength of rock increases markedly at higher confining pressure (Heard, 1960). In contrast, temperature has a large effect on ductile processes such as dislocation motion and diffusion-assisted deformation, whereas brittle strength shows little dependence on temperature (Heard, 1963). In laboratory experiments, rocks are markedly weaker during ductile deformation at high temperature which means that $\sigma_d$ is much lower in the warmer portions of the **plastosphere** and in regions of the **schizosphere** dominated by ductile deformation. Semibrittle behavior occurs in the transition from pressure-dominated brittle deformation to temperature-dominated ductile deformation (Tullis and Yund, 1977). Semibrittle behavior is found in a transition zone of the lithosphere where $cr$, gradually drops from a maximum at depths of 10-15 km to smaller values at depths near the Moho. This brittle-ductile transition zone is the boundary between the schizosphere and plastosphere. The Moho apparently represents a mechanical discontinuity within the lithosphere where the underlying peridotite is stronger than rocks of the lower crust (Kirby, 1985). As is the case with the lower crust, strength of the mantle drops as temperature increases with depth.

The limits within which lithospheric stress must lie on the basis of the assumptions (1) that rocks are fractured and that friction on fractures controls the stress at shallow depths, (2) that the creep properties of quartz or olivine control the stress below about 15 or 25 km, respectively, and (3) that the effective stress principle operates for friction but not for creep.

Difference between maximum or minimum horizontal stress and the vertical stress as a function of depth. Values of $\lambda$ give pore pressure level.
A compilation of data from the upper tier of the ductile-flow regime reveals that while $\sigma_d$ near fault zones approaches the frictional strength of the schizosphere (i.e., Zoback 2007), there is plenty of evidence indicating that the heart of foreland fold-thrust belts was generally at $\sigma_d$, much less than required for frictional slip. Data from the Glarus Thrust in Switzerland indicate that $\sigma_d$ was near the frictional strength of the upper crust (Briegel and Goetze, 1978). Within the Marquette Synclinorium, Michigan, $\sigma_d$ varied from 1 to 36 MPa (Kappmeyer and Wiltschko, 1984). $\sigma_d$ near other major faults was lower, ranging from 125 MPa near the McConnell Thrust of the Canadian Rockies to 2.5 MPa along the Appalachian Plateau decollement. Fold-thrust belts riding atop a salt decollement (e.g., the Appalachian Plateau, the Franklin Mountains of Canada, the Jura of the Alps) are extremely wide and characterized by a small cross-sectional taper angle (Davis and Engelder, 1985). The width of these fold-thrust belts is attributed to the extremely weak nature of the salt decollement and a relatively low $\sigma_d$ within the elastic wedges. All of this means that earthquakes, a frictional phenomenon responding to higher $\sigma_d$, are not common within the elastic wedges of fold-thrust belts in the schizosphere and that ductile flow at relatively low $\sigma_d$ plays a major role in foreland deformation (Groshong, 1988).
If $\sigma_d$ was too low for brittle frictional slip during certain phases of mountain building, what is the explanation of the pervasive faults throughout forelands? Part of the explanation is tied to the nature of faulting which is characterized by slickensides coated with secondary mineral growth. The *slickenlines* on these surfaces are not brittle wear grooves indicative of high frictional stresses but rather fibrous minerals indicative of local dissolution and mineral growth. Technically, slip by a ductile mechanism is also frictional slip, however in the context of this book the frictional-slip regime refers to $\sigma_d$ developed as a consequence of brittle wear. The Umbrian Apennine fold-thrust belt contains an array of faults displaying slickenlines. Such faults have a preferred orientation and fall into one of two clusters forming conjugate sets with a 90° dihedral angle (Marshak et al., 1982). A conjugate set of faults forming at a 90° dihedral angle is unusual for brittle behavior. A 90° dihedral angle suggests that the faults slipped at the maximum $\tau$, a situation indicative of a ductile creep mechanism (i.e., stress solution) rather than a brittle friction. If so, these fault surfaces slipped under relatively low $\tau$ in the upper tier of the ductile-flow regime. In other mountain belts $\sigma_d$ is sufficiently low so that the crack propagation under high pore pressure is the favored brittle deformation mechanism. During the Alleghanian Orogeny, not only was brittle deformation of elastic sediments of the Appalachian Plateau restricted to crack propagation (e.g., Babat and Engelder, 1984), but also the Alleghanian joints were not reactivated in frictional slip after the stress field rotation to subject early joints to $\tau$ (Geiser and Engelder, 1983). This further suggests that $\sigma_d$ remained relatively low throughout large portions of the Appalachian Plateau.

The strata of the northern Umbrian Apennine fold belt are cut by an array of mesoscopic faults that generally display strike- or oblique-slip offset. The majority of these faults have traces less than a few metres long and represent displacements of < 10 cm. Fault surfaces are associated with stylolites and are coated with elongate calcite fibers, suggesting that movement occurred by the mechanism of pressure-solution slip. There is a great range among fault attitudes, but two clusters forming a conjugate set with about a 90° dihedral angle stand out (Marshak et al., 1982).
Stress in the middle tier of the ductile-flow regime is largely known from fault zones in quartz-bearing rocks $\sigma_d$ ranges from as low as 20-40 MPa along the Ikertoq shear zone in Greenland (Kohlstedt et al., 1979) to as high as 130 MPa in the Massif Central of France (Burg and Laurent, 1978). Some of the fault zones come from depths as great as 30 km where temperatures range up to 800°C. Work on calcite from the Glarus mylonite and within the Intrahelvetic complex yield estimates of 25-260 MPa and 42-100, respectively (Pfiffner, 1982). Although stresses as high as 200 MPa are reported for the Mullen Creek-Nash Fork shear zone in Wyoming (e.g., Weathers et al., 1979), these data are based on free dislocation density which is notoriously sensitive to late-stage stress changes. In some fault-zone samples, a low $P_{dis}$ or large recrystallization grain size may reflect a statically annealed state associated with stresses generated during uplift, erosion, and cooling (Kohlstedt and Weathers, 1980). Samples from the Ikertoq shear zone, for example, indicate a relatively low $a$, which, Kohlstedt and Weathers (1980) suggest, does not represent the major episode of deformation within the fault zone but rather some other point in a complicated history during uplift and erosion.
Lower tier of the ductile flow regime

The ductile-flow regime within the lithosphere has three tiers which correspond to the schizosphere, the brittle-ductile transition, and the plastosphere. Within all three tiers ductile flow serves to modulate $\sigma_d$ which otherwise would climb until the point of inducing either frictional slip or shear rupture. From the upper to lower tiers, the dominant deformation mechanisms are diffusion mass transfer, restricted dislocation glide, and dislocation creep, respectively. The upper tier of the ductile-flow regime is found in that portion of the lithosphere characterized by the three brittle-stress regimes. In foreland settings there is apparently a symbiosis between the upper tier of the ductile-flow regime and the crack-propagation regime such that ductile flow suppresses $\sigma_d$ to favor crack propagation upon increase in $P_p$ above hydrostatic pressure. In a sense, the upper tier of the ductile-flow regime can occupy the same space as the frictional-slip or shear-rupture regimes, but the ductile-flow and high-stress regimes do not occupy that space concurrently. In the middle and lower tiers of the ductile-flow regime, brittle deformation is suppressed by high confining pressure in favor of dislocation glide (slip, twinning), dislocation creep (climb, cross-slip), and other high-temperature diffusion mechanisms.

Estimates for $\sigma_d$ in the Basin and Range of the United States using recrystallization grain size (adapted from Mercier, 1980). Estimates are obtained by using Ross's et al. (1980) grain boundary migration piezometer. These data are characteristic of the stress-depth trend for the lower tier of the ductile-flow regime.
3.1.5 – Ductile Shear Zones

An AAPG Short Course by
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Physical factors likely to affect the genesis of the various fault rocks—frictional properties, temperature, effective stress normal to the fault and differential stress—are examined in relation to the energy budget of fault zones, the main velocity modes of faulting and the type of faulting, whether thrust, wrench, or normal. In a conceptual model of a major fault zone cutting crystalline quartzo-feldspathic crust, a zone of elastico-frictional (EF) behaviour generating random-fabric fault rocks (gouge—breccia—cataclasite series—pseudotachylyte) overlies a region where quasi-plastic (QP) processes of rock deformation operate in ductile shear zones with the production of mylonite series rocks possessing strong tectonite fabrics. In some cases, fault rocks developed by transient seismic faulting can be distinguished from those generated by slow aseismic shear. Random-fabric fault rocks may form as a result of seismic faulting within the ductile shear zones from time to time, but tend to be obliterated by continued shearing. Resistance to shear within the fault zone reaches a peak value (greatest for thrusts and least for normal faults) around the EF/OP transition level, which for normal geothermal gradients and an adequate supply of water, occurs at depths of 10-15 km.
Incipient cataclasis
Cataclastic deformation band
Mylonites
Moine Thrust, Stack of Glencoul, Scotland, United Kingdom

Grid Reference: NC 28882876
Approximate horizontal and vertical scale
Mylonites
Moine Thrust, Stack of Glencoul, Scotland, United Kingdom

Mylonitic foliation

Moine Thrust

R.D. Law et al., 2010
White Quartz Mylonites
Moine Thrust, Stack of Glen Coule, Scotland, United Kingdom
Lewisian Rocks
Moine Thrust, Stack of Glen Coule, Scotland, United Kingdom
Lewisian Rocks
Moine Thrust, Stack of Glen Coule, Scotland, United Kingdom
3.1.6 – Structures of the Plastosphere

An AAPG Short Course by
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Passive folding
Bronson Hill Anticlinorium, Massachusetts, USA
Bronson Hill Anticlinorium
Nappes and Gneiss Domes, Massachusetts, USA

Thompson, et al., 1968
Peninsula of Tovqussaq nuna
Greenland

Myers & Kroner, 1994 after Berthelsen, 1960

Legend:
- Quartzo-feldsparic gneiss
- Anorthosite
- Amphibolite

- Dip and strike of layering
- Direction and plunge of fold axis
3.2.1 Cleavage

An AAPG Short Course by
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The mechanical compaction of shale is arrested with the onset of compaction disequilibrium which leaves a porosity above 20%. Yet, shale at depth is fully cemented with a low porosity. One source of this cement comes from grain-grain pressure solution which stops the further compaction of shale as indicated by bedding that wraps around concretions. While shale compaction stops, pressure solution at depth enhances the tendency for carbonates to compact.

Sandstone compaction:
*Note the role of pressure solution in developing closer packing!*
A stylolitic finger capped by a relatively insoluble inclusion (arrow) allows the darker layer to penetrate upward into an overlying layer which otherwise competes evenly to pressure solve the adjacent layer. This interpenetration indicates that at least a 4 cm of layer of rock has been removed by pressure solution.
Bedding plane stylolites: Illustrates overburden compaction
Indiana Limestone Fm, Bloomington, Indiana, USA

From Simon Katterhorn, University of Idaho
The Pennsylvanian age Purgatory Conglomerate of Rhode Island was deformed under late Paleozoic lower greenschist facies conditions. Two mechanisms are invoked to account for the deformation. Pressure solution resulted in the formation of a highly flattened and elongated pebble fabric. This fabric was enhanced by slip and concurrent pebble rotation on intra-pebble shear fractures. Adjacent quartzite pebbles are molded against each other and usually exhibit distinct pressure shadow over-growths in the direction of pebble elongation. The matrix between pebbles, exclusive of pressure shadow overgrowths, is predominately micaceous.

**Important point:** Micaceous material is less soluble relative to either quartz or carbonate and thus is left as a residue where $\text{SiO}_2$ and $\text{CaCO}_3$ are dissolved under stress.
Examples of interpenetration of quartz pebbles and flattening of the pebbles

Mosher, 1976
Important Point: Pressure solution and brittle fracture co-exist in the upper brittle crust where geological structures are most important in the quest to extract fossil fuels.

Quartzite pebble with evenly spaced intra-pebble shear fractures. Note the offsetting of the pebble margin.
The term 'cleavage', as used here, refers to a set of closely spaced secondary, planar, fabric elements that impart mechanical anisotropy to a rock without apparent loss of cohesion. The fabric elements, which are commonly called 'domains', are zones in which the original fabric of the rock has been altered by the deformation processes responsible for creation of the cleavage, mainly pressure solution in the brittle crust. Cleavage domains have been variously referred to as bands, films, folia, selvages and stripes. Uncleaved, or less strongly cleaved, bands of rock that lie between adjacent domains are called microlithons (Borradaile et al. 1982). 'Disjunctive' cleavage domains, in contrast to 'crenulation' domains, cut across pre-existing (sedimentary) layering in the rock without reorienting the layering (Powell 1979). The principal characteristics that have been used to classify disjunctive cleavage are; (1) the spacing of cleavage domains and (2) the morphology of the domain (Borradaile et al. 1982).

Pressure solution refers to the process of dissolution at grain-to-grain contacts under conditions that cause the rate of dissolution to be controlled by the magnitude of normal stress across the contact; grains dissolve most rapidly where the normal stress is greatest. Subsequent to removal from the grain surface, dissolved ions then diffuse through fluid films away from the point of dissolution and either precipitate at sites of lower normal stress, or pass into the pore fluid of the rock. The rate-controlling step of the pressure-solution process is thought to be this diffusion through fluid films (Rutter 1976), and based on thermodynamic arguments, the driving force for the diffusion accompanying pressure solution is a chemical potential gradient created by variations in the magnitude of normal stress at grain-grain contacts.

Patterns of disjunctive (domains can be seen with the naked eye) cleavage cutting beds.

