A New Model of CBM Permeability and Sorption-Induced Strain Based on Open-System Geomechanics*

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Search and Discovery Article #80262 (2012)**
Posted October 22, 2012

*Adapted from extended abstract prepared in conjunction with oral presentation at AAPG International Convention and Exhibition, Singapore, 16-19 September 2012, AAPG©2012

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Abstract

The sorption-induced porosity and permeability alteration of coal beds as pore pressure and in-situ stress are varied can be couched within the frameworks of 'open thermodynamic system' and 'open-system geomechanics' (OSG). Production from coal bed methane (CBM) reservoir changes not only the fluid mass (gas and water) but also solid mass as a result of methane desorption from rock matrix. The latter manifests itself as rock matrix shrinkage which often leads to permeability enhancement. This approach captures the fundamental process of mass changes in the CBM as a result of methane production. In this formulation, the use of fluid and solid mass as elementary variables is beneficial because they directly impact the variation of strain and stress as fluid and 'solid' are removed from the reservoir. This new model provides a fundamental, fully coupled description of stress-strain, solid and fluid masses evolution instigated by sorption/desorption transition. It involves a new constitutive relationship which describes the evolution of sorption-induced strain and heat transfer equations.

As a result, the OSG approach expresses explicitly the sorption-induced volumetric strain via the Skempton coefficient of coal beds. It elegantly leads to a new model of coal-bed methane (CBM) permeability description as a function of pore pressure and fundamental measureable properties such as Skempton coefficient, fluid (gas) and solid (coal matrix) bulk moduli, Biot's constant, and initial porosity.

The prediction of CBM permeability using this new approach is compared to published experimental data and existing theoretical models. Initial investigation is encouraging and the comparison shows substantial improvement over previous models.

Introduction

The rocks, where phase transformations or chemical reactions between solid component and fluid (gas) component take place, cannot be described in the framework of conventional theory of poroelasticity because solid mass is not constant in the system. Coal bed methane reservoirs belong to this class of systems because of sorption-desorption processes which change the mass of coal matrix. For example, as it was shown recently (Liu and Harpalani, 2012), the attempts to fit experimental data of CBM samples testing by conventional poroelasticity lead to inconsistencies, e.g. unreasonably high Biot's coefficient.

We generalized formulation of theory of poroelasticity for the case of non-constant solid mass (Geilikman and Wong, 2008; Geilikman and Wong, 2007; Geilikman and Wong, 2012). In this case use of so-called of un-drained variables (Rice and Cleary, 1976), that is mass of fluid rather than pressure, seems beneficial in geo-mechanical treatment of any open porous system (compaction/depletion of conventional reservoirs, injection of external fluid in the reservoir etc) because variation of strain and stress depends on the amount of mass in the system.

The so-called eigen-volume and eigen-strain are those, which system assumes in unstressed (free) state. We derived the following expression for eigen-strain in the case when solid mass is not constant (Geilikman and Wong, 2008; Geilikman and Wong, 2007; Geilikman and Wong, 2012):

$$\varepsilon_{eg} = B[m_s(\frac{1}{\rho_g} - \frac{1}{\rho_s}) + \frac{m_g}{\rho_g}] + \beta_u \Delta T \qquad (1)$$

where $m_{s,p} = \Delta M_{s,g}/V$, with ΔM_s is a mass of solid converted into fluid (gas) state in pores, and ΔM_g is a mass of fluid (gas), B is Skempton coefficient, ρ_g is gas density, ρ_s is density of fluid (gas) molecules in solid state, β_u is undrained coefficient of thermal expansion, T temperature, and V is initial volume of porous medium. If we put $m_s = 0$, all equations coincide with conventional mass/heat transport equations of chemically inert porous medium (Rice and Cleary, 1976).

Incremental mean stress induced by variation of eigen-strain is equal to:

$$\Delta\sigma_m = K_u(\varepsilon_v - \varepsilon_{eg}) \qquad (2)$$

where ε_v is volumetric strain, and K_u is undrained bulk modulus of porous medium: so that when the volumetric strain equals to eigen-strain the mean stress turns to zero, as it should be.

Relationship between drained bulk modulus K_b and undrained bulk modulus K_u is well known (Charlez, 1991):

$$K_b = K_u(1 - \alpha B) \qquad (3)$$

where Biot's coefficient $\alpha = 1 - K_b/K_s$, K_s is bulk modulus of solid component.

Variation of porosity can be expressed via mean stress and fluid pressure as follows (Rice and Cleary, 1976; Charlez, 1991):

$$\Delta\phi = (\alpha - \phi_0) \cdot \frac{\Delta p - \Delta\sigma_m}{K_b} \tag{4}$$

where p is pore pressure, ϕ_0 is initial porosity. After utilization of Equation (2) we obtain for variation of porosity:

$$\Delta\phi = \frac{(\alpha - \phi_0)}{K_b} \cdot [\Delta p - K_u (\varepsilon_v - \varepsilon_{eg})] \tag{5}$$

Actual volume of rock is different from the eigen-volume (i.e. the free one at zero stress) due to the stress exerted by interaction with surrounding rock. If mechanical properties (Young's modulus and Poisson's ratio) are homogeneous in space then the problem can be solved similarly to that of thermal strain even if the eigen-strain is coordinate-dependent. As a result, volumetric strain, ε_v, is expressed via eigen-strain as follows (Geilikman and Wong, 2012):

$$\varepsilon_v = s\,\varepsilon_{eg} \tag{6}$$

where

$$s = \frac{(1+\nu)}{3(1-\nu)} \tag{7}$$

where ν is un-drained Poisson's ratio. Using Equations (6), (7), expression for porosity (5) can be re-written as follows:

$$\Delta\phi = \frac{(\alpha - \phi_0)}{K_b} \cdot [\Delta p + \frac{2(1-2\nu)}{3(1-\nu)} \cdot K_u\,\varepsilon_{eg}] \tag{8}$$

For homogeneous porous media the interval for Biot's constant variation is (Charlez, 1991):

$\phi < \alpha < 1$.

