# Elastic Wave Propagation in Fractured Media Using the Discontinuous Galerkin Method*

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## Abstract

A realistic model of the subsurface in reservoir modeling should include fractures. We have developed a novel method to include discrete fractures into 2D or 3D elastic models in order to gain insight into the wave phenomena related to the presence of fractures.

In order to determine the presence of fractures in the subsurface one has to rely on seismic data. In particular, azimuthal velocity anisotropy has been observed in many regions and this has been attributed to aligned fractures. Other wave phenomena related to the presence of fractures are phase-shifting, frequency-filtering and scattering of the reflected, transmitted and converted waves. Furthermore, fracture interface waves have also been observed in practice.

There are two main approaches to incorporate the effects of fractures: Using equivalent medium theories or using a numerical scheme to simulate the fractures. There have been many theories proposed in the literature that predict the effective media parameters associated with a particular fracture distribution. All of these models make different assumptions about the fractures, in particular they usually assume small, circular, non-intersecting cracks. The advantage of the equivalent medium theories is that they provide analytic expressions for the media parameters as a function of the fracture parameters. On the other hand, they have limited applicability because of the large number of assumptions.

Regarding the numerical schemes to incorporate the fractures, there are many approaches that have been proposed in the literature. The main advantage of the numerical schemes is that they require few assumptions and therefore they have a broad applicability and are useful to validate the equivalent medium theories. In particular, the approaches based on the linear-slip model require the least number of assumptions.

We propose a new scheme that incorporates fractures using the linear-slip model into a discontinuous Galerkin method. This approach can be used to simulate a wide variety of wave phenomena related to fractures. We validate our method using a set of parallel fractures and compare

the results with those obtained using an equivalent medium. We show results for a single elongated horizontal fracture and show numerical examples using 2D and 3D models with one fracture and two orthogonal fractures.

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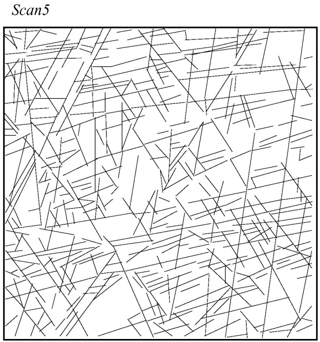
# Outline

# Outline

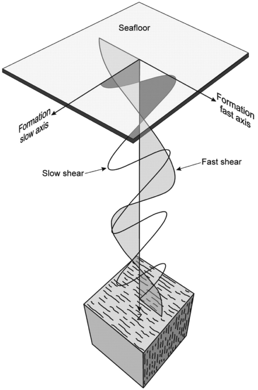
# Motivation

- Fractures are a **common feature in the subsurface**
- Observed in many scales, from faults to micro-cracks
- Modeling of fractured media has important practical applications, for example:
  1. In the oil and gas industry (Sayers, 2007)
  2. In the geothermal industry (Wu et al., 2002)




http://www.leeds.ac.uk/StochasticRockFractures/

# Fractures and Wave Propagation

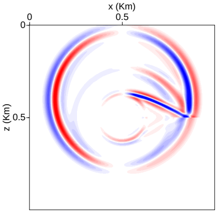



(Barton, 2007)

- To determine the presence of fractures in the subsurface we have to rely on seismic data
- Parallel micro-cracks introduce **seismic anisotropy**
- Other wave phenomena related to fractures:
  - Phase shifting
  - Frequency filtering
  - Dispersion of the reflected and transmitted waves
  - Fracture interface waves

(Schoenberg and Douma, 1988; Carcione, 1996; Pyrak-Nolte et al., 1996)

# Numerical Strategies

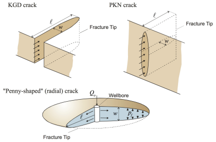



Two approaches to incorporate the effects of fractures in wave propagation:

1. Using equivalent media theories
2. Incorporating discrete fractures in a numerical scheme

# Equivalent Media Theories

- Equivalent Media Theories predict the **effective elastic properties** of fractured media given some fracture parameters.
- Common assumptions:
  - Idealized crack shape,
  - small aspect ratio and crack density compared to wavelength,
  - Cracks are isolated with respect to fluid flow.
- Examples of Effective Media Theories:
  - Kuster-Toksöz,
  - Differential Effective Medium,
  - Hudson,
  - Eshelby-Cheng.




http://www.cefor.umn.edu

(Mavko et al., 1998; Saenger et al., 2004, and references therein)

# Numerical Approaches

- Numerical approaches that have been proposed in the literature:
  - Use locally an effective medium (Vlastos et al., 2003),
  - Incorporate locally a low velocity and low density inclusion into a finite difference scheme (Saenger and Shapiro, 2002; Saenger et al., 2004), and
  - Explicitly use a displacement discontinuity condition using the linear-slip model (Zhang, 2005; Zhang and Gao, 2009).

- The advantage: they require few assumptions and therefore they have a broad applicability and are useful to validate the equivalent medium theories.

- Approaches based on the linear-slip model require the least number of assumptions.

# The Linear Slip Model

## The Linear-Slip Model (LSM)

Prescribes a linear relation between the traction vector and jump in the displacement:

$$[\boldsymbol{u}] = \boldsymbol{Z}\boldsymbol{\tau},$$

where
$[\boldsymbol{u}]$ is the jump of the displacement,
$\boldsymbol{\tau}$ is the traction vector at the fracture and
$\boldsymbol{Z}$ is the fracture compliance matrix.

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where
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$\boldsymbol{\tau}$ is the traction vector at the fracture and
$\boldsymbol{Z}$ is the fracture compliance matrix.

For a fracture with rotational symmetry about the normal, the fracture compliance matrix is given by (Schoenberg and Douma, 1988; Zhang and Gao, 2009)

$$Z_{ij} = Z_N n_i n_j + Z_T(\delta_{ij} - n_i n_j),$$

where $Z_T$ and $Z_N$ are the tangential and normal components of the compliance matrix.

# Outline

# Discontinuous Galerkin Method

The Discontinuous Galerkin Method (DGM) is a generalization of FEM that allows for the basis functions to be **discontinuous** at the element interfaces.

The interior-penalty formulations (Rivière, 2008):

SIPG: Symmetric Interior-Penalty Galerkin

NIPG: Non-symmetric

IIPG: Incomplete

Advantages

- it can accommodate discontinuities in the wave field
- it can be energy conservative
- it can handle more general meshes
- it is suitable for local time stepping and parallel implementations

# Interior-Penalty Weak Formulation

Find $\boldsymbol{u} \in \boldsymbol{X}$ such that for all $\boldsymbol{v} \in \boldsymbol{X}$

$$\sum_{E\in\Omega_h} \Big( (\rho\partial_{tt}\boldsymbol{u}, \boldsymbol{v})_E + \boldsymbol{B}_E(\boldsymbol{u}, \boldsymbol{v}) \Big) + \sum_{\gamma\in\Gamma_h} \boldsymbol{J}^c_\gamma(\boldsymbol{u}, \boldsymbol{v}; S, R) = \sum_{E\in\Omega_h} (\boldsymbol{f}, \boldsymbol{v})_E$$

where $\boldsymbol{X} = \Big\{ \varphi \mid \varphi \in \boldsymbol{H}^1(E)\ \forall\ E \in \Omega_h,\ \boldsymbol{\varphi} = 0 \text{ on } \Gamma_D \Big\}$,

$$\begin{aligned}
\boldsymbol{B}_E(\boldsymbol{u}, \boldsymbol{v}) &= \int_E \Big( \lambda\partial_i u_i \partial_j v_j + \mu(\partial_j u_i + \partial_i u_j)\partial_j v_i \Big)\, d\Omega, \\
\boldsymbol{J}^c_\gamma(\boldsymbol{u}, \boldsymbol{v}; S, R) &= -\int_\gamma \{\tau_i(\boldsymbol{u})\}[v_i]\, d\gamma + S\int_\gamma \{\tau_i(\boldsymbol{v})\}[u_i]\, d\gamma \\
&+ R\int_\gamma \{\lambda + 2\mu\}[u_i][v_i]\, d\gamma,
\end{aligned}$$