Engelder & Marshak, 1985
Tectonic stylolites (sutured cleavage) showing NE-SW compression
Upper Jurassic Limestone, Rabac, Croatia

Looking down on a horizontal bedding surface.
Maximum penetration is about 1 cm
Dinaride Thust Belt
Croatia

From Tomašić et al., 2011
Layer-parallel shortening stylolites with millifractures
Madison Group, Teton Anticline, Sawtooth Range, Montana, USA
Disjunctive cleavage has been subdivided into two principal types based on domain spacing. Spaced cleavage is recognized where domains and microlithons can be distinguished in hand specimen or in outcrop, and slaty cleavage where adjacent domains cannot be distinguished without the aid of optical microscopy; Powell (1979) noted that most slaty cleavage has a domain spacing of less than 1 mm. Spaced cleavages can be further labeled, based on domain spacing, as weak, moderate, strong and very strong (Alvarez et al. 1978).

Spaced cleavage has been variously referred to as tectonic striping, solution cleavage, fracture cleavage, stylolitic cleavage, zonal cleavage, fissuring, reticulate cleavage, false cleavage, and spaced-solution cleavage. Very widely spaced domains, which commonly have pitted surfaces, are also called stylolites. The variety of terms listed above can be confusing, for some of these terms imply a mechanism of origin and others indicate morphological characteristics. The term 'fracture cleavage', for example, implies that disjunctive cleavage is a consequence of brittle failure of rock, which is generally not the case. Usually, the non-genetic term 'spaced cleavage' can be used in preference to the others.
Pencil cleavage in shale of the Upper Devonian Canadaway Group and along the Vandermark Creek 4 km east of Scio, New York. The view is a vertical face looking parallel to the strike of the pencil structures (N60°E). Bedding is horizontal, and the parting that forms the pencil structures is vertical.

The top part of the Tulley Limestone interfingering with shale of the overlying Genesee Group at Taughannock Falls State Park near Ithaca, New York. A zone of stylolitic solution cleavage developed in the limestone grades downward into a zone of pencil cleavage developed in a 20-cm-thick bed of shale. Strike of pencil cleavage parallel to fold axes.
Edge of a crinoid columnal removed by pressure solution, the mechanism responsible pencil cleavage throughout the Appalachian Plateau.
The distribution of layer parallel shortening fabrics across the Appalachian plateau to the Hudson Valley. The trend-line map was prepared by connecting data points (thick lines) with nearly parallel cleavage planes. The orientation of the cleavage planes is shown by a plot of the strike of cleavage planes.
Cleavage distribution is strongly controlled by limestone composition; limestones containing greater than 10% clay-quartz matrix develop widespread cleavage, whereas limestones with less than 10% clay-quartz matrix develop isolated tectonic stylolites. This dependence appears to be textural, and may reflect the role that clay plays in providing interconnectivity between the sites of dissolution along grain boundaries and the free-fluid system. Cleavage morphology is also controlled by limestone composition; tectonic stylolites in pure limestones have a sutured morphology, whereas cleavage domains in impure limestones have a non-sutured morphology.

Layer-parallel shortening (LPS) is the same on all rock units but the mechanism is calcite twinning in low clay rocks and cleavage development (few calcite twins) in the high clay rocks.

Marshak & Engelder, 1985
Weathering accents disjunctive cleavage (core taken from fold limb)
Middle Devonian Mahantango Formation, Sunbury, Pennsylvania, USA
X-ray tomography shows that cleavage is an early layer-parallel shortening fabric before fold limbs grow.
Disjunctive cleavage in a sample of the Silurian Mifflintown micritic limestone from the central portion of the Valley and Ridge, Pennsylvania. This is a region stacked fault-related folding where rocks have passed through more than one fold hinge. This stylolitic surface contains insoluble residue 2-3 mm thick. Passing through fold hinges also places the limestone bed into extension as reflected by the calcite filling that follows the insoluble residue.
Cross cutting slaty cleavage of the Ordovician Martinsburg Formation from the Great Valley of Pennsylvania.
Slaty cleavage
Devonian Esopus Formation, Catskill, New York, USA
Crack extension (opening) passes in en echelon fashion to maintain a constant crack-parallel extension throughout the area of the sample.
Buckle folds developed at the scale of crenulation cleavage
Marcellus gas shale, Newton-Hamilton, Pennsylvania, USA
3.2.2 - Folding

An AAPG Short Course by
Terry Engelder
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There was a time when geologists did not automatically connect folds in foreland basins with unseen faults at depth. One of the first to recognize the connection was Gwinn (1964). He wrote, “New data, illustrating complex subsurface structure as revealed by recent exploration for natural gas in the Appalachian basin, are combined with surface geological data to present evidence that folds of the Plateau and western Valley and Ridge provinces are surficial, compressional wrinkles developed above deep non-outcropping bedding-plane thrust faults of large displacement, which continue northwestward beneath the Plateau from the western margin of the Valley and Ridge province”. Our understanding of modern structural geology owes a great debt to the connection that Gwinn made between folding at the surface of the Appalachian Plateau and faulting at depth. The following is an example of what Gwinn saw at depth, largely based on the correlation of well logs.

Imbricate thrust faulting, Giffin Dome, Chestnut Ridge anticline, Westmoreland County, Pennsylvania; at the level of the lower Devonian, the northwest flank of the anticline is an imbricately faulted, overturned fold. (Don-h = Devonian Onondaga-Oriskany-Helderberg sequence (stippled); T = Tully limestone; solid line near the surface is the base of the Mississippian Pocono Sandstone.)

The Gwinn cross section of Chestnut Ridge served as a solution to the famous ‘room’ problem for folding in compressional mountain belts.
Before we go on to further establish the connection between faulting at depth and folds, we must develop a terminology for folds. Folds are most commonly marked by bedding, a surface referred to as $S_0$ in structural geology. Folds are characterized by axial surfaces, $S_1$, and a fold axis. The plane perpendicular to the fold axis is that surface on which the profile of a fold is projected. A fold that has a planar axial surface is said to be a plane fold, otherwise it is a nonplanar fold.

One ideal fold is cylindrical with any profile the same as any other profile. Real folds are rarely cylindrical but rather conical in shape. The ideal fold can be divided into the hinge area where the radius of curvature of the fold is smallest and the limb area where the radius of curvature of the fold is largest. A profile or cross section through a set of cylindrical folds constituting a wave train. The Amplitude and wave length of the wave train are shown.

Folds are sometimes overturned. In this example the sandstone is younger than the limestone, yet in the overturned limb the older limestone appears over the younger sandstone. The direction of younging is said to be downward. The enveloping surface of the folds as well as the crest and trough are indicated.
Folds have many shapes in cross section depending on their intralimb angle. Four general classes of folds based on intralimb angle are: Gentle, Open, Tight, and Isoclinal. Other shapes of folds include Rounded, Angular, Chevron, and Box.

Folds may be classified based on the orientation of their hinge line and axial surface. Turner and Weiss (1963) have identified seven types of folds including: horizontal normal, plunging normal, horizontal inclined, plunging inclined, reclined, vertical, and recumbent.
Tight chevron fold – Horizontal normal fold
Scaglia Rosa Limestone, Gubbio, Italy
Inclined axial plane with horizontal fold axis – Tight chevron fold
Loughskinny, Ireland

overturned limb

horizontal inclined
Vertical axial plane with plunging fold axis - Open chevron fold
Loughskinny, Ireland
Horizontal axial plane with horizontal fold axis
Col de Tourmelet, Pyrenees Mtns, France
Refolded Vertical Folds
Connecticut, USA
Ramsay's (1967) classification of folds is based on the dip isogon. A dip isogon is the line joining points of equal dip. Two common types of folds are the similar and parallel folds. In similar folds the bedding shape remains the same from bed to bed whereas in parallel folds the bedding thickness remains unaltered from bed to bed. Parallel folds lead to the famous room problem that was addressed by Gwinn (1964).

In order to maintain similarity from bed to bed, material within the beds must be moved out of the limbs and into the hinges.

In parallel folds the shape of the beds must change from layer to layer in order the maintain constant bed thickness throughout.
The mechanism of folding that will operate during deformation is a function of a) the nature of inherent anisotropy in the rock b) the ductilities of the involved rocks.

As ductility of the rocks increases, the effect of layering decreases.

Increasing depth of burial favors lower ductility contrast and high mean ductility.

<table>
<thead>
<tr>
<th>Class</th>
<th>Type</th>
<th>Predominant mechanism</th>
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</thead>
<tbody>
<tr>
<td>Flexural</td>
<td>Flexural slip</td>
<td>Slip between flexed layers</td>
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<tr>
<td></td>
<td>Flexural flow</td>
<td>Flow within flexed layers</td>
</tr>
<tr>
<td>Passive</td>
<td>Passive flow</td>
<td>Flow across layer boundaries</td>
</tr>
<tr>
<td></td>
<td>Passive slip</td>
<td>Slip across layer boundaries</td>
</tr>
<tr>
<td>Quasi-flexural</td>
<td></td>
<td>Irregular flow within and across layers</td>
</tr>
</tbody>
</table>

Donath and Parker, 1964
-Movement Horizontal surface, generally parallel to bedding, along which movement has taken place during flexural slip, and which is commonly marked by thin quartz-fibre sheets or veins.

-Flexural folds form as a result of the slip of one layer over another as the limb dip increases in response to lateral shortening.

Tanner, 1989
Buckle Folding (single beds)

The mechanics of folding of a single layer has evolved from the model for the folding of an elastic plate. The elastic plate has a thickness of 2y. The plate is folded upon the application of a force $P$ distributed on the area $\Delta A$ at the ends of the plate. As the plate is folded the upper portion of the plate is stretched (stippled area) whereas the lower portion of the plate is compressed. A plane between the compressed and stretched portion of the plate is folded but neither stretched nor compressed. This plane is referred to as the neutral surface. A plane normal to the fold axis cuts the neutral surface to form a line called the neutral fiber. The neutral fiber has the length $l$. A line a distance of $y$ above the neutral fiber has the length $l + \Delta l$. The stretch $\Delta l$ is proportional to

$$\frac{\Delta l}{l} = \frac{y}{R}.$$ 

If this is an elastic fold it is known as a buckle. The force $P$ necessary to cause the buckle is proportional to the distance from the neutral fiber. For the force per unit area on a small strip at $y$ from the neutral fiber

$$\frac{\Delta P}{\Delta A} = \frac{E \cdot y}{R} \quad \sigma = E \varepsilon \quad \varepsilon = \frac{\Delta l}{l},$$

where $E$ is the Young's modulus. For elastic plates the bending moment ($M$) is defined as the torque about the neutral fiber.

$$M = \int_{\text{cross section}} y dP \quad dP = \frac{E \cdot y}{R} dA \quad M = \frac{E}{R} \int_{\text{cross section}} y^2 dA$$

Now the moment of inertia ($I$) about a horizontal axis through its "unit mass" is

$$I = \int y^2 dA \quad M = \frac{E}{RI}.$$
Stress distribution in an elastic fold
Curvature is defined as the rate of change of a curve (i.e. the neutral fiber) in the vertical direction (y) as a function of distance in the horizontal direction

\[ \frac{1}{R} \approx -\left(\frac{d^2y}{dx^2}\right). \]

Folds can also be defined in terms of a bending moment at x where the equation for a beam is

\[ M(x) = EI \left(\frac{d^2y}{dx^2}\right) = Py \]

Rearranging this equation we can construct a homogeneous 2nd order differential equation

\[ \frac{Py}{Ei} \frac{d^2y}{dx^2} = -\frac{d^2y}{dx^2} \]

The same equation for the buckling of a plate is

\[ M(x) = \frac{Ei}{(1-v)} \cdot \frac{d^2y}{dx^2}. \]

For small deflections the shape of a buckled beam can be represented by an equation for a sine curve. Taking the second derivative for this sine curve, the second order differential equation for the bending moment can be written in terms of the force necessary to cause the buckle. Taking the homogeneous 2nd order differential equation from above and rewriting it in terms of P, we can derive

\[ P = \frac{\frac{d^2y}{dx^2} \cdot \frac{Ei}{y}}{P} = \frac{\pi^2Ei}{L^2}. \]

If the end of the buckle fold is fixed

\[ P = \frac{4\pi^2Ei}{L^2}. \]

The physical significance of this force P is that if P is less than a certain value there will be no bending but if the force is slightly greater, the beam will bend suddenly where \[ \frac{\pi^2Ei}{L^2} = \text{the Euler force at which a beam will buckle.} \]
Rocks layers differ from the single member which is constrained neither on top of bottom. Rather anytime that a rock layer folds, that layer has to contend with moving aside the rock layers above and below. In modeling folds these adjacent layers are handled in several manners. One approach is to consider the situation where both the layer of interest and those above and below are elastic. The layer in question is considered competent and located in an incompetent elastic media. Johnson (1970) considers the problem of a confined member loaded axially. Johnson (1970) derived the value to the Euler wave length.

\[ L_e = 2\pi \sqrt{\frac{E I}{p}}. \]

A buckle folding might include the following:
A. Undeformed layer

B. Buckling by orthogonal flexure

C. Buckling by flexural shear

D. Buckling by volume-loss flexure

B. Bending by orthogonal flexure

C. Bending by flexural shear

D. Bending by volume-loss flexure
Kink bands occur in strongly anisotropic rock where the anisotropy is either beds with a finite thickness or foliation with very thin layers. These two are fundamentally different types of geological material that kink in different manners. The first is a well cleaved schist where individual microlithons are very thin. This material resembles a pack of computer cards where folding is very sharp at the hinges. In the second geological material the bedding has a finite thickness where any folding at the hinges has to respond to the thickness of the hinges. Kinks in bedded material resemble the common chevron folds.

