The Theory Behind the Carman-Kozeny Equation in the Quest for Permeability of Rocks*

Carlos Haro[1]

Search and Discovery Article #41344 (2014)
Posted May 12, 2014

*Adapted from extended abstract prepared in conjunction with oral presentation at CSPG/CSEG/CWLS GeoConvention 2013, (Integration: Geoscience engineering Partnership) Calgary TELUS Convention Centre & ERCB Core Research Centre, Calgary, AB, Canada, 6-12 May 2013, AAPG/CSPG©2014

[1]Occidental Oil and Gas, Houston, USA (carlos_haro@oxy.com)

Abstract

Permeability is one of the most fundamental petrophysical parameters in petroleum engineering calculations. The Carman-Kozeny equation has been used to obtain permeability (k). This equation results from the mixture of Darcy's and Poiseuille's laws. While Darcy's law macroscopically quantifies fluid flow, Poiseuille's law describes the parabolic displacement of a viscous fluid in a straight-circular tube. This denotes a common scheme in hydraulics (like the boundary layer theory) that, for the sake of simplification, solves separately for inviscid (zero viscosity) and viscous effects. Yet, in some cases this is only a good approximation (White 2003).

Darcy's law, intended for hydraulic systems, has been catalogued as equivalent to Ohm's law, best suited for electric problems. Ohm's law has been proven insufficient in porous media because it is correct only for the straight paths of the electric current, not for the meandering routes encountered in the rocks (Haro 2010). Thus, by similarity, Darcy's law also requires modification to compensate for the tortuous paths. It should be derived by use of the streamlines that describe the hydraulic flow distribution within the porous network. This law should invoke a corner-angle solution of the Laplace differential equation, to include an exponent (A), associated with the corresponding cross section, that considers the velocity variations in direction caused by the hydraulic tortuosity of the rock (White 2003).

The mathematical similarity between Darcy's and Ohm's laws permits us to connect Carman-Kozeny's and Archie-Haro's (modified Archie's equation, Haro, 2010) equations, which are also mathematically analogous (Haro, 2004, 2006). This means that resistivity log measurements can help us find the unknowns in Carman-Kozeny's equation. However, the electric tortuosity exponent (m) differs from the hydraulic tortuosity exponent (A) because of fluid viscosity and the inertial forces created by the intricacy of the rock. Although a rough estimate might be (m + 2), hydraulic tortuosity is mainly uncertain because flow is not always laminar Darcy flow everywhere within the pores. In fact, hydraulic tortuosity is the main unknown to find k.

This theory in conjunction with a database approach (lookup table) facilitates enhanced permeability modeling (Haro, 2004, 2006). This method enables the creation of static and dynamic synthetic production logs, which can assist effectively in fluid allocation, production forecasting, history matching, and well completion (Haro, 2012). These are critical variables to succeed in reservoir management.

Introduction

Innumerable methods have been created for permeability modeling. Nelson in 1994 did a comprehensive compilation of the methods available at the time. However, he was unable to explain the differences in the slope of the data in a log-log plot of permeability vs. porosity. Among the methods, he mentioned Amaefule's, Winland's, and Lucia's. These are some of the most popular ones in the engineering and geoscientist communities that can be represented with two unknowns. While the first two methods use a fixed porosity exponent (around 3 and 2, respectively) for lack of enough equations, the third one employs a variable exponent (between 4 and 9) because it is based on two equations. Civan (2002) proposed a new permeability method based on fractal analysis that also supports a variable exponent. This recommendation coincides with the analysis presented in this article. Indeed, the various data slopes in log-log plots confirm that the exponent A (Eq. 1) should be variable.

According to the literature, Carman-Kozeny's equation was derived assuming straight, parallel, non-communicating conduits (bundle of parallel capillary tubes), rather than true granular media with topology and tortuosity. For this reason, the porosity exponent in this equation is fixed and equal to 3. This is equivalent to $m = 1$ in Archie-Haro's equation. Again, a porous medium requires a variable exponent to be theoretically sound, which is essential for improved permeability modeling.

Theory and Method

The Laplace differential equation in 2D adequately describes the streamlines and potentials when analyzing fluid mechanics of inviscid flow. Its corner angle solution contains an exponent that can be associated with the cross section of the hydraulic paths (areal porosity). Moreover, the exponent has a mathematical connection with the corner angle that delineates the meandering routes within the porous rock. For these reasons, the Carman-Kozeny equation must have a variable porosity exponent and direct geometric ties with the hydraulic trails, as expected. Although there are various forms of the Carman-Kozeny equation, they can be reduced to the following form (Haro, 2004, 2006):

$$\mathrm{Log(k)} = [\mathrm{A}\log(\phi)] + B \qquad (1)$$

where ϕ is porosity of the porous medium, and A and B are calibration parameters. The Archie-Haro equation reduces to the same mathematical form (Haro, 2004, 2006).