$\tau_i(\boldsymbol{u}) = \lambda u_{k,k} n_i + \mu(u_{i,j} + u_{j,i})n_j$ is the traction vector, $\Omega_h$ is the set of elements and $\Gamma_h$ is the set of all faces between elements. The parameter $R$ is the penalty, and $S$ is a parameter that takes the values $S = 0$ for IIPG, $S = -1$ for SIPG and $S = 1$ for NIPG.

# Accuracy and Stability of DGM

- Grid dispersion and stability analyzed in De Basabe et al. (2008) and De Basabe and Sen (2010).
- **Superconvergence** of the grid-dispersion error with respect to the sampling ratio for the symmetric formulation and nodal basis functions,
- The numerical **anisotropy** is negligible for basis of degree 4 or greater,
- **Stability** condition in 2D given by

$$\frac{\alpha \Delta t}{\Delta x} \leq 0.25,$$

where $\Delta x$ is the smallest spatial increment, $\Delta t$ is the size of the time step and $\alpha$ is the largest P-wave velocity.

# Proposed Numerical Scheme

Find $\boldsymbol{u} \in \boldsymbol{X}$ such that for all $\boldsymbol{v} \in \boldsymbol{X}$

$$\sum_{E \in \Omega_h} \Big( (\rho \partial_{tt} \boldsymbol{u}, \boldsymbol{v})_E + \boldsymbol{B}_E(\boldsymbol{u}, \boldsymbol{v}) \Big) + \sum_{\gamma \in \Gamma_c} \boldsymbol{J}^c_\gamma(\boldsymbol{u}, \boldsymbol{v}) + \sum_{\gamma \in \Gamma_f} \boldsymbol{J}^f_\gamma(\boldsymbol{u}, \boldsymbol{v}) = \sum_{E \in \Omega_h} (\boldsymbol{f}, \boldsymbol{v})_E$$

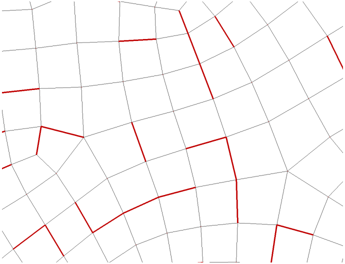
where $\Gamma_c \subset \Gamma_h$ is the subset of all faces where the displacement field is continuous, $\Gamma_f \subset \Gamma_h$ is the subset of faces that represent fractures, and

$$\boldsymbol{J}^f_\gamma(\boldsymbol{u}, \boldsymbol{v}) = \int_\gamma Z^{-1}_{ij} [u_j][v_i] \, d\gamma.$$

The linear slip condition is weakly imposed through this term. (De Basabe et al., 2016)

# Advantages of the Proposed Scheme

- It does not require mesh refinements near the fractures. There is no loss of accuracy because the fractures are at the element interfaces.
- It can be used to simulate fractures of any shape or orientation
- Intersecting fractures can be included without special treatment
- Fracture parameters are taken into account through the fracture compliances

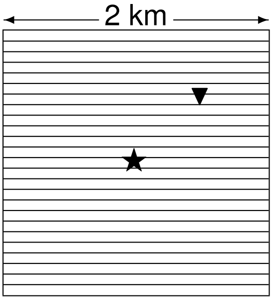


# Fracture-Induced Anisotropy

For this numerical experiment, we compare the wave fields computed using an equivalent anisotropic model based on Schoenberg and Douma (1988) and using a fractured elastic model.