The terminology for kink bands includes the kink band which is that zone within the beds or foliation has been rotated. In this case the geological material has an anisotropy developed by the bedding planes. Within the kink band the beds have been rotated from horizontal to a steeply dipping position. The kink plane is the boundary between the rotated and unrotated portion of the beds. The kink axis is the same as a fold axis with the strike of the axis parallel to the strike of the kink plane provided the beds were initially horizontal.
Kink folding

Kink-bands and kink-like folds may develop by flexural slip or continuous simple shear. The deformation within a kink band can be described in terms of an external rotation ($\varphi$) and an internal rotation ($\theta$). The external rotation takes the bedding planes or foliation and rotates them through an angle $\varphi$ so that the beds or foliation are rotated into the obtuse angle sector between the kink band boundary (kink plane) and the foliation outside the band. The internal rotation ($\theta$) is indicated as the rotation of the radius of an undeformed circle to the deformed ellipse. The strain ellipse within the kink band shows the opposite sense of rotation from the shear couple parallel to the kink-band boundary. The shear $S$ indicated by the internal rotation ($\theta$) can be measured according to the rule.

$$S = \tan \theta = \frac{1 - \cos (\alpha + \varphi) \sec \alpha}{\sin (\alpha + \varphi) \sec \alpha}$$

Compressive strain parallel to the foliation is

$$\varepsilon_f = 1 - \cos \varphi$$
Kink folding
Silurian Wills Creek Formation, Huntingdon, Pennsylvania, USA
Kink bands develop one of two senses of rotation known as normal and reverse respectively. Largely the difference is determined by the acute angle between the foliation and the kink-band boundary. For the normal kink band the external rotation is opposite in sign from the acute angle measured from the kink plane to the foliation. In the reverse kink band the external rotation has the same sign. The limbs of kinks are distinguished on the bases of whether the kink band constitutes the long limb or short limb. Often the rotated limb is the short limb.

Chevron folds resemble kink bands except there is generally one fold plane where kinks are distinguished by a pair of kink planes. For three beds the nature of deformation is associated with both the limbs and hinge area. The sandstone in the hinge is above the neutral fiber and as a consequence extends along fractures as shown in black. The hinge area in this fold is not as sharp as was the case for the kink bands. As the hinge develops there are room problems where holes open and other portions of the fold wants to overlap. Usually in the areas of overlap the folds take the overlap by pressure solution where part of the bed is dissolved and carried away in solution or redeposited in the form of veins.
Three types of rotation can be distinguished in association with a chevron fold. The rotation #1 is an external rotation as indicated by the change in orientation of the bedding. This is $\varphi$ in our analysis of kink folds. Rotation #2 is an internal rotation and is shown on the strain ellipse by $\theta$. Rotation #3 develops off of beds of limestone that have been shortened by cleavage development. Each block of limestone is bounded by pressure solution cleavage that dissolves the beds and permits shortening without the total rotation shown as #1 for the sandstone beds.

Fault-bend folds, versions of kink folds, are a consequence of décollement tectonics where bedding parallel detachments climb section by cutting across several beds (Suppe, 1983). A model of fault-bend folding involves a detachment fault where the thrust plane comes from the right, cuts upward across bedding, and continues parallel with bedding off to the left. Upon the initiation of the fault across bedding two kink bands develop at the bends in the footwall because these bends cause folding in the upper sheet of the thrust system. The axial surfaces $A$ and $B$ terminate at the bends in the detachment where it changes from bedding parallel to cross cutting. The axial surfaces $A'$ and $B'$ terminate at the transitions between crosscutting and bedding-plane fault segments in the hanging wall at points $X'$ and $Y'$ which match points $X$ and $Y$ in the footwall. As slip continues kink bands $A-A'$ and $B-B'$ grow in with and the anticline above the crosscutting fault grows in height.
Axial surfaces A' and B' move with the thrust sheet because they are fixed to the hanging wall cutoffs. Because the axial surfaces A and B are fixed to the footwall cutoffs X and Y, the beds must move through the axial surfaces, first bending and then unbending.

The motion of the fault-bend fold is complex because when the hanging-wall cutoff, Y', reaches the footwall cutoff, X, the fold has reached its maximum amplitude, which is the height of the step in décollement. At this point the deformation, axial surface A, which has been fixed to the footwall is suddenly released to move with the hanging-wall cutoff, Y', whereas axial surface B', which has been moving with the hanging wall, is suddenly locked to the footwall cutoff, X.

**Fault-propagation folds** form as a part of the process of fault propagation. In such a fold the thrust fault cuts across section but does not extend in the subsurface in front of the fold as was the case for a fault bend-fold. Like the fault-bend fold, as the thrust starts to step up through the section two kink bands immediately develop. One axial surface is pinned to the footwall cutoff and the beds roll through the fold by first folding and then unfolding. Another axial surface is pinned to the tip of the fold and beds roll through it was the fault propagates forward. A third axial surface touching the fault plane moves with the velocity of the thrust sheet and is the surface where the two initial kink bands have merged below the surface. Commonly, fault-propagation folds will become locked because the bending resistance of some formation may be too great. In this case the fault may propagate along the anticlinal or synclinal axial surfaces or somewhere in between.
Polydeformed Archean aged turbidites and sandstone dykes spectacularly exposed near Rankin Inlet, Nunavut, Canada.

Photo courtesy of Chris Studnicki-Gizbert vis Clark Burchfiel, MIT
Passive folding: Refolded folds
Connecticut, USA
Faulted folds in the Great Kavir Desert 75 km NNW of Jandaq, Iran

Terry Engelder
3.2.3 Strain during Folding

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Mud pellets in an Appalachian Limestone
Appalachian Mountains, Pennsylvania, USA

Bedding Plane

Cross Section of a Bed
Deformed Oolites in the Cambrian Conococheague Limestone
South Mountain Anticline, Hagerstown, Maryland, USA

Bedding

Cleavage-Bedding Intersection
Deformed Oolites in the Cambrian Conococheague Limestone
South Mountain Anticline, Hagerstown, Maryland, USA

Cleavage face showing bed-normal stretching (lineation)

Cloos 1947
Cleavage and Oölites in cross section

Figure 5.—Relation of bedding to oöid extension

\[ \text{Cloos 1947} \]
Figure 16.—Fracture zones normal to fold axes
Deformed Oolites in the Cambrian Conococheague Limestone
South Mountain Anticline, Hagerstown, Maryland, USA

Cloos 1947
3.2.4 - Salt Tectonics

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
A very wide variety of rocks have remarkably similar frictional strengths under conditions which are common in the top 10 km or so of the crust (Byerlee, 1978). Exceptions to this rule include clays and shales, which are relatively weak. However, evaporites (and, in particular, rock salt) are vastly weaker than any other common rock type. Salt is below its brittle-ductile transition even in the top few kilometers of the crust and flows at geologically important strain rates in response to shear stresses of less than 1 MPa (Carter and Hansen, 1983). Because overthrusts appear to seek out horizons which provide even a modest reduction in resistance to slip, it is logical to expect that salt may have a major effect upon the deformation in a fold-and-thrust belt.

Let us now consider the effect of a weak basal layer upon the mechanics of fold-and-thrust belts. The overall mechanics of thin-skinned fold-and-thrust belts and their marine analogs, accretionary prisms, have been modeled as analogous to that of a wedge of soil in front of an advancing bulldozer (Davis et al., 1983; Dahlen et al., 1984). The critical taper is that cross-sectional wedge taper maintained when the entire thrust belt is on the verge of horizontal compressive failure. A wedge that is too narrowly tapered (such as one which has undergone erosion) will not slide along its basal decollement when pushed from behind. Instead, the failure criterion is exceeded within a narrowed portion of the wedge, leading to internal thrust faulting which shortens and thickens the wedge to reestablish the critical taper.

The failure criterion used to calculate the critical taper for fold-and-thrust belts consisting of rocks that have normal stress dependent strengths (Byerlee, 1978) is the Coulomb criterion. This criterion for shear traction \( \tau \) at failure is of the form

\[
\tau = S_0 + \mu \sigma_n (1 - \lambda)
\]

where \( S_0 \) is cohesion, \( \mu \) is the friction coefficient, \( \sigma_n \) is the normal stress, and \( \lambda \) is the pore-fluid pressure ratio, as defined by Hubbert and Rubey (1959). The Coulomb failure criterion, with \( \rho \) between roughly 0.6 and 0.85 and with \( S_0 \), between roughly 5 and 20 MPa,
Experimental deformation of salt (e.g., Carter and Hansen, 1983) demonstrates a different sort of behavior which is not suitably modeled by the Coulomb criterion. Even at the temperature and pressure conditions found at shallow depths in the earth, salt is in the ductile regime. At typical depths for a basal detachment (2-6 km), geological strain rates ($10^{-14}$) and geothermal gradients ($20^\circ$-$35^\circ$C km$^{-1}$), the yield strength $\tau_0$ for salt is between roughly 100 kPa and 1 MPa (Carter and Hansen, 1983). In a salt-dominated basal decollement, the normal stress dependence disappears. Instead, it becomes more appropriate to write:

$$\tau = \tau_0 \leq 1 \text{ MPa}$$

Thus, under conditions which are appropriate to a basal detachment, salt is between 1 and 2 orders of magnitude weaker than most other rocks.

If the basal decollement occurs along the salt horizon, the critical taper may be very small. Let us assume a pressure-independent (i.e., plastic) yield stress $\tau_0$ along the basal decollement, with the overlying rock having a strength dominated by a friction coefficient $\mu = \tan \phi$ and with $S_0$ negligible. If $\tau_0 = 1 \text{ MPa}$ a few km deep at the base of a fold-and-thrust belt, then essentially no taper ($\approx 1^\circ$) is required for the overlying sediments to be pushed horizontally toward the foreland.
Plot of log subgrain size (μm) versus log $\sigma_d$ (MPa) for creep experiments on Avery Island domal salt. Horizontal bars show maximum and minimum stress levels recorded during experiments and vertical bars show one standard deviation from mean subgrain diameter. Data fit the dashed line with a correlation coefficient of 0.9775. Also shown are mean subgrain sizes and corresponding $\sigma_d$ for seven naturally bedded, domal, and anticlinal rock-salts. Vertical bars show one standard deviation from mean subgrain diameter (adapted from Carter et al., 1982).
Appalachian Plateau

NY
15% Layer-Parallel Shortening

Extent of Silurian Salt

Juniata Culmination

Catskill Mts.

Lackawanna syncline

Albany

Harrisburg

PA

OH

WV

50 0 50 100

Kilometers
Layer-Parallel Shortening:

- Brittle collapse leads to anticlinal growth
- Ramp Thrusts
- Wedge Faulting

Deformation within Appalachian Plateau Detachment Sheet

3km

Silurian Salt

Wedge Faults

Ramp Faults
Another version of the Davis & Engelder (1985) tapered wedge diagram.
Block diagrams showing three basic types of salt canopy formed by coalescence of salt stocks, salt walls and salt tongues, respectively. The degree of coalescence increases towards the front of each group. The sutures between coalesced salt structures can be recognized by lenses of country rock between salt, by cuspate synforms in the overburden above, and by pairs of raised, appressed antiforms in the margins of the coalesced salt sheets.
Block diagrams showing the schematic shapes of known classes of salt structures

Jackson & Talbot, 1994
Sedimentary record of salt flow during extension (above) and shortening (below). The prekinematic layer was deposited before salt flow began. The synkinematic layer accumulated during salt flow and may include internal onlap or truncation. The postkinematic layer was deposited after salt flow ceased; its base may onlap or truncate originally opencast (exposed) structures in the synkinematic layer.
3.2.5 Nappes

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Fold nappes of the Montagne Noire
Nappe du Mont-Peyroux, Haut-Languedoc, France

Carte géologique générale de la Montagne noire (modifié d'après Demange, 1998).

Coupe générale synthétique de la Montagne noire (modifié d'après Demange, 1998).
Fold nappes of the Montagne Noire
Nappe du Mont-Peyroux, Haut-Languedoc, France

schéma général (Engel et al., 1978)

coupe dans la partie occidentale de la nappe (Demange, 1997)
Lithospheric section of north-western Alps - 1) Austroalpine: Sesia-Lanzo inlier (sl) and Dent Blanche nappe s.l. (db), including Matterhorn (Ma); 2) Penninic domain (P): Piedmont ophiolitic units (po), Monte Rosa (mr) and Grand St. Bernard (sb) nappes, underlain by lower Penninic and outer Penninic Valais zone (va), Penninic klippen (Pk), Penninic frontal thrust (pft); 3) Helvetic basement slices and cover nappes (H); 4) Molasse foredeep (M); 5) Jura belt (J); 6) buried wedge (BW) of European mantle or eclogitized crustal units; 7) European lithosphere: continental crust (EC) and mantle (EM); asthenosphere (AS); 8) Adriatic lithosphere: antithetic belt of Southern Alps (SA) and mantle (AM); Periadriatic fault system (pl); 9) Padane-Adriatic foreland (PA)
In the Helvitic Nappes the appearance of calcite twin lamellae changes systematically as a function of deformation temperature. Microtwins and straight narrow (< 1/μm) lamellae are characteristic of very low temperatures where no other slip system competes and where the absence of effective recovery mechanisms prohibits large strains by twinning. Above ca 150°C, thicker (> 1-5/μm) but fewer twins are developed. Above approximately 200°C, curved twins, twinned twins and completely twinned grains indicate the progressive importance of other slip systems, and larger intracrystalline strains are possible. At ca 250°C and above ancient straight twin lamellae are modified into irregular geometries by recrystallization and grain-boundary migration.
Pyrite framboids with stress shadow

Layer-parallel shortening

Appalachian Valley & Ridge
Selensgrove, PA, USA
Pyrite framboids with stress shadow

Layer-parallel shortening

Cleavage and the development of a pressure shadow
Hsuehshan Range, Taiwan
Mesozoic cover spit out between Aar and Gotthard Massifs
Swiss Alps near Andermatt, Switzerland

Pyrite pressure shadows

Terry Engelder
Mesozoic cover spit out between Aar and Gotthard Massifs
Swiss Alps near Andermatt, Switzerland

Cracks in silt layer indicate up dip stretching

Bedding to cleavage = 2.5°
Silt layer dip = 85°

silt
shale
Nummulites with cleavage from the compression of the Helvetic Nappes
Swiss Alps near Andermatt, Switzerland
Overturned limb: Lebanon Nappe, Great Valley
Appalachian Mountains, Elizabethtown, Pennsylvania, USA

Terry Engelder
4.1.1 Properties of Faults

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Classification of Faults

Faults

- Normal
- Reverse
- Strike-slip
Reverse Fault to Detachment (Décollement)
Normal Fault

Normal Fault, Utah

Terry Engelder
Faults

- Normal
- Reverse
- Strike-slip

San Andreas Fault, California

Right-lateral Strike-slip Fault
Andersonian Classification of Faults

Fault Regimes in the Lithosphere

Normal-Fault Regime

\[ S_v = \sigma_1 \]
\[ S_h = \sigma_3 \]

Strike-slip Fault Regime

\[ S_v = \sigma_2 \]
\[ S_h = \sigma_3 \]
\[ S_H = \sigma_1 \]

Thrust-Fault Regime

\[ S_v = \sigma_3 \]
\[ S_h = \sigma_2 \]
\[ S_H = \sigma_1 \]
Fault-related rocks: A terminology

COHERENT BUT UNFOLIATED ROCKS PRODUCED BY MICRO- AND/OR MACRO-FRACTURING AND SHOWING LITTLE OR NO FRICTIONALLY PRODUCED THERMAL EFFECTS.