In generating a good permeability transform with cores and well logs, rank correlation is usually employed to find the proper well log parameters. This could be a potential problem if the correlation coefficients are low. In most cases, this is a time-consuming trial and error exercise. With the proposed method, irreducible water saturation (Swi), ϕ, and deep resistivity (Rt), which are integral parts of Archie-Haro's equation, are always used. Historically, these parameters have been reported to have the best correlations with permeability, so it makes a lot of sense to use them. Besides, these parameters should always be available for wells with sufficient data to generate a standard petrophysical analysis.

The combination of the Archie-Haro and Carman-Kozeny equations enables us to use a rock type definition (Eq. 2) that is equal to Buckles' when $m = n$ (where n is the saturation exponent). This permits a direct connection with water cut, because coherent Buckles patterns should correspond to zero water production, according to the theory. In addition, *Swi* becomes a mathematical link with capillary pressure, absolute and relative permeability, and fractional flow, because *Swi* constitutes one of the endpoints and part of the locus of those equations.

A cross section of a pore is depicted in Figure 1, where irreducible water is displayed attached to the pore walls (water-wet condition). The figure illustrates that the greater the *Swi*, the smaller the space for the hydrocarbons to pass through, therefore permeability to oil diminishes. This applies both to the pore throats and pore bodies. Accordingly, *Swi* has a good theoretical correlation with k. Furthermore, the larger the hydraulic paths are in the rock, the longer the time it will take for the fluids to travel across. So, topology and tortuosity have an impact on permeability magnitude, and they should be ingredients or controls of k. Thus, the following relationships (obtained from combining Archie-Haro's and Carman- Kozeny's equations) apply (Haro, 2006):

$$\frac{B}{A} = (1/m)\log\left(\frac{R_t}{abR_w}\right) = \log\frac{1}{\left(S_{wi}^{n/m}\right)\phi} \qquad (2)$$

$$(kS^2)^{\frac{m}{A}} = \frac{1}{S_{wi}^{\;n}} \qquad (3)$$

where a and b are the tortuosity factors (Haro, 2010), and S is the specific surface area, obtainable from various sources (Haro, 2006). Both equations are dimensionally homogeneous and should be resolved simultaneously. Eq. 2 is intended for rock typing, while Eq. 3 serves to find permeability. The right hand side of Eq. 2 depends only on log values, thus rock typing can be performed in cored and uncored sections/wells. Since Eq. 3 is parameterized using hydraulic tortuosity, this equation is geared towards fluid permeability, not to air permeability, which is generally measured during core analysis. Thus, the corresponding conversion factor is necessary.

The proposed permeability model complies with important characteristics, which are considered advantageous in the literature (Jensen, 2000):

• It is theory based. Besides, there is a physical reason for resistivity (measurement) and permeability (unknown) to be related. These items avoid “happenstance.”

• The usage of microscopic relations for Darcy’s, Poiseuille’s and Archie’s equations, which honor respectively the internal distribution of flow, fluid laminae and electric currents travelling through the pore passages, makes them volumetrically compatible with permeability, which also depends on topology, tortuosity and the internal structure of the rock, as specified before.

• A rock type definition that employs *Swi* and ϕ refers to sample volumes generally consistent with the geological scales. Therefore, it is possible to take advantage of the semi-repetitive nature of geological events that occur in similar rock units. However, some incompatibility persists in laminations or features below the resolution of the logging tools, which should be resolved before attempting permeability estimations.

Example

In the quest for permeability, lookup tables are used with the proposed method. This type of procedure is generally used in engineering (e.g. to match PVT data) when there is no evident correlation. Since Eqs. 2 and 3 have a good theoretical foundation, they overcome the limitations reported by Nelson (1994). This methodology can be compared with a cloud transform, fuzzy logic or conditional expectation. These techniques are widely used in reservoir characterization.

The main advantage of a database approach (Delfiner, 2007) is that the high values of permeability are honored and not replaced by lower values coming from a correlating line. This observation signifies that permeability multipliers are not needed to match production, whereas it is common to require multipliers of 5 or more to get a history match, especially in simulators (Haro, 2012). The best way to show this type of agreement is by means of a synthetic production log (SPLT), as the one illustrated in Figure 2. SPLTs are obtained using a radial flow equation in vertical wells and summing all the fluid contributions at every depth. Of course this equation uses permeability. Therefore, the conformity between an SPLT and an actual production log means that permeability is correctly calculated and vertically distributed, and therefore it can be used for production allocation. In addition, good permeability values are a requisite to obtain accurate DSPLTs (dynamic synthetic production logs obtained over time), which shall permit history matching and reliable production forecasts in a well. This has already been demonstrated in a previous article (Haro, 2012). For reference, Amaefule et al. (1993) mention that they also used a database approach in the implementation of their flow zone indicator model (FZI) in uncored sections.