Fracture spacing and compliances are given as follows:

| NF | $h_f$ (m) | $Z_T$ (m/Pa) | $Z_N$ (m/Pa) |
|---|---|---|---|
| 199 | 20 | $16\times10^{11}$ | $12\times10^{11}$ |
| 397 | 10 | $8\times10^{11}$ | $6\times10^{11}$ |
| 793 | 5 | $4\times10^{11}$ | $3\times10^{11}$ |



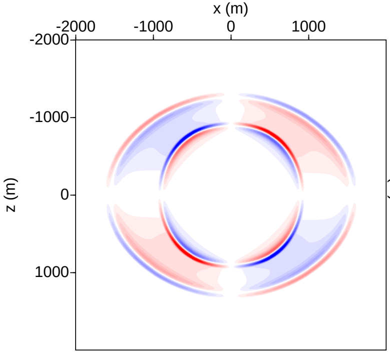

$\Omega = 2 \times 2$ km

$\alpha = 5.8$ km/s

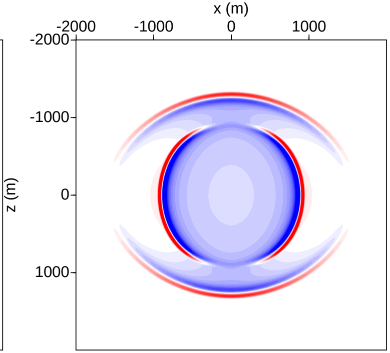
$\beta = 3.8$ km/s

# Fracture-Induced Anisotropy

## Anisotropic model



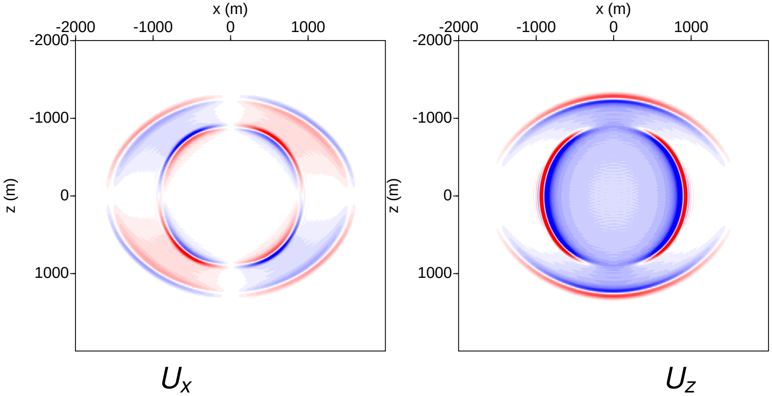

$U_x$




$U_z$

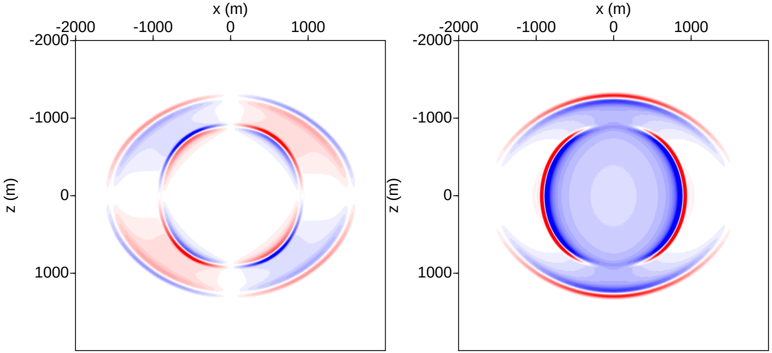
# Fracture-Induced Anisotropy

$\Delta = 20$ m, $Z_T = 16 \times 10^{-11}$, and $Z_N = 12 \times 10^{-11}$



# Fracture-Induced Anisotropy

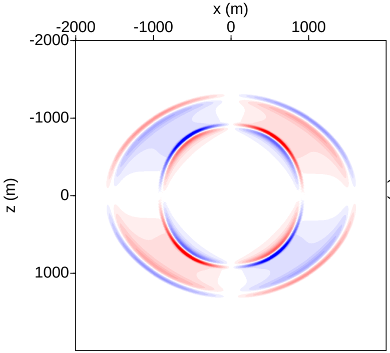
## $\Delta = 10$ m, $Z_T = 8 \times 10^{-11}$, and $Z_N = 6 \times 10^{-11}$