NON-FOLIATED, BUT WITH SOME FRICTIONALLY PRODUCED GLASS CEMENTING A MICROBRECCIA.

MATRIX PRODUCED BY SYNTECTONIC CRYSTAL-PLASTIC PROCESSES, HAS AT LEAST MINOR MYLONITIC FOLIATION, LITTLE RECOVERY, AND ALMOST NO ANNEALING, WITH APPROPRIATE MINERAL CONTRASTS, SURVIVOR MEGACRYSTS COMPRIZE MORE THAN 50% OF THE ROCK.

MATRIX PRODUCED BY SYNTECTONIC CRYSTAL-PLASTIC PROCESSES, SHOWS STRONG RECOVERY, POSSIBLY WITH SOME ANNEALING. STRONG MYLONITIC FOLIATION COMMON. WITH APPROPRIATE MINERAL CONTRASTS, SURVIVOR MEGACRYSTS COMPRIZE 10 TO 50% OF THE ROCK.

MATRIX PRODUCED BY SYNTECTONIC CRYSTAL-PLASTIC PROCESSES, SHOWS PERVASIVE RECOVERY, POSSIBLY WITH EXTENSIVE ANNEALING. SURVIVOR MEGACRYSTS COMPRISE LESS THAN 10% OF THE ROCK. MATRIX GRAINS ARE LESS THAN 0.5 MM IN DIAMETER.

PERVASIVE RECOVERY, INCLUDING ANNEALING OF SYNTECTONICALLY PRODUCED MATRIX WITH MATRIX GRAINS INCREASING ABOVE 0.5 MM. MEGACRYSTS MAY INVOLVE SYNKINEMATIC OR ANNEALING GROWTH, EITHER AS NEOCRYSTS OR AS OVERGROWTHS ON PORPHYROCLASTS.

MATRIX RECRYSTALLIZATION INCREASES AVERAGE GRAIN SIZE TO EQUAL OR EXCEED THAT OF THE PROTOLITH.

RATE OF STRAIN

RATE OF RECOVERY
The evolutionary history of a rock in the vicinity of a major fault zone might follow the path illustrated here. The main rock mass might pass through a series of deformations involving generally low strain rates in going to high metamorphic grade and back to surface conditions. Superimposed on this general pattern could be a number of brief pulses of high strain rates, as indicated by the spikes in Figure 2. Frictional heating at higher strain rates might cause temporary, slightly increased recovery rates, as suggested by curvature of the spikes to the right. Early-formed breccia and gouge (A) or mylonite (B) would be homogenized and in part camouflaged by later metamorphism and ductile flowage. Mylonitic and cataclastic rocks produced after the metamorphic peak would be much more likely to survive in recognizable form. Some of the early-formed mylonites (D) would be likely to have a variety of younger deformational features superimposed on them, such as foliation, kink bands, and passive and/or flexural folds, or they might be slickensided or brecciated by late fault motions (E). Thus, a typical mylonitic specimen should be considered the end result of a long history of these types of deformations and metamorphisms under a variety of pressure, temperature, and strain conditions.

Wise et al., 1984
Faulting is affected by changes in mechanism from brittle to ductile behavior as evidenced by exhumed fault zones which were once active at great depths. Some famous examples include the Moine Thrust of Scotland, the Alpine Fault of New Zealand, the Outer Hebrides Thrusts of Scotland, the Greenland Shear Zones, and the mantled gneiss domes of the southwestern United States. A correlation is seen between depth of burial and several parameters including temperature, confining pressure, metamorphic grade, and general type of fault rock (Sibson, 1977, 1986). Fault rock is characterized by grain reduction through either brittle or ductile mechanisms with the three main types of fault rock being gouge, cataclasite, and mylonite. **Gouge**, a product of frictional slip, is a cataclastic material without fabric or cohesion; grain-size reduction is dominated by brittle fracture. A **cataclasite** is a nonfoliated fault-zone material with cohesion, whereas a **mylonite** is a foliated fault-zone material with cohesion. Both cataclasites and mylonites are the product of semibrittle deformation with differences depending, in part, on strain rate versus recovery rate of grains. As a general rule, higher $\sigma_{dr}$ is associated with higher strain rate.
4.1.2 – Faulted Surfaces

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Classification of faulted surfaces

- Means, 1987
  - scratches
  - tails
  - spikes

- Petit, 1987
  - streaks
  - fibers

Diagram:
- Striating element
- Crystallization
Classification of faulted surfaces

- Doblas, 1998

<table>
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<th>CM</th>
<th>ST</th>
<th>FR</th>
<th>IS</th>
<th>TM</th>
<th>AE</th>
<th>DE</th>
<th>PW</th>
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<tr>
<td>&quot;V&quot; or crescentic markings</td>
<td>Steps</td>
<td>Fractures</td>
<td>Trains of inclined planar structures</td>
<td>Trailed material</td>
<td>Asymmetric elevations</td>
<td>Deformed elements</td>
<td>Asymmetric plan-view features</td>
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Legend:
- 1
- 2
- a
- b
- c
- d
Classification of faulted surfaces

- Brittle:
  - Asperity ploughing
  - Debris ploughing
  - Step-like slip

- Ductile:
  - Fiber growth
  - Amorphous surfaces
  - Shiny surfaces

Aydin & Engelder, 2014
53 cm thick cleavage duplex 2 m above the Selinsgrove Limestone in the Marcellus Formation at Newtown Hamilton.
Cleavage duplex in Union Springs at Selinsgrove Junction, PA (Handiboe Core).
Cleavage duplexes in Utica = High pore pressure fluid (gas & water)
Thickness (m)

Distance above Selinsgrove (m)

Number of cleavage duplexes

Distance above Selinsgrove (m)

Number of cleavage duplexes

Thickness (m)

Number of cleavage duplexes
Bedding-parallel slip surfaces in the Lock Haven Formation north of Williamsport, PA.

A mirror slip surface.

Green chlorite over white quartz over ‘black’ graywacke with ridge-in-groove striation.

A mirror slip surface that appears black when green chlorite sits directly on ‘black’ graywacke.
(Right) Ridge-in-groove striation on bedding slip surface in Lock Haven Formation showing the olive green color of a chlorite film on white quartz fibers.

(Left) Mirror slip surface of a chlorite film on a greywacke matrix. Olive green light refracts from the mirror.
The morphology of intraformational slip surfaces (ISS) in the Mahantango-Marcellus section

- slip fibers
- ridge-in-groove striations

Mahantango (Bilger 117.49 m)
Oatka Creek (Handiboe 145.57 m)
Union Springs (Handiboe 2.36 m)
Union Springs (Erb 4.30 m)
Progressive mineral development on slip surfaces.

Pyrite and quartz entrained in fibers.

Mahantango (Bilger, 108.81 m)

Union Springs (Handiboe, 10.57 m)

Union Springs (Handiboe, 31.84 m)

Calcite and chlorite entrained in a mirror surface.

Pyrite entrained in a ridge-in-grove striations of chlorite.

A matrix breccia entrained in calcite fibers.
Distribution of ISS in four ABBSG cores
4.1.3 Fault Seals

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Fault Terminology

- **Hangingwall**
- **Footwall**
- **Offset marker horizon**
- **Dip separation**
- **Fault**
- **Throw** $T$
- **Heave** $H$
- **Horizontal component of throw**
- **Vertical component of throw**
- **Dip** $\phi$
Allan diagram showing fluid migration pathways through permeable beds separated by impermeable beds; footwall *shaded*. Arrows give migration routes of fluids that are lighter than water. Oil accumulations are *solid black*; gas accumulation is indicated by *vertical lines*. (from Allan 1989)
Fault Properties

• Shale gouge ratio (SGR), i.e. the proportion of shale which has moved past each point on a fault

• Gouge ratio is an estimate of the proportion of fine-grained material entrained into the fault gouge from the wall rocks.

• Smear factor methods (including clay smear potential and shale smear factor) estimate the profile thickness of a shale drawn along the fault zone during faulting.
• Effect of rock properties and stress on relative permeability of fault zone rock.

Suzuki, 2003
Conventional fault seal analyses estimate fault zone shale content (i.e., shale/gouge ratio) which is considered to impart a first order control on fault rock capillary properties.

Yielding et al., 1997
• Normal fault with low SGR

Triassic sandstones, High Atlas, Morocco

www.dstu.univ-montp2.fr
- Offshore Trinidad: Dynamic seal across siliciclastic faults as a function of shale/gouge ratio. 

Gibson, 1994
• Differential pressure across a seal as a function of SGR

Gibson, 1994
4.1.4 – Normal Fault Systems

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Small Normal Fault
preCambrian Uinta Mountain Group, north flank Uinta Mountains, Utah, USA
While thrust faults are characteristic of convergent margins of continental crust, normal faults are characteristic of the extension of continental crust. Normal faulting is commonly found within continental crust that has grown warm and stretched. The Basin and Range province of the western United States is finest examples in the world of crustal extension over a large region. Central Africa is cut by extensional or rift basins and a consequence of the thermal heating of continental crust. Some rift basins eventually grow to become filled with basalts and sheeted dike complexes that are the first signs of oceanic crust. The passive margins of most continents are bounded by rift basins without extending to the point that oceanic crust has formed in the rifts.

A classic rift zone of relatively young age is the Red Sea rift. A cross section through the Red Sea rift shows a series of listric normal faults dipping toward the sea.
African Plate Triple Junction

A triple junction (or triple point), where three plates are pulling away from one another: the Arabian Plate, and the two parts of the African Plate (the Nubian and the Somalian) splitting along the East African Rift Zone. In general the divergent plate margins are regions where new crustal material is formed from the mantle. As seen from the diagrams above the plumes from the mantle bring up material from the mantle to form new crust.
There have been a number of interesting models to explain the geometry of rifting at the edge of continents. The most difficult problem involves providing an explanation for the faulting that does not open large gaps in the crust. The listric fault geometry accomplishes this purpose. Another model that seems to work in some cases is called the domino model (Wernicke & Burchfiel, 1982).

Domino-style normal faulting

Listric normal faulting with reverse drag

Imbricate listric normal faulting

The Basin and Range of the western United State is characterized by rift faulting with the level of erosion reaching well into basement rocks. Core complexes are believed to be a manifestation of listric faulting reaching mid- to lower crustal levels (Lister & Davis, 1989).
Very elegant clay models have been constructed that capture the shape of faulting at continental margins. An example from Ken McClay is shown below.
Most normal faults are concave upward, or listric. This shape can be produced by geometric constraints, either because the faults reactivate curved thrusts, or because they must be curved to accommodate rotations. Another effect which will produce curved faults is the variation of rheology with depth: brittle failure at shallow depths produces less fault rotation than does distributed creep in the lower part of the crust. An important geometric feature of normal faulting is the uplift of the footwall. The amount of such uplift is related not only to the elastic properties of the lithosphere, but also to the throw and dip of the fault (Jackson & McKenzie, 1983).

The uplift which forms the trap of the Beatrice oil field was produced by the unloading of a normal fault. Because the displacement on the fault has a maximum, and decreases both to the east and the west, the uplift also dies away along strike. In the southeastern direction, the structure is closed by the downthrown shales, and in the northwestern direction by the decay of the uplift with increasing distance from the fault. Hence, the uplift associated with the normal fault produces closure in all four directions to form a structural trap.

Jackson & McKenzie, 1983
Once continents have rifted apart each continent is bounded by a series of half grabens that dip oceanward. A classic example of this is found between the Texas Gulf Coast and the Gulf of Mexico. Seismic sections show a very elaborate network of normal faults. The master faults are listric in nature but secondary faults are antithetic with dips back toward the continent. The largest listric faults on a continental margin may cut down to the Moho. In the Texas Gulf Coast many of the listric faults cut down to a detachment at the level of a salt layer in the Jurassic.

The source of these figures is unknown but Dula (1991) is offered as a reference.
Block diagrams illustrating the development of normal faults and relay ramps in a limestone–mudrock sequence. (a) Veins initiate in the brittle limestones. (b) Extension continues, with shear across the veins and connection of veins across the mudrocks. Relay ramps develop at steps between faults in map view. Displacement minima typically occur at steps. (c) Linkage occurs between segments, with breaching of relay ramps.
Relay Ramps for Normal Faults
Blue Lias, Lilstock Beach, Bristol Channel, United Kingdom

Normal fault cuts brittle limestone layers

Terry Engelder
Examples of an open relays with displacement profile at relay ramps and associated separation profiles (distance between faults normal to their trace along overlap). Indisplacement profile, broken lines represent aggregate profiles at the overlap zone. Displacement profiles of each segment are projected following an axis perpendicular to fault segments. Error bars are labelled on profiles.
A simplified block diagram and structural map of the Devil's Lane graben. Two major relay ramp features are present, dipping in opposing directions.
Relay Ramp Forms and Normal Fault Linkage
Canyonlands National Park, Utah, USA
The relationship between the maximum finite displacement on a fault ($D$) and its maximum dimensions ($W$) is $D = cW^n$ where $c$ is a constant and $n$ ranges between 1.0 and 2 (Walsh and Watterson, 1988; Gibson et al., 1989; Marrett and Allmendinger, 1991; Cowie and Scholz, 1992).