Conclusions

The Laplace differential equation and its corner angle solution justify the use of a variable exponent for the porosity term in the Carman-Kozeny equation. The exponent, which represents hydraulic tortuosity, has geometrical implications as well as viscosity compensation. Hydraulic tortuosity (*A*) is different from electrical tortuosity (*m*) due to the viscosity effects of the hydraulic flow.

A database approach proves to be a superior technique compared to traditional ones (k-ϕ lines) to find permeability. However, a database approach must be solidly based on the theory.

Acknowledgments

The author wishes to thank Occidental Oil and Gas for granting permission to publish this paper.

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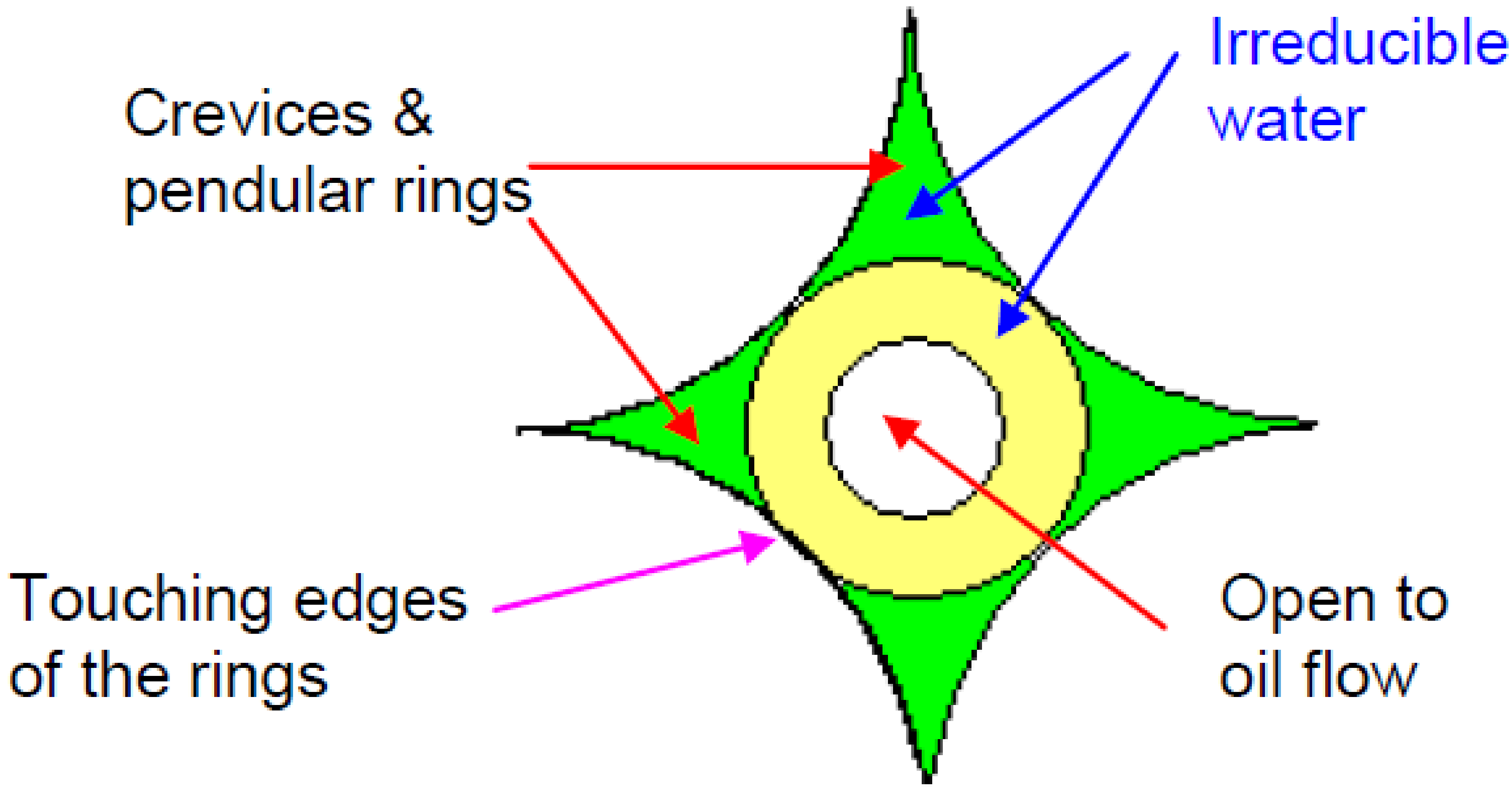

Figure 1. Cross sectional view of a pore: Irreducible water is occupying a space, crevices (green) and films (yellow) that otherwise should be available to non-wetting fluid flow.