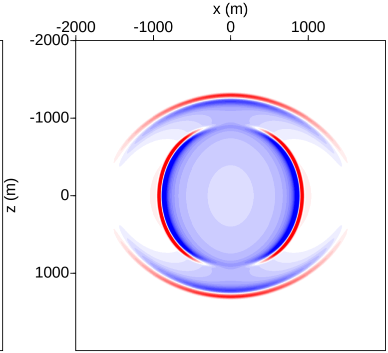

$U_x$ $U_z$

# Fracture-Induced Anisotropy

$\Delta = 5$ m, $Z_T = 4 \times 10^{-11}$, and $Z_N = 3 \times 10^{-11}$



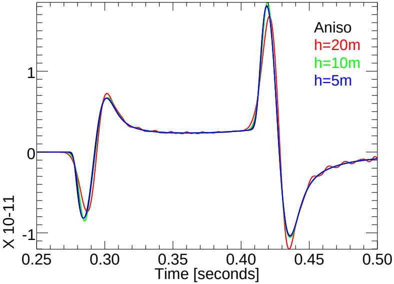

$U_x$



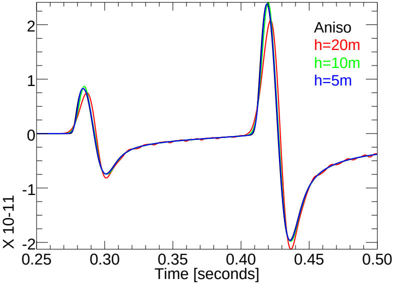

$U_z$

# Fracture-Induced Anisotropy

## Seismogram 1 km above the fracture with a horizontal offset of 1 km




$U_x$




$U_z$

# Random Vertical Fractures

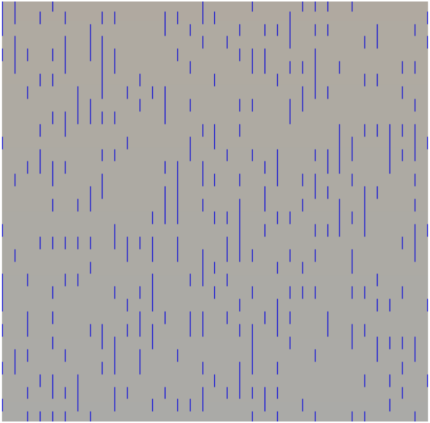
Background medium

- $V_p$= 3310 m/s
- $V_s$= 1620 m/s
- $\rho$= 2500 Kg/m$^3$

Dry fractures with

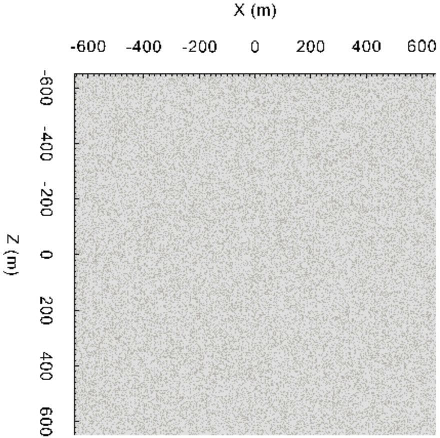
- $Z_T = 5.137 \times 10^{-11}$
- $Z_N = 4.258 \times 10^{-11}$

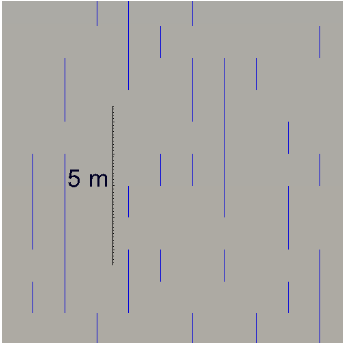
The source is at the center with a peak frequency of 45 hz.