Examples of basin geometries assuming boundary fault propagation occurs during basin formation; based on a model by Schlische and Anders (1996). $L =$ fault length; other letters are the same as equation 1.
Watterson (1986) and Walsh and Watterson (1989) proposed an empirical growth model based on combined data that they interpreted as $D \alpha L^2$. Their model assumes that brittle faults grow in elliptical slip events in which the fault length (which they call width) increases by a constant increment. Marrett and Allmendinger (1991) interpreted a similar combined data set as $D \alpha L^{1.5}$. Dawers et al., (1993) find that the ratio $D_{\text{avg}}/L$ for the Volcanic Tableland faults is $10^{-2}$. 

Normalized distance along fault

Normalized throw

- $L = 696 \text{ m}$, $D_{\text{max}} = 8.40 \text{ m}$, $D_{\text{avg}} = 5.30 \text{ m}$
- $L = 740 \text{ m}$, $D_{\text{max}} = 10.20 \text{ m}$, $D_{\text{avg}} = 5.81 \text{ m}$
- $L = 780 \text{ m}$, $D_{\text{max}} = 10.10 \text{ m}$, $D_{\text{avg}} = 7.12 \text{ m}$
- $L = 866 \text{ m}$, $D_{\text{max}} = 8.30 \text{ m}$, $D_{\text{avg}} = 5.14 \text{ m}$
- $L = 1820 \text{ m}$, $D_{\text{max}} = 18.50 \text{ m}$, $D_{\text{avg}} = 10.06 \text{ m}$
- $L = 1630 \text{ m}$, $D_{\text{max}} = 17.30 \text{ m}$, $D_{\text{avg}} = 11.96 \text{ m}$
- $L = 2210 \text{ m}$, $D_{\text{max}} = 31.00 \text{ m}$, $D_{\text{avg}} = 21.81 \text{ m}$
Digital elevation map of the Basin and Range
Domino normal faults in the Newfoundland Mountains, northwest Utah. These faults offset a Pennsylvanian-Devonian unconformity in a top to the east (left) sense of shear. Note that the faults, although presently nearly horizontal, cut the steeply dipping bedding at high angles.

From Rick Allmendinger, Cornell
4.2.1 - The Overthrust Problem

The Hubbert-Rubey (1959) solution to Escher’s “colossal over-shove” problem

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Fold nappes represent behavior in the middle crust where ductile deformation enables for most of the tectonic displacement. In the upper crust brittle behavior mainly through frictional slip enable most of the tectonics behavior. Even in the brittle crust pressure solution is a ductile mechanism that accompanies frictional slip. Overthrust terrains, which operate in the brittle crust, can be divided into two general classes: 1.) Those where the decollement surface is parallel to bedding within the sedimentary section. The central Appalachian Valley and Ridge and the Canadian Rockies are representative examples. 2.) Those terrains where a horizontal thrust sheet develops in crystalline basement and breaks upward through the sedimentary overburden. The Wyoming Province of the U.S. Rockies is a type example of structures in sedimentary rocks developing over crystalline upthrusts. The outer crystalline thrusts of the Blue Ridge in the southern Appalachians are believed to be other examples.

Three general classes of thrust faults develop within thrust sheets whose decollement is within the sedimentary section. Geiser (198) identified these structures as: Break Thrusts, Fold Thrusts, and Shortening Thrusts. Each of the three has a different mechanical behavior as explained below.

Break thrusts develop where the Coulomb-Mohr failure criterion is favored over folding and shortening. Fold-thrust belts developed in a thick but reasonably homogeneous stratigraphic section seem to favor the break thrust style. The southern Appalachians is one example of the break thrust style. This style seems to be the most common throughout the upper crust.
Fold thrusts develop where the viscosity contrast between a sheet of sediments ($\eta_1$) and the underlying rock ($\eta$) is large.

$$\eta > \eta_1$$

In this case the Euler Buckling Force is relatively low compared with the Coulomb-Mohr failure criterion.

$$\frac{\pi EI}{L^2} < S_0 + \mu_\sigma$$

A fine example of fold thrusts may be found in the Swiss Jura where thin box folds form over a Mesozoic salt.

Shortening thrusts develop in association with volume loss strain. An example is the Appalachian Plateau where the plateau is folded slightly over blind thrusts but the dominant mechanism for absorbing the decollement slip layer parallel shortening. The mechanism for shortening is the development of solution cleavage where the dissolved rock is carried out of the section in circulating water. Shortening occurs because the yield stress for pressure solution cleavage is less than the Euler Buckling Force and the Coulomb-Mohr failure criterion.
Escher (1841) identified the Glarus thrust as, “Folge einer kolossalen Ueberschiebung oder eines Umbiegens der Schichten” (an episode of colossal over-shove or over-fold).

Bailey (1935) says of Escher’s interpretation, “he admits that this interpretation lands one into great difficulties”
Glarus Thrust
Alps, Switzerland
The difficulty with Escher’s interpretation led the H.D. Rogers (1842) to write of the Appalachian Valley and Ridge, “they result from an onward, billowy movement proceeding from beneath, and not folding due simply to some great horizontal or lateral compression”.

View of anthracite grade folding at Bear Valley near Shomkin, PA
Observations of the “colossal over-shove” by lateral compression lasted almost 70 years before Escher’s (1841) great difficulty was understood in terms of mechanics.

Moine – Scotland: Nicol (1861)
Scandinavia – Norway: Tornebohm (1883)
Rocky Mountains – Canada: McConnell (1887)
Himalayans – India: Oldam (1893)

Eldon (Cambrian)
Belly River (Cretaceous)

View of the McConnell Thrust from west of Calgary, Canadian Rockies
Granite has the strength to support a column 3 km high. Thus, sliding on a fault with a friction of $\approx 0.6$, granite has the strength to support the stress necessary to slide a block $\approx 5$ km long.

Drawing from Hubbert and Rubey (1959)
Hubbert and Rubey (1959) point out that thickness of the block adds strength so that blocks may get longer by 2.6 km per km of thickness.

The density hoops supporting a silo is an analog for stress in the earth and depth-related increase in rock strength.
Reasonable crustal thicknesses can account for up to 10 to 15 km of over-shove on a detachment of normal friction!

http://www.nps.gov/history/history/online_books/glac/3/images/fig3.jpg
The Glarus thrust has a minimum displacement of 40 km and the Scandinavian over thrusts have a displacement of 130 km.
Three solutions to Escher’s colossal over-shove

1. A high temperature detachment
2. A strong tectonic wedge
3. A low-strength detachment

Under the heading of “Swiss Tectonic Arena Sardona”, Switzerland asks UNESCO support for the “Glarus overthrust” World Heritage site.
The contrasting "double-fold" of Escher and Heim with the single Glarus thrust sheet proposed by Bertrand (E.B. Bailey, 1935).
Strong Tectonic Wedge

Davis et al, 1985

Rockies, Montana to Canada
Engelder, 1979

Low Strength Detachment

Layer-Parallel Shortening within Detachment Sheet

230 km

15\% Layer-Parallel Shortening (Allegheny Front to Buffalo)
High Temperature – No
Wedge Strength – No
Salt Weakness - No

Pine Mountain Block

1.5 km

50 km

43 km
ROLE OF FLUID PRESSURE IN MECHANICS OF OVERTHRUST FAULTING

1. Mechanics of Fluid-Filled Porous Solids and Its Application to Overthrust Faulting

BY M. KING HUBBERT AND WILLIAM W. RUBFY

ABSTRACT

Promise of resolving the paradox of overthrust faulting arises from a consideration of the influence of the pressure of interstitial fluids upon the effective stresses in rocks. If, in a porous rock filled with a fluid at pressure \( p \), the normal and shear components of total stress across any given plane are \( S \) and \( T \), then

\[
\sigma = S - p, \tag{1}
\]

\[
\tau = T, \tag{2}
\]

are the corresponding components of the effective stress in the solid alone.
Three necessary geological conditions for the Hubbert-Rubey mechanism for overthrust faulting:

1. Requires a mechanism(s) for generating abnormal pore pressure.
   - High permeability rock

2. Requires a mechanism(s) for delivering abnormal pressure to the slip surface.
   - Expansion of cold air
   - The popped top

3. Requires a mechanism(s) for maintaining pressure even when leakage occurs.
   - Low permeability rock
   - Time-dependent expansion
Mechanisms for Generating Overpressure

• Stress-Related Mechanisms ★
  • Disequilibrium Compaction
  • Tectonic Compaction

• Fluid Volume Increase Mechanisms
  • Aquathermal Expansion
  • Water release during mineral diagenesis
  • Hydrocarbon generation

• Fluid Dynamic Mechanisms ★
  • Potentiometric head
  • Hydrocarbon buoyancy

Most effective generation mechanisms

Swarbrick & Osborne, 1998
Two (2) Hubbert-Rubey (1959) mechanisms for abnormal pore pressure (mechanical compaction = stress mechanism - Orange) and (buoyancy = fluid dynamic mechanism - Red).
Deformation associated with the movement of the Muddy Mountain overthrust in the Buffington window, southeastern Nevada

WILLIAM G. BROCK  AMOCO Production Company, Security Life Building, Denver, Colorado 80202
TERRY ENGELDER  Lamont-Doherty Geological Observatory, Palisades, New York 10964

ABSTRACT

The Muddy Mountain overthrust, exposed in the Buffington window, southeastern Nevada, consists of a Paleozoic carbonate sheet thrust over Mesozoic Aztec Sandstone, with a molasse filling topographic lows. Evidence suggests that the thrust sheet moved across an erosional surface and that the molasse may have been a layer. on a fault when the shear stress on this plane exceeds the sum of the cohesive strength and the product of the coefficient of friction and the effective normal stress. The Hubbert-Rubey hypothesis suggests a mechanism for reducing the effective normal stress, whereas the Wilson hypothesis suggests a way to reduce the coefficient of friction.

We wished to study deformation associated with overthrust
(Turonian - Coniacian: 93-88 Ma)

26.2 km displacement
26.2 km total shortening
3.5 km crustal thickening

elevation = 2.8 km.

(Aptian-Early Albian?: >110 Ma)

FUTURE PXT
The Muddy Mountain Thrust

Topography (DeCelles & Coogan, 2006)

Sea Level

Aztec Sandstone with fore-thrust debris

Scale 1:1

3 km 10 km

Thrust distance without rear-end collapse (normal friction)
Longwell (1922) discovered the Muddy Mountain overthrust in southeastern Nevada, in which a block of Paleozoic strata with a stratigraphic thickness of about 25,000 feet had overridden the same section for about 15 miles.
Buffington Window
Looking west (1971)
99 Ma fore-thrust debris (Fleck and Carr, 1990)
How can a pore pressure be maintained when the detachment rides over the land surface?

“It can’t!” (Brock and Engelder, 1977)
What does rock look like at the sliding surface when slip = 56 km??
STRUCTURE OF THE NORTHERN MUDDY MOUNTAIN AREA, NEVADA

BY CHESTER R. LONGWELL
Valley of Fire

Ja Aztec Sandstone
Pz_c Lower Paleozoic Carbonates
Zion National Park from Checkerboard Mesa

Dune cross-strata: High permeability

No Hematite

Oil-Water Contact?

Hematite

Horizontal bedding: Low permeability zones that restrict fluid flow

Navajo Sandstone

Beitler et al., 2003
Bleaching patterns indicate reducing fluid was buoyant

In SW Utah
Glen Canyon Sandstones
Six Uplifts = 1.8 x 10^{12} bbl
Ghawar = 80 x 10^{9} bbl

Photography by Tanya
Angel's Landing
Checkerboard Mesa
Oil-water contact
Valley of Fire

Seal

Reservoir

$J_a$

$K_{wt}$

Oil-water contact

1 km
Interpretation: Secor (1965) believed these were natural hydraulic fractures

Engelder et al., (2009)
Smoluchowski’s Dilemma Revisited: A Note on the Fluid Pressure History of the Central Appalachian Fold-Thrust Belt

TERRY ENGELDER
The Pennsylvania State University

Abstract

Cross-fold joints in the Central Appalachian fold-thrust belt propagated during periods of abnormally high fluid pressure prior to tectonic compaction and the development of first-order Alleghanian structures in the valley and ridge. These early joints, found in...
Hydrodynamic Pressure

Toth, 1962

Oliver, 1986

Oliver, 1986

Hydrodynamic Pressure

Toth, 1962
Rule #1: Methane at pressure drives joints!

Gradual increase in $K_i$ and instantaneous velocity
Propagation returns to subcritical region S3 prior to the next hesitation

Lacazette & Engelder, 1992

Savalli & Engelder, 2005
Rule #2: Orientation of NHF controlled by superposition of gravity-related stress and tectonic stress yoked to pore pressure!

Engelder, & Fischer, 1994

Bell, 1990

SCOTIAN SHELF, CANADA
Rule #2: Orientation of NHF controlled by superposition of gravity-related stress and tectonic stress yoked to pore pressure!