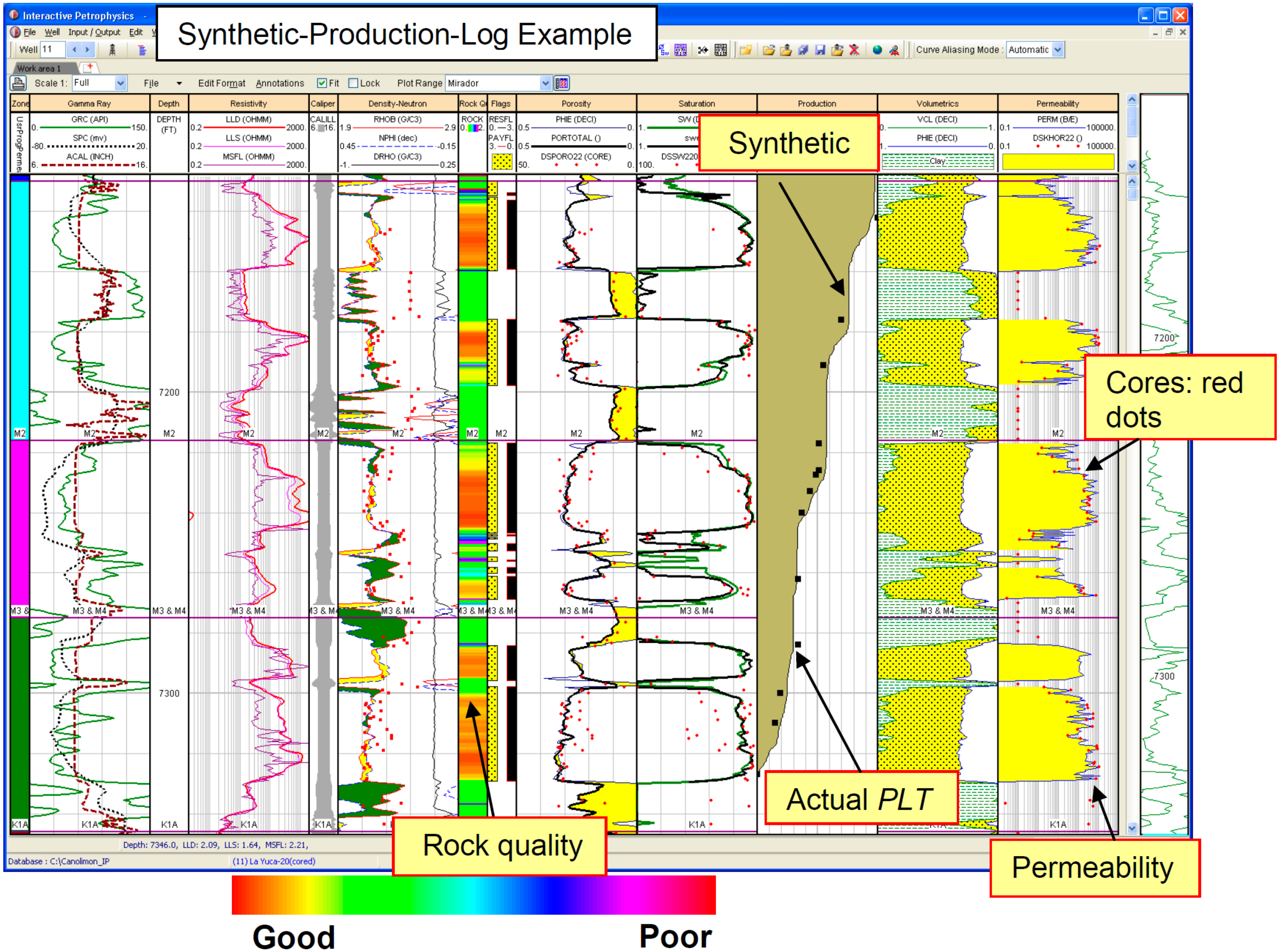

Figure 2. Comparison of a synthetic production log in a vertical well with zero water cut (continuous line) and an actual production log (dots). With good agreement, the method can be used confidently for production allocation.