# Random Vertical Fractures

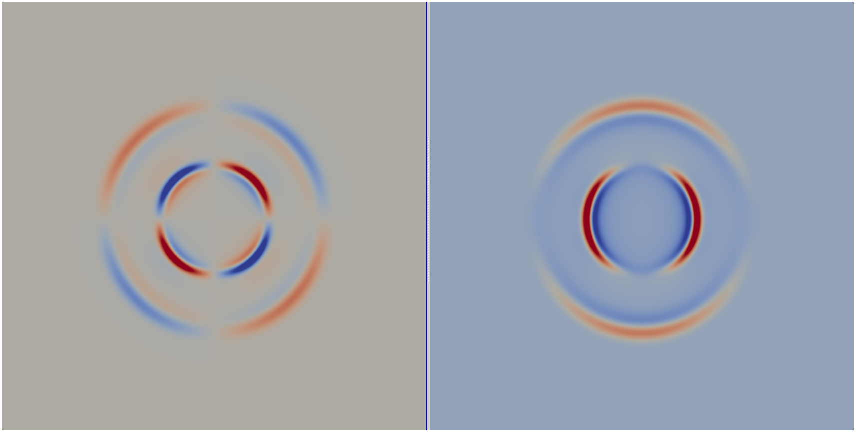
## Vertical fractures of 1 to 5 m of length



# Random Vertical Fractures

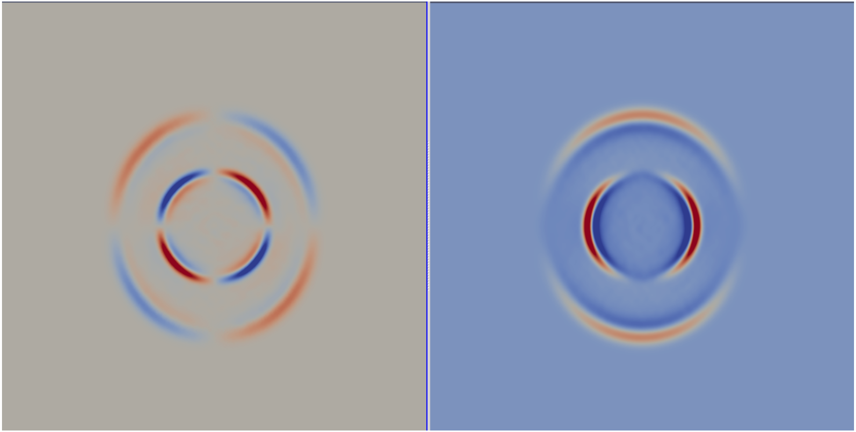
## Wavefield at $t = 0.16$ s using an Equivalent Anisotropic Medium



540,384 Fractures – Hudson model

# Random Vertical Fractures

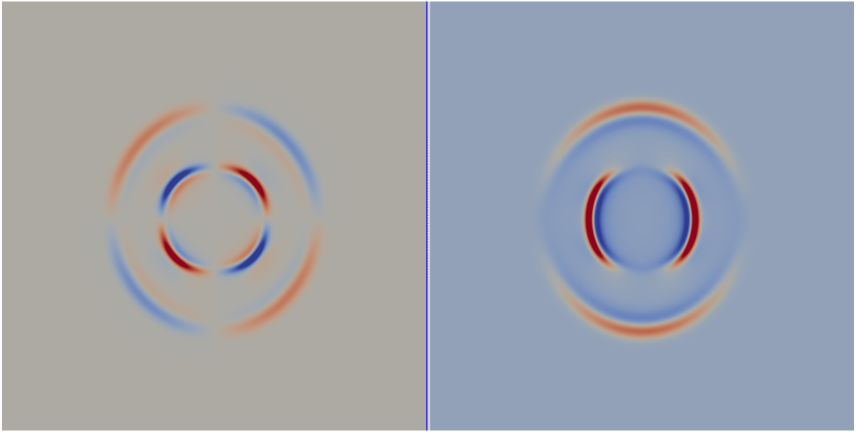
## Wavefield at $t = 0.16$ s using Discrete Fractures