Bell, 1990

SCOTIAN SHELF, CANADA

Vertical NHF if $S_h < P_p < S_v$

Horizontal NHF if $S_v < P_p < S_h$

Bell, 1990
ROLE OF FLUID PRESSURE IN MECHANICS OF OVERTHRUST FAULTING

I. Mechanics of Fluid-Filled Porous Solids and Its Application to Overthrust Faulting

By M. King Hubbert and William W. Rubey

Journal of Structural Geology 69 (2014) 519–537

Revisiting the Hubbert–Rubey pore pressure model for overthrust faulting: Inferences from bedding-parallel detachment surfaces within Middle Devonian gas shale, the Appalachian Basin, USA

Murat G. Aydin¹, Terry Engelder*
Marcellus Shale Asset Optimization through Increased Geological Understanding
Yang, Bowman, Morris, Zagorski (2013)
AAPG ACE

- Marcellus not created equal
  - Gas in place
  - Reservoir quality
  - Completions quality
- Engineering practices
  - Drilling
  - Completions
Another major Appalachian overthrust is the Pine Mountain overthrust and the Cumberland thrust block first described by Wentworth (1921a; 1921b) and subsequently studied in more detail by Butts (1927), Rich (1934), and Hubbert and Rubey (1959) said:

According to Young (1957), the fault surface has been intersected by numerous gas wells which show that it occurs near the base of a Devonian shale at an average depth of about 5500 feet, or a little more than a mile. It is also of interest that high pressures encountered in the fault zone cause troublesome blowouts while drilling. No pressure measurements were given, but since the drilling was with cable tools this does not necessarily imply that the pressure was abnormally high.
Chattanooga (Upper Devonian) black shale

Rome-Conasauga (Cambrian) shale

24 km of foreland slip

28 km of continuous rupture in the Chattanooga

6 km

Mitra, 1988, GSAB
extent to Silurian salt

extent of Ohio Shale
4.2.2 – Fault Bend Folding

An AAPG Short Course by
Terry Engelder
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BLIND AUTOCHTHONOUS ROOF DUPLEX

EMERGENT AUTOCHTHONOUS ROOF DUPLEX

After Boyer & Elliott, 1982

After Banks & Warburton, 1985

Geiser, 1988
These are the types of structures that gave rise to the fault-bend fold model of Suppe (1983). The fault-bend fold model was greatly simplified by cutting some of the section out with a hanging wall ramp.

The effect of the hanging wall ramp was to over steepen the beds of the leading syncline so that the fault-bend fold became asymmetrical with the nose of the fold dipping on the order of $65^\circ$ while the trailing hanging wall flat dips toward the hinterland at about $25^\circ$. 
Dip Domains
Appalachian Valley and Ridge

Domain C
Domain B
Domain A

Axial trace
Boundary between different rotation domains
Outcrop trace
Sense of rotation

ROTATION DOMAINS

MAP VIEW

M. & U. Silurian
L. & M. Devonian
U. Devonian

Google Earth

Faill, 1973
Dip Domains
Appalachian Valley and Ridge

Faill, 1973
Explanation for the original dip-domain model as first put forward by Faill (1973)

3D Model
(Fault-Bend Anticline)

from Medwedeff, Lacazette, 2000
Suppe’s (1983) original study of fault-bend folds gave the Powell Valley Anticline as a ‘type’ example of such folds.
Fault-related fold
Cumberland Plateau, Southern Appalachian Mtns, Tennessee, USA

Ohmacher & Aydin, 1997
As early as the first survey of Pennsylvania, geologists realized that the anticlines of the state were asymmetric with the dip of the foreland limb being more than twice as steep as the dip of the hinterland limb (Rogers and Rogers 1843).
Nittany Mountain Syncline
Foreland Limb, Nittany Mountain Syncline

Hinterland limb – Nittany Mountain syncline
4.2.3 – Structural Validation

An AAPG Short Course by
Terry Engelder
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The Pennsylvania State University
Validating structural interpretations is necessary to extract additional information, such as the shape of the structure between wells and seismic lines, predicting the presence of structures too small to be seen at the resolution of seismic lines and surface outcrops, and determining the structural evolution over time. Most of the techniques of structural restoration, balance and prediction are related to one another by use of a common set of kinematic models.

A restorable structure can be returned to its original, pre-deformation geometry with a perfect or near-perfect fit of all the segments in their correct pre-deformation order. Restoration is a fundamental test of the validity of the interpretation. A restorable structure is internally consistent and therefore has a topologically possible geometry. An unrestorable structure is topologically impossible and therefore is geologically not possible (Dahlstrom 1969). An interpretation based on a large amount of hard data, such as a complete exposure, many wells, or good seismic depth sections controlled by wells, is nearly always restorable, whereas interpretations based on sparse data are rarely restorable. This is the empirical evidence that validates restoration as a validation technique.
Kinematic models represent simplified descriptions of the mechanical processes that form structures. The deformation in some structures is more complex than can be fit by one of the simple kinematic models. For these structures the more general area balancing methods can be appropriate. Using the relationship between displaced area and depth, a structure can be tested for area balance and its lower detachment predicted without performing a restoration or a model-based prediction. Layer-parallel strain is treated here because it is an intrinsic part of both the kinematic models and the area-depth relationship and because it provides a tool for predicting sub-resolution structure (i.e., folds and faults too small to be seen at the resolution of the data) and is another tool for validating the structural interpretation.

One of the most common kinematic behaviors in layered rock is flexural slip. There are a number of validation models that assume that bed lengths do not change during deformation, particularly involving flexural slip. Given the evidence for layer-parallel shortening in forelands, especially the Appalachian Plateau, this assumption may be questioned. Never-the-less a first cut at restoration can assume constant bed length so that a pin line and loose line remain intact during restoration. Below are examples of both a perfect and imperfect restorations.
Structural Validation

Flexural-slip restoration is based on the model that bed lengths do not change during deformation (Chamberlin 1910; Dahlstrom 1969; Woodward et al. 1985, 1989). Internal deformation is assumed to occur mainly by layer-parallel simple shear (Fig. 11.3b). For the area to remain constant, the bed thicknesses must be unchanged by the deformation as well. This is the constant bed length, constant bed thickness (constant BLT) model. Flexural-slip restoration is particularly suitable where the beds are folded and structurally induced thickness changes are small, the style of deformation in many compressional structures.

Restoration may be used for fault-shape prediction. Based on the assumption of constant BLT on a cross section bounded by vertical pin lines, there is a unique relationship between the hangingwall shape caused by movement on a fault and the shape of the fault itself. In the technique developed by Geiser et al. (1988), a complete deformed-state cross section is constructed while simultaneously producing a restored cross section. The method is based on:

1. constant bed length and bed thickness,
2. slip parallel to bedding,
3. fixed pin lines in the hangingwall and footwall of the fault, and
4. the hangingwall geometry is controlled by the fault shape.

The pin lines are chosen to be perpendicular to bedding. The data required to use the method are the location of a reference bed and the hangingwall and footwall fault cutoff locations of the reference bed. The original regional of the reference surface is not required. The method produces a cross section that is length balanced, has constant bed thickness and has bedding-normal pin lines at both ends. The technique is as follows:

1. Define the reference bed, its fault cutoffs, and the shape of the fault between the cutoffs (top).
2. Place the pin lines in the deformed-state cross section. Usually they are chosen to be perpendicular to bedding and beyond the limits of the structure of interest (top).
3. Measure the bed length of the reference horizon between the pin lines and draw the restored-state section (bottom).
4..... Construct one or more constant thickness beds in the hangingwall between the hangingwall pin and the fault (top).
5..... Measure all the bed lengths in the hangingwall between the hangingwall pin and the fault and place them on the restored-state cross section (Fig. 11.36b, bottom). This defines the shape of the footwall beds in the restored state.

6..... Draw the restored-state footwall on the deformed-state cross section (top). This gives a new hangingwall cutoff.

7..... Continue the cycle of steps 4–6 until the section is complete.
4.2.4 – Fault-related Folding Models

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
A small thrust in the Chilean Andes, showing a classic footwall ramp and flat geometry, overlain by a hangingwall flat.

From Rick Allmendinger, Cornell
Chevron fold – Fault-bend folds

Three types of rotation can be distinguished in association with a chevron fold. The rotation #1 is an external rotation as indicated by the change in orientation of the bedding. This is $\varphi$ in our analysis of kink folds. Rotation #2 is an internal rotation and is shown on the strain ellipse by $\theta$. Rotation #3 develops off of beds of limestone that have been shortened by cleavage development. Each block of limestone is bounded by pressure solution cleavage that dissolves the beds and permits shortening without the total rotation shown as #1 for the sandstone beds.

Fault-bend folds, versions of kink folds, are a consequence of décollement tectonics where bedding parallelDetachments climb section by cutting across several beds (Suppe, 1983). A model of fault-bend folding involves a detachment fault where the thrust plane comes from the right, cuts upward across bedding, and continues parallel with bedding off to the left. Upon the initiation of the fault across bedding two kink bands develop at the bends in the footwall because these bends cause folding in the upper sheet of the thrust system. The axial surfaces A and B terminate at the bends in the detachment where it changes from bedding parallel to cross cutting. The axial surfaces A' and B' terminate at the transitions between crosscutting and bedding-plane fault segments in the hanging wall at points X' and Y' which match points X and Y in the footwall. As slip continues, kink bands A-A' and B-B' grow in with and the anticline above the crosscutting fault grows in height.
Axial surfaces A' and B' move with the thrust sheet because they are fixed to the hanging wall cutoffs. Because the axial surfaces A and B are fixed to the footwall cutoffs X and Y, the beds must move through the axial surfaces, first bending and then unbending.

The motion of the fault-bend fold is complex because when the hanging-wall cutoff, Y', reaches the footwall cutoff, X, the fold has reached its maximum amplitude, which is the height of the step in décollement. At this point the deformation, axial surface A, which has been fixed to the footwall is suddenly released to move with the hanging-wall cutoff, Y', whereas axial surface B', which has been moving with the hanging wall, is suddenly locked to the footwall cutoff, X.

**Fault-propagation folds** form as a part of the process of fault propagation. In such a fold the thrust fault cuts across section but does not extend in the subsurface in front of the fold as was the case for a fault bend-fold. Like the fault-bend fold, as the thrust starts to step up through the section two kink bands immediately develop. One axial surface is pinned to the footwall cutoff and the beds roll through the fold by first folding and then unfolding. Another axial surface is pinned to the tip of the fold and beds roll through it as the fault propagates forward. A third axial surface touching the fault plane moves with the velocity of the thrust sheet and is the surface where the two initial kink bands have merged below the surface. Commonly, fault-propagation folds will become locked because the bending resistance of some formation may be too great. In this case the fault may propagate along the anticlinal or synclinal axial surfaces or somewhere in between.
Fault-related folding

Retrodeformable model of the progressive development of a simple-step fault-propagation fold (from Suppe & Medwedeff, 1990). The drawing is for a step-up angle of 29°, for which axial-surface A is fixed relative to the material. Under these conditions the constant-thickness and fixed-axis theories predict identical foldshapes.

Cross-section of Meilin anticline, western Taiwan overthrust belt near Chiavi, a fault-propagation fold.
Planar rollover panels above a normal fault that is composed of planar segments. Rollover panels are bounded on the left by active axial surfaces, which are pinned to fault bends (from Shaw et al., 1997).
Numerous models have been developed that relate fold shape to fault shape for various flexural-slip structural styles related to fault-related folding. Many of these have adopted planar or ramp-flat fault shapes, leading to *dip-domain cross-section styles*. Models like the fault-bend fold, fault-propagation fold and fault-tip fault-propagation fold are based on the assumption of constant bed length and constant bed thickness (BLT) throughout and so yield unique relationships between the fold shape and the fault shape. Such models can be used to predict the entire structure from a very small amount of hard data. Other models, like the simple-shear, pure shear, and detachment folds maintain constant BLT in the upper part of the structure but are area balanced in the decollement zone at the base. The latter style of model admits a wider range of geometries and so more must be known before the complete structure can be predicted. Other variants allow thickness changes in the steep limb, which creates even more degrees of freedom in the interpretation. A discussion of all the models and their variants is beyond the scope of this book. The objective here is to introduce the basic concepts and provide a guide to additional sources of information.

Fault-related fold models. Unshaded: constant-thickness units; shaded: variable-thickness unit.

a…. Fault-bend fold (after Suppe 1983).
b…. Fault-propagation fold (after Suppe 1985; Suppe and Medwedeff 1990).
c…. Fault-propagation fold at the tip of a long ramp (after Chester and Chester 1990).
d…. Simple-shear fault-bend fold (after Suppe et al. 2004).
e…. Pure-shear fault-bend fold (after Suppe et al. 2004).
f…. Detachment fold (after Jamison 1987)
The starting point for a model-based interpretation is to determine which model is appropriate. Constant BLT fault-bend and fault-propagation folds have analytical relationships between forelimb and backlimb dips that can be expressed graphically. This graph is a plot of forelimb dip versus backlimb dip, with the dips being measured relative to the regional dip outside the fold. A field example (next page) can be plotted on the graph to see if falls on either the fault-bend or fault-propagation fold curve. If the point falls on one of the lines, then there is a high probability that the fold fits the model and the model can be used to predict the fold geometry.
As an example of the methodology, the deep structure of this fold is interpreted. The regional dip is inferred to be parallel to the planar domain between the two limbs and is confirmed because the same dip is seen outside the structure. The limb dips are measured with respect to regional and plotted. The points fall on the fault-bend fold curve, indicating the model to be used. The interlimb angles are measured and bisected to find the axial-surfaces. The backlimb axial surfaces (1 and 2) are parallel, as are the forelimb axial surfaces (3 and 4). Beds must return to regional dip outside the anticline, allowing the location where the forelimb flattens into regional dip (axial surface 4) to be determined. The fault ramp must be below and parallel to the backlimb. The exact location of the fault requires additional information about the position of the lower and upper detachments. Bed truncations in the forelimb of the hangingwall or against the ramp in the footwall may serve to locate the detachments. Alternatively, the stratigraphic section may contain known detachment horizons that can be utilized in the interpretation. Additional controls from the model are that axial surfaces 1 and 2 always terminate at the upper detachment, and axial surface 1 always terminates at the base of the ramp. Axial surface 3 can be on the ramp or at the top of the ramp. Having determined the best location for the fault, the dip domains can be filled in with the appropriate stratigraphy and the interpretation is complete.

**Construction of a complete fault-bend fold starting with a single horizon.**

- **a.** Starting fold geometry with dips measured from regional dip.
- **b.** Axial surfaces bisect hinges.
- **c.** Location of the fault. Ud: upper detachment; f: fault ramp; Ld: lower detachment.
- **d.** Final geometry.
4.2.5 – Fold-related Stresses during Fault Bend Folding

An AAPG Short Course by

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Bedding-parallel (BPV), strike (SV), and cross-fold (CFV) veins represent a sequence of polyphase fracturing during the development of a fault-bend fold in lower Paleozoic carbonate beds in the Appalachian Valley and Ridge province, Pennsylvania. Brecciation and layer-parallel shearing played important roles in the development of the earliest vein set (BPV) which propagated prior to folding. Fluid inclusions in calcite BPV are highly saline brines (23.4 wt% NaO) trapped at conditions close to lithostatic (P < 180 MPa, T < 267°C). While passing through the lower kink plane of a fault-bend fold, strike joints propagated in dolomitic beds located on the extensional side of neutral surfaces. Stylolitization of strike joint surfaces accompanied slip of the hanging wall up a ramp. Renewed extension upon passing through a second kink plane led to propagation of antitaxial SV along the stylolitized joints by the crack-seal process. Slightly less saline fluids (22.4 wt% NaO) were trapped in SV at fluid pressures < 144 MPa and temperatures < 217 °C. With the concurrent formation of lateral ramps, the carbonates moved to the upper Oat and were subjected to strike parallel extension as manifested by the propagation of antitaxial CFV. Due to further mixing of fresh waters, the salinity of fluids forming CFV decreased (20.5 wt% NaO) with trapping conditions at p < 116 MPa and T < 179°C. The fluid evolution path from BPV to CFV through SV shows a modest decrease in salinity with a sharp decrease in possible trapping pressures.
Vein Sequence
• Bedding Parallel
• Strike
• Cross-fold

strike vein with insoluble residue

cross-fold vein cuts strike vein

Srivastava & Engelder, 1990
Strike Veins

- Joint surface morphology
- Accented by stylolite development
- Stylolites covered with insoluble residue
- Vein filling over stylolitic residue

Srivastava & Engelder, 1990

Stylolitic teeth with residue
Later vein fill
Strike vein in thin section

Srivastava & Engelder, 1990
Strike Vein History

- Extension, Shortening, Extension
Fluid Inclusions: Frozen methane with crystal face shrinkage crack with frozen CO$_2$.

CO$_2$ melts at 56$^\circ$C & CH$_4$ melts at 182$^\circ$C

Srivastava & Engelder, 1990
BEDDING PARALLEL VEINS
N = 57

STRIKE VEINS (STYLOLITIC)
N = 19

STRIKE VEINS (NON-STYLOLITIC)
N = 33

CROSS-FOLD VEINS
N = 65

Srivastava & Engelder, 1990
Bedding parallel veins

Srivastava & Engelder, 1990

Strike veins (non-stylocytic)

Cross-fold veins

Nittany Mountain Syncline

Tussey Ridge

Srivastava & Engelder, 1990
Fluid inclusions indicate that bedding-parallel veins happened first and cross fold veins last.

Srivastava & Engelder, 1990
Phase Diagram

**Isochore** – line connecting points of constant volume

**Homogenization**

Temperature

Srivastava & Engelder, 1990

**Diagram Details**

- **Axes**
  - Vertical: Pressure (P)
  - Horizontal: Temperature (T)

- **Lines**
  - **a** and **b**
  - Isochore
  - Methane
  - Methane + brine
  - Brine

- **Key Points**
  - Homogenization temperature (Tt)
• LOCHT provide constraints on trapping conditions and hence fluid pressure at the time of crack propagation.

Lines of Constant Homogenization Temperatures

Srivastava & Engelder, 1990
BEDDING PARALLEL VEINS

STRIKE VEINS

CROSS-FOLD VEINS

Srivastava & Engelder, 1990
5.1.1 – Basement-involved thrust-generated Folds.

An AAPG Short Course by
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Sevier Orogeny is the manifestation of a back-arc thrust belt on an Andean convergent margin. The Sevier fold-thrust belt developed during the mid Cretaceous about the time of the formation of the Sierra Nevada batholith in California.
Laramide basement thrusts

Laramide is a period of deformation in the Rocky Mountain Region dating between the Late Cretaceous (80 Ma until the Paleocene (50 Ma).

*Laramide is a period of deformation in the Rocky Mountain Region dating between the Late Cretaceous (80 Ma until the Paleocene (50 Ma).
Shallow slab subduction and Laramide block faulting
Basement-involved thrusting
Laramide Orogeny, Wyoming, USA

Stone, 1993
Folding over basement involved thrusts
Rocky Mountain Foreland, USA

Stone, 1993
Beer Mug Anticline
place
Circle Ridge Anticline
Wind River Basin, Wyoming, USA

Seismic interpretation Anderson and O'Connell, 1993
Laramide basement-involved thrusting
Rattlesnake Mountain, Wyoming, USA

Neely & Erslev, 2009
Laramide basement-involved thrusting
Rattlesnake Mountain, Wyoming, USA

Neely & Erslev, 2009
Folding of a massive reservoir sandstone
Weber Sandstone, Split Mountain Anticline, Utah, USA
Basement Block Uplift
Big Horn Mountains, Wyoming, USA

Cover folding over basement
Large fault cutting cover rock and basement.

New Mexico
Colorado
Utah
Arizona
Colorado

National Monument

Geological Map of Colorado
Basement thrust folding cover rock
Colorado National Monument, Colorado, USA

Jamison & Stearns, 1982
5.1.2 - Trishear Folding

An AAPG Short Course by
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Trishear Folding

- All the material points in the hanging wall of the fault move with constant velocity and all points in the footwall are fixed.
- To conserve area, the triangular shear zone at the tip of the fault is symmetric with respect to the fault.
- Within this zone, all points on a ray which goes through the tip line experience the same velocity.
Hardy and Ford (1997) showed that there was spectrum of possible ratios of propagation to slip (P/S). Footwall-fixed trishear zones have P/S = 0 and hanging wall fixed zones have P/S = 1.
• the range of possible P/S is virtually infinite.
Six parameters control the geometry of a trishear fold: fault ramp angle, the fault slip, the X and Y positions of the tip line, the trishear apical angle and P/S (Figs. 1 and 2). Several of these parameters are relatively easy to determine if the final geometry of the structure is well known, but the physical characterization of the trishear apical angle and the P/S ratio has proved elusive.
By allowing one or several of the trishear parameters to change during movement of a single fault, virtually any fold shape can be produced. We model the deformation over ramps as kink-style fault-bend folds.
where $P/S > 1$—the migration of the hanging wall boundary of the trishear zone through the material produces a growth axial surface
5.1.3 – Strike-slip Faulting

An AAPG Short Course by
Terry Engelder
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Mid Ocean Ridges

Marie Tharp’s great discovery (ca. 1952): Rift valleys bisecting the Mid-Atlantic Ridge were structurally similar to the Great Rift Valleys of Africa (Heezen et al., 1959).

The rifts were cut by transform faults which slipped in the opposite direction as the offset of rift valleys might indicate. This problem was solved by Tuzo Wilson at the beginning of the era of plate tectonics theory.
Three types of transform Faults

The San Andreas fault is a ridge-ridge transform

After Wilson, 1965

https://learning.uonbi.ac.ke/courses/SGL301/scormPackages/path_2/71_plate_tectonics.html
San Andreas fault (looking south) with a Google Earth image showing the same area.
San Andreas Fault from the Air
While thrust faulting is characteristic of convergent margins of continental crust and normal faults are characteristic of divergent margins where rift basins have formed, strike-slip faults are characteristic of margins where lithospheric plates slide past each other. Some of the most significant strike-slip margins are associated with large strike-slip faults including the San Andreas Fault, the Alpine Fault (New Zealand), Chaman Fault (Pakistan), the Motagua Fault (Guatemala), the Dead Sea Transform (Israel-Jordan), the Altyn Tagh Fault (Tibet), Kunlun Fault (China), Herat Fault (Afghanistan), and the Anatolian Fault (Turkey).

Strike-slip faults are rarely planar. This is best illustrated by the big bend on the San Andreas Fault just north of Los Angeles. The San Andreas is a right lateral strike-slip fault with Pacific Plate moving north relative to the North American Plate. This means that Los Angeles and San Francisco on the Pacific Plate are moving north relative to the rest of North America. The big bend north of Los Angeles is called a restraining bend largely because the westward jump in the fault when tracing the fault to the north, leaves rocks in the way for further slip. These ‘extra’ rocks must be moved out of the way for further slip. Depending on the shape of the irregularities along strike-slip faults relative to the motion along the fault, there may also be releasing bends. These are places where cavities open because opposite sides of an irregularity move away from each other.
Restraining and releasing bends produce diagnostic structures. The restraining bend produces a ‘flower structure’ which is a series of vertical faults of compressional duplexes that are subparallel to the parent fault. The vertical faults divide upthrown blocks the spread near the surface in a manner resembling a flower. The upthrown blocks are sometimes called popups. Releasing bends produce pull-apart basins. Like the restraining bend, a series of faults that form subparallel to the parent fault produce a structure that in cross section is called a ‘flower structure’.

A. A planar strike-slip fault. B. Strike-slip movement along irregularly curved faults producing gaps at releasing bends and crowding at restraining bends (from Crowell, 1974).

Restraining bend with a compressional duplex (left) and extensional duplexes at releasing bends (right) (from Twiss and Moores, 1992).
Schematic interpretation of the formation of the positive flower structure and contractional duplex by transpression.
Example of a restraining bends is found along the San Andreas Fault where the contraction expected at a left bend in a dextral fault is seen as the block uplifts on the Transverse Ranges. The restraining bend of the San Andreas is reflected in the blocks bounded by east-west thrust faults. Extensional basins are also present including the Salton Sea trough between the San Andreas and San Jacinto faults (blue arrow).

The San Andreas Fault system showing the block uplifts (shaded areas) of the Transverse Ranges of southern California (from Twiss and Moores, 1992).
Damage Zones Associated with Faulting

(a) Wing crack

(b) Horsetail splay or pinnate fracture

(c) Synthetic branch fault

(d) Antithetic fault

(e) Horsetail splay + Antithetic fault

(f) Branch fault + Antithetic fault

(g) Mixed mode tip (II & III)

(a) Mode II tip propagation

(b) En echelon vein

(c) Extension fracture

(d) Synthetic fault

(e) Antithetic fault

(f) Block rotation or joint drag

(a) Extension fracture

(b) Pull-apart

(c) Rotated block

(d) Isolated lens

(e) Rotated block

(f) Connecting fault

(g) Isolated lens

The main types of tip damage zones.

Damage zones within the walls of faults.

Linking damage zones.

Kim et al., 2004
5.1.4 – Balancing Sections

An AAPG Short Course by
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The original concept of a balanced cross section is that the deformed-state and restored cross sections maintain constant area and so would balance on a beam balance. This concept is the constant volume criterion. In many structures there is little or no deformation along the axis of the structure, and so in practice the third dimension can often be temporarily ignored and constancy of volume can be applied to a cross section. Units which maintain constant bed length are said to be length balanced and units that maintain constant area but not constant bed length or bed thickness are said to be area balanced. A balanced cross section is generally understood to be one which is restorable to a geologically reasonable pre-deformation geometry, as well as maintaining constant area.

**Boundaries**

Ideally, the boundaries of a section to be restored are chosen so that the section will restore to a rectangle. The side boundaries are pin lines, and the upper boundary is a reference bed that will be returned to its original depositional geometry (Dahlstrom 1969; Elliott in Geiser 1988; Marshak and Woodward 1988). The original position of a horizon, including both its shape and elevation, is known as the regional datum, commonly shortened to just the regional. The restored positions of all other horizons are determined with respect to the reference horizon.
The base of the section is normally either a stratigraphic marker or a detachment horizon, but may be simply the lowest visible unit. The bounding pin lines are leading and trailing pins, according to their position in the structure with respect to the transport direction. A locally balanced structure is one in which bed-normal pins on either side of the structure of interest define a region in which the area has remained constant. If material has been transported across the chosen pin lines by displacement on an upper detachment or into or out of the region by layer parallel simple shear, the structure should be regionally balanced, even if the region of interest is not locally balanced. Thus for a regionally balanced structure a vertical pin line may represent a working pin line that will restore with an offset or a tilt. A valid cross section might fail to restore to a rectangle because of unrecognized transport across a working pin line. The restoration reveals these otherwise hidden displacements.

Pin lines bounding a region of interest. LPL: leading pin line; TPL: trailing pin line; WPL: working pin line. a Undeformed. b Locally balanced. c Regionally balanced with transport of material out of the structure. d Regionally balanced with simple-shear transport of material into the structure.
Area Restoration

The first predictive uses of the concept of area balance were to determine the depth to detachment of the structure. Area uplifted above the regional as a result of compressional deformation is termed the excess area. Area that drops below regional as a result of extensional deformation is termed lost area. More generally, these can be called the displaced areas. The classical displaced area method is designed to find the detachment depth from the excess or lost area of one horizon in a structure. The displaced area is produced by displacement along the lower detachment such that \( S = D \times H \), where \( S \) = area above or below the regional, \( D \) = displacement, and \( H \) = depth to detachment from regional. A unique depth to detachment can be calculated from a measurement of the excess area if the displacement that formed the structure is known. It is assumed that bed length remains constant and so the displacement is the difference between the curved bed length of the marker horizon and its length at regional: \( D = L_0 - W \), where \( D \) = displacement, \( L_0 \) = curved-bed length (assumed equal to original bed length) and \( W \) = width of structure at regional. Substituting we get \( H_c = S / (L_0 - W) \), where \( H_c \) indicates the detachment depth if bed length is constant.
Area-balance terminology. $S$: Excess or lost area; $D$: displacement on the lower detachment; $H$: distance from the lower detachment to the regional; $h$: elevation of the regional above or below the reference level; $L_1$: bed length after deformation. a Extension (after Groshong 1996). b Contraction (after Groshong and Epard 1994)
5.2.1 – Quality Control

An AAPG Short Course by
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Quality control means locating and correcting errors in the data and in the interpretation. Problems can arise from data transcription errors, incomplete exposure in the field, interpolation uncertainties between wells and seismic profiles, and missing or misleading information in seismic interpretations. The quality-control issues discussed in this chapter can be broadly categorized as trend incompatibilities, data errors and contouring artifacts, bed thickness anomalies, and impossible fault shapes.