540,384 Fractures – DG

# Random Vertical Fractures

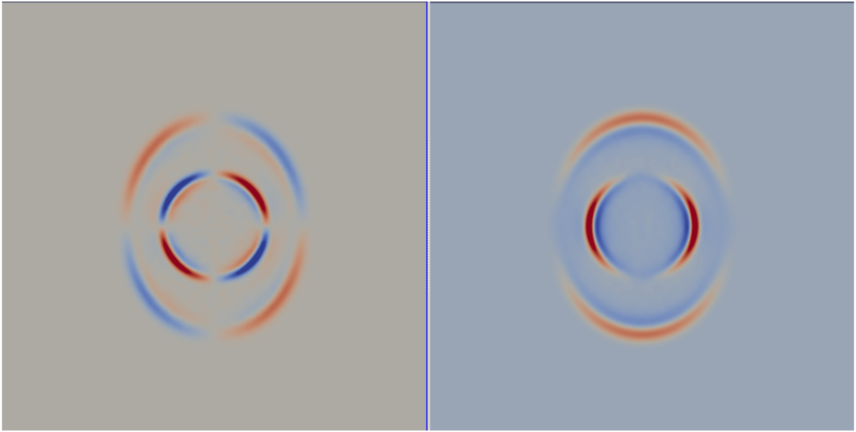
## Wavefield at $t = 0.16$ s using an Equivalent Anisotropic Medium



1,013,220 Fractures – Hudson model

# Random Vertical Fractures

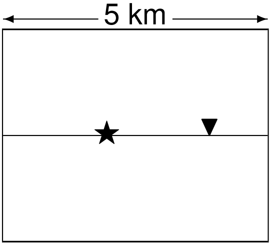
## Wavefield at $t = 0.08$ s using Discrete Fractures



1,013,220 Fractures – DG

# Fracture Interface Waves

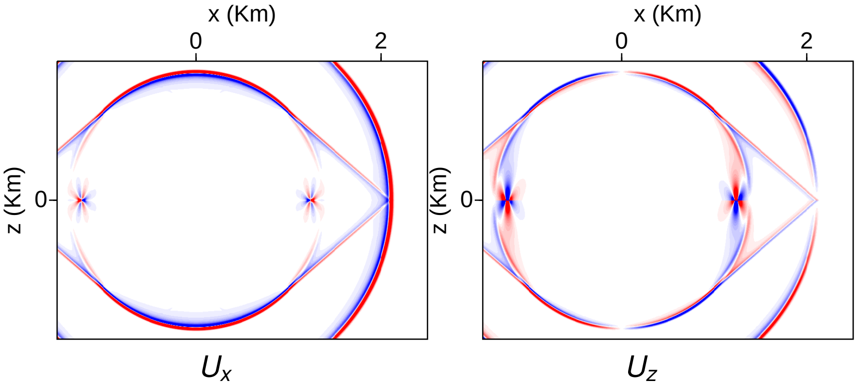
- There are two types of fracture interface waves (Pyrak-Nolte et al., 1996):
  - The fast interface wave is exited by the component of the source that is parallel to the fracture.
  - The slow interface wave is exited by the component of the source that is orthogonal to the fracture.
- Their velocity is bounded above by the S-wave velocity and below by the Rayleigh wave velocity. These waves converge to the Rayleigh wave for $\boldsymbol{Z} = \infty$.
- The amplitude of these waves is maximized with the source at the fracture.




$\Omega = 5 \times 4$ km
$\alpha = 5.8$ km/s
$\beta = 3.8$ km/s
$\rho = 2.6$ g/cm$^3$
100 Hz source
Mesh is 500×400 elements

# Fracture Interface Waves

## Horizontal source at the fracture, $t = 0.37\text{s}$, $Z_T = Z_N = 10^{-8}$

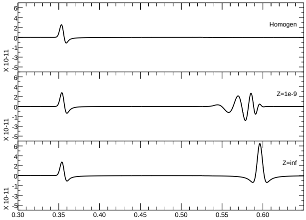



Fast interface wave (Pyrak-Nolte et al., 1996; Gu et al., 1996).

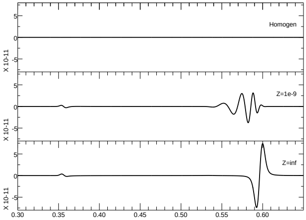
# Fracture Interface Waves

## Horizontal source at the fracture

Synthetic seismograms at a horizontal offset of 1.8 km using $Z = 0$, $Z = 10^{-9}$ and $Z = \infty$




$U_x$

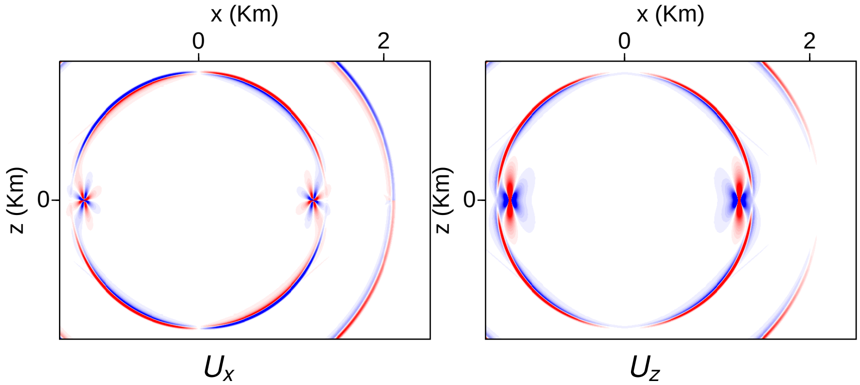



$U_z$

Interface wave travels faster than the Rayleigh wave.

# Fracture Interface Waves

Vertical source at the fracture, $t = 0.37\text{s}$, $Z_T = Z_N = 10^{-8}$

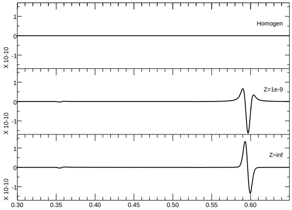


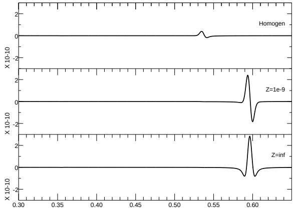

Slow interface wave (Pyrak-Nolte et al., 1996; Gu et al., 1996).

# Fracture Interface Waves

## Vertical source at the fracture

Synthetic seismograms at a horizontal offset of 1.8 km using $Z = 0$, $Z = 10^{-9}$ and $Z = \infty$

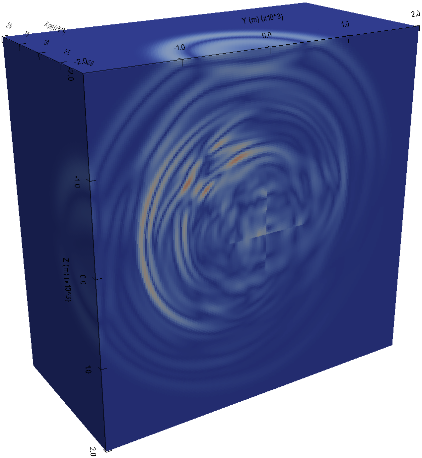



$U_x$ $U_z$

Interface wave traveling approximately at the Rayleigh-wave velocity.

# Finite Fracture in 3D



- 1 $km \times 1$ $km$ orthogonal fractures.
- Source 0.1 $km$ above the fracture.

# Outline

# Conclusions

- The numerical results show that parallel elongated fractures introduce numerical anisotropy. Furthermore, fracture-induced anisotropy converges to the analytic solution obtained by Backus averaging.

- The high resolution of the method allows the simulation of a wide variety of wave phenomena related to fractures, including interface waves.

- The proposed method is not restricted to simplified models, it can be applied to:
  - Intersecting fractures,
  - Fracture sets (not necessarily parallel),
  - Models with topography and any type of heterogeneities,
  - 3D models.

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