**Trend incompatibilities**

Geologic map. Units from youngest to oldest are Mh, Mpm, Mtfp. Topographic elevations are in feet and the scale bar is 1000 ft. a Structure contours (heavy lines) determined from the intersection of mapped formation boundaries with the topographic contours. b Revised geologic and structure-contour map.

Section 5.2.1 is taken exclusively from R.H. Groshong’s book, 3-D Structural Geology. Paragraphs following Groshong’s words, however inexact, are presented in italics.
Bad points can arise from a deviated well that is mistakenly interpreted as being vertical, causing both location and thickness anomalies. The log depth to a formation boundary is larger in a well that deviates down dip than in a vertical well at the same surface location. If the formation boundary is plotted vertically beneath the well location, its depth will be too great. If an apparent dip is then determined between this well and a correctly located formation top in another well, the apparent dip will be wrong, perhaps even in the wrong direction. If the dip is determined from a core in a deviated well that is mistakenly thought to be vertical, then the inferred dip will be too large. The apparent thicknesses are too large in both situations. A well that deviates up dip will result in an apparent steepening of the dip between two wells and thicknesses (if mistakenly corrected for dip) that will be too small.
Use of the expansion index to test unit correlations. An expansion index of less than one implies reverse fault movement or a miscorrelation across a normal fault.
Impossible fault shapes

The geometry of the cutoff lines of marker surfaces against the fault provides a test of the quality of the interpretation. The relationships are nicely shown on an Allan diagram.

A stratigraphic separation diagram shows the fault separation in terms of the units juxtaposed across the fault along a single line, for example along the map trace of the fault. The curves for the hangingwall and footwall are not expected to cross as shown in this diagram, because this implies that either (1) the fault changes from a thrust to a normal fault or from a normal fault to a thrust, or (2) the fault is out of the normal evolutionary sequence and cuts an older fold or fault. A separation anomaly requires careful attention to the correlation of the fault cuts and to the interpretation of the fault if the correlation is accepted. A strike-slip fault might have an apparent separation anomaly like this figure.
5.2.2 – The Niger Delta: A Case Study

An AAPG Short Course by
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Location map of the Niger Delta region showing the main sedimentary basins and tectonic features.
Schematic diagram of the regional stratigraphy of the Niger Delta and variable density seismic display of the main stratigraphic units in the outer fold and thrust belt and main reflectors, including (1) top of the Agbada Formation, (2) top of the Akata Formation, (3) mid-Akata reflection, (4) speculated top of the synrift clastic deposits, and (5) top of the oceanic crust. Main detachment levels are highlighted with red arrows.
Figure IGA

1. Extensional province
2. Mud-diapir province
3. Inner fold and thrust belt
4. Detachment fold province
5. Outer fold and thrust belt
Patterns of growth sedimentation, where the uplift rates along the fault ramp are equal to the rates of growth sedimentation, indicate fold growth by combined limb rotation and kink-band migration.
A simple-shear fault-bend fold. The interpretation reflects some important characteristics of the structure, including the fold and fault shapes and the onlaps and fanning of limb dips in growth strata. Notice how the back limb dips much less than the fault ramp. Decreasing dips upward in growth strata above the backlimb suggest folding by limb rotation. Growth strata onlap above the forelimb, suggesting low to equal growth sedimentation rates compared with structural uplift rates. This simple-shear fault bend fold has a planar backlimb synclinal axial surface.
Seismic profile showing a simple-shear fault-bend fold and a pure-shear wedge in the leading edge culmination of the outer fold belt. Note the common detachment level for both the forethrust and backthrust near the top of the Akata Formation.
A: Break-forward thrusting

- Incipient thrust fault
- Axial surfaces
- Increase in accommodation space

B: Break-backward thrusting

- Incipient thrust fault
- Axial surfaces
- Increase in accommodation space
Seismic section across a break-forward imbricate system with forward distributed transfer of shear, showing the patterns of growth sedimentation
5.2.3 – Seismic Interpretations

An AAPG Short Course by
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Folds and bedding thickness

Non-parallel fold model

$T_2 > T_1$

Non-parallel fold, anticlinal axial surface

$T_2 > T_1$

Parallel fold model

Layer thickness is conserved: Bed thickness $T_1$ equals bed thickness $T_2$.

Bisecting axial surfaces: Interlimb angle $\gamma_1$ equals interlimb angle $\gamma_2$.

Parallel fold, synclinal axial surface

$T_2 = T_1$
Fault-bend folds form as hanging wall-rocks move over bends in an underlying fault.
Contractional Fault-Related Folds
Seismic Interpretation

**Folding by kink-band migration**
- Pre-growth strata only
- Sedimentation > uplift
- Sedimentation < uplift

**Folding by progressive limb rotation**
- Pre-growth strata only
- Sedimentation > uplift
- Sedimentation < uplift

Seismic Example: offshore Angola

- Fanning of limb dips
- Salt mound
- Detachment

Shaw et al., 2005
Synclinal Fault-Bend Fold

Kinematic Model

Seismic Example: Argentina

Data courtesy of BHP

Shaw et al., 2005
Anticlinal Fault-Bend Fold

**Kinematic Model**

1. **0**
   - Initial state
   - Active axial surface pinned to fault bend

2. **1**
   - Evolution of the fold
   - Axial surface indicated

3. **2**
   - Final state
   - Active axial surface pinned to fault bend

**Seismic Example: Niger Delta**

- Axial surface
- Fault

Data courtesy of Mabone Limited

Shaw et al., 2005
Growth or syntectonic strata are stratigraphic intervals that were deposited during deformation. The ages of growth strata therefore define the timing of deformations. In contractional fault-related folds, growth strata typically thin across fold limbs toward structural highs. The geometries of growth structures are controlled primarily by the folding mechanism and the relative rates of sedimentation and uplift. Thus, growth fold patterns imaged in seismic data are often considered diagnostic of folding mechanism and sediment-to-uplift ratio. In this section, we describe common patterns of growth strata in fault-related folds that are imaged in seismic reflection data.
Fault-propagation folds form at the tips of faults and consume slip. These folds are generally asymmetric, with forelimbs that are much steeper and narrower than their corresponding backlimbs. Several modes of folding at fault tips have been described to explain these structures, including: constant thickness and fixed axis fault propagation folding (Suppe and Medwedeff, 1990); trishear folding (Erslev, 1991; Hardy and Ford, 1997; Allmendinger, 1998); and basement-involved (triple junction) folding (Narr and Suppe, 1994).

**FPF terminology**

The following terms are used in the derivation and graphs that describe fault-propagation folds.

- $\theta_1 =$ hanging wall cut-off (lower fault segment)
- $\theta_2 =$ footwall cut-off (upper fault segment)
- $\phi =$ change in fault dip
- $\gamma =$ forelimb syncline interlimb angle
- $\gamma^*$ = anticlinal interlimb angle
- $\delta_b =$ backlimb dip
- $\delta_f =$ forelimb dip

Shaw et al., 2005
Trishear fault-propagation folds

Seismic section

1 km

Trishear interpretation

4 km

Data courtesy of CNPC

Shaw et al., 2005
**Imbricate structures** form by the stacking of two or more thrust sheets and are common in fold and thrust belts worldwide. Imbricate structures can form by break-forward propagation of thrust sheets, by break-backward thrusting, or with coeval motion on both deep and shallow faults. In this section, we describe the basic characteristics of imbricate structures, and outline an approach to interpret these structures in seismic profiles using imbricate fault-bend fold theory (Suppe, 1983; Shaw et al., 1999).

![Break-forward imbricate](image1)

![Break-backward imbricate](image2)

Common characteristics of Imbricate fault-bend folds typically contain:
1) Two or more vertically stacked thrust ramps;
2) Bedding dips that change across thrust ramps; and
3) Fold limbs at high structural levels with multiple dip domains, reflecting refolding caused by multiple ramps.
Shear fault-bend folding produces ramp anticlines with very distinctive shapes that reflect a significant non-flexural-slip component to the deformation. The structural style typically shows long back-limbs that dip less than the fault ramp, in contrast with classical fault-bend folding. These ramp anticlines also commonly show front limbs that are quite narrow relative to their long back limbs.
Detachment folds form as displacement along a bedding-parallel fault is transferred into folding of the hanging wall layers. Although detachment folds may share some geometric similarities with fault-bend and fault-propagation folds, they differ from these structures because they are not directly related to thrust ramps but rather to distributed deformation above detachments.

Common characteristics: Detachment folds generally share the following characteristics:
1) An incompetent, ductile basal unit thickened in core of fold, with no visible thrust ramp.
2) A detachment that defines the downward termination of the fold.
3) Competent pregrowth units that, if present, generally maintain layer thickness.
4) Growth units, if present, that thin onto the fold crest and exhibit a fanning of limb dips.

Detachment folds are common in outcrop and at scales typically imaged by seismic reflection data. They have been documented in the foreland of fold and thrust belts such as the Jura, Appalachian Plateau (Wiltschko and Chapple, 1977), and Tian Shan. Detachment folds are also common in passive margin fold belts, such as the Mississippi Fan (Rowan, 1997) and Perdido Fold Belts in the Gulf of Mexico, and in the Campos Basin, Brazil, (Demercian et al., 1993), and the Niger Delta.
**Structural wedges** contain two connected fault segments that bound a triangular, or wedge-shaped fault block. The two fault segments, which typically include two ramps or one ramp and one detachment, merge at the tip of the wedge. Slip on both faults accommodates propagation of the wedge tip and causes faulting (Medwedeff, 1989). Wedges occur at the variety of scales. At the large scales associated with mountain fronts, wedges are typically referred to as triangle zones (Bally et al., 1966).

Common characteristics: Wedges exhibit a wide range of geometries. However, several characteristics are common to most wedge structure, including:

1) presence of coeval fore- and back-thrusts;
2) folding localized along an active axial surface pinned to the wedge tip; and
3) folds may exist in the footwall of the back thrust that produce structural relief.
Interference structures form when two or more monoclinal kink bands intersect, often yielding distinctive patterns in cross section with anticlines perched above synclines. Interference structures have been documented in the field and laboratory (e.g., Dewey, 1965; Paterson and Weiss, 1966; Stewart and Alvarez, 1991), and have been proposed as the origin of structures imaged in seismic profiles (e.g., Mount, 1989; Novoa et al., 1998). In this section, we describe a simple style of interference structure comprised of two kink bands with opposing dips, and present examples of these structures imaged in seismic sections.
Seismic section and interpretation of a hybrid fault-zone containing both planar normal and listric normal fault geometry. The fault also shows migration of the thermal subsidence phase planar normal type faults into the footwall and an associated footwall uplift unconformity.
Interpretation of structural style within the wedge and imbrication zones of the detachment sheet as found in seismic profile in this migrated time section through Laurel Hill anticline. Schematic line drawings are shown at 3:1 vertical exaggeration, as portrayed in seismic sections, and with no vertical exaggeration.
3D reflection seismic data acquired offshore of southeast Japan as part of the Nankai Trough Seismogenic Zone Experiment provides a unique opportunity to study active accretionary prism processes. The 3D seismic volume revealed complex interactions between active sedimentation and tectonics within multiple slope basins above the accretionary prism.
The Sequatchie anticline is the frontal structure of the Southern Appalachian thrust belt in southern Tennessee, Georgia, and Alabama (see regional map in frontispiece). The structure has a low relief and exposes Mississippian to Pennsylvanian units on the crest of the structure. Farther south, the relief increases, and Middle Ordovician to Devonian units are exposed at the surface (Hardeman et al., 1966; Harris and Milici, 1977).

Regional fault ramps on basement hinge (red arrow) in much the same manner as the Laural Hill and Chestnut Ridge anticlines of the Central Appalachian Plateau are associated with basement faults (Scanlin and Engelder, 2003) (see frontispiece).
This cross section of the Sequatchie Anticline was restored using line-length balancing. The restoration shows that the shortening for the base of the Rome Formation is approximately 5100 ft. The fault displacement decreases from 5100 ft at the base of the Rome Formation to 4500 ft at the top of the Knox Group and 2200 ft at the top of the Mississippian units. The forward shear of the loose line in the restored section suggests a small amount of differential penetrative strain at the mesoscopic and microscopic scales within the Silurian to Mississippian units. This is equivalent to the layer-parallel shortening recorded throughout the Central Appalachian Plateau (Engelder and Engelder, 1977). This inclined shear profile and proposed penetrative deformation is consistent with the steep front limb of the fold, and the small fault displacement in the Mississippian units.
5.2.4 – The Structural Geology of Petroleum Migration

An AAPG Short Course by
Terry Engelder
Professor of Geosciences
The Pennsylvania State University
Devonian Catskill Delta

The structural geology of petroleum migration

The Pennsylvania Source, primary migration
reservoir rock

Devonian Catskill Delta

The structural geology of petroleum migration

- Leakoff Test

- Incipient joint in seal rock

- Pore pressure profile through a seal

- Secondary migration

- Fluid-driven joint breaching seal

- Distinct normal faults breaching seal

source rock

60 meters
Scaling the Properties of Fractures: Fractal Behavior

Map

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Fit > 0.99

$r = \text{length}$

bitumen-filled cracks
source-rock joints
reservoir-rock joints

Barton, 1995
References

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The Pennsylvania State University


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