New Model of CBM Permeability

Let us consider isothermal case, and using Equation (1) we obtain for sorption-induced eigen-strain:

$$\varepsilon_{eg} = B\left(\frac{1}{\rho_g} - \frac{1}{\rho_a}\right) m_s \qquad (9)$$

where m_s is a change of mass of adsorbed molecules per total volume, ρ_a now has a sense of density of adsorbed molecules (e.g. methane) per unit volume occupied by methane in adsorbed state, which is of the order of condensed phase density. The eigen-strain Equation (9) describes variation of volume due to transfer of methane molecules between gaseous and adsorbed states.
After utilizing a well-known expression for Langmuir isotherm for adsorbed gas mass:

$$m_s = m_a \frac{p}{p + p_L} \qquad (10)$$

(where m_a and p_L are Langmuir constants) we arrive at the following explicit expression for sorption-induced volumetric strain:

$$\varepsilon_{eg} = \left[B\left(\frac{1}{\rho_g} - \frac{1}{\rho_a}\right) m_a\right] \frac{p}{p + p_L} \qquad (11)$$

Expression for sorption-induced volumetric strain was based previously on experimental measurements (Harpalani and Schraunfnagel, 1990; Cui and Bustin, 2005; Robertson and Christiansen, 2005):

$$\varepsilon_s = \varepsilon_L \frac{p}{p + p_L} \qquad (12)$$

where ε_L was previously an experimentally measured constant, and/or production history matching (Shi and Durucan, 2004) (parameter ε_l).

Langmuir isotherm is frequently expressed in term of volume (not mass) of gas per unit mass of coal:

$$c_s = V_L \frac{p}{p + p_L} \qquad (13)$$

where V_L is a constant. Using previous results (Saghafi et al., 2007; Zhang et al., 2008) the mass adsorption constant, m_a, can be expressed via volumetric constant, V_L, as follows:

$$m_a = \rho_{ga} \rho_c V_L / (1 - \phi) \qquad (14)$$

where ρ_c is density of coal, and ρ_{ga} is gas density at standard conditions.

As a result our approach allows us to find the sorption- strain constant ε_L of Equation (12) as a function of Skempton coefficient, gas density, and mass Langmuir constant:

$$\varepsilon_L = B\left(\frac{1}{\rho_g} - \frac{1}{\rho_a}\right) m_a \qquad (15)$$

Explicit expression for Skempton coefficient via elastic properties of porous media components is well-known (Charlez, 1991):

$$B = \frac{\alpha}{\alpha + \phi\left(\frac{K_b}{K_g} + \alpha - 1\right)} \qquad (16)$$

Now we can find variation of porosity from Equation (8) using expression for sorption-induced strain given by Equations (9) - (15):

$$\phi \approx \phi_e + \frac{(\alpha - \phi_e)}{K_b} \cdot (p - p_e) \cdot \left[1 - \frac{2(1-2\nu)K_u}{3(1-\nu)} \cdot \frac{B m_a \left(\frac{1}{\rho_g} - \frac{1}{\rho_a}\right) p_L}{(p + p_L)(p_e + p_L)}\right] \qquad (17)$$

where p_e is pressure at which porosity is equal to ϕ_e.

Permeability vs. pore pressure, $k(p)$, can be found using conventional Reiss expression (Reiss, 1980):

$$k(p) = k_e \left(\frac{\phi}{\phi_e}\right)^3 \qquad (18)$$

Equations (17) and (18) constitute a new model of CBM permeability based on Open-System Geomechanics approach which allows us to explicitly find parameters of sorption-induced strain via mechanical properties of porous medium such as Skempton coefficient and elastic parameters.

Expression for Skempton coefficient can be simplified further taking into account that gas bulk modulus is much lower than that of the coal:

$$B \approx \frac{\alpha K_g}{\phi K_b} \tag{19}$$

As we can see from Equation (19), $B << 1$, because in gas-saturated porous media external stress is supported mainly by its solid frame.

If we also consider gas in the pores as an ideal one with equation of state:

$$\rho_g \approx \frac{M_g}{RT} p \tag{20}$$

where M_g is molecular weight of gas molecules, then the gas bulk modulus $K_g = p$.

If we also take into account that gas density is much lower than that of adsorbed molecules we obtain for eigen-strain:

$$\varepsilon_{eg} \approx \frac{B}{\rho_g} m_s \approx \frac{\alpha K_g}{\phi K_b \rho_g} \cdot \frac{m_a p}{p + p_L} \tag{21}$$

By utilizing gas equation of state (20), we can also express gas density in Equation (17) via pressure. As a result we obtain for eigen-strain:

$$\varepsilon_{eg} \approx \frac{\alpha RT}{\phi M_g K_b} \cdot \frac{m_a p}{p + p_L} \tag{22}$$

After these simplifications, expression for porosity (17) reads:

$$\phi \approx \phi_e + \frac{(\alpha - \phi_e)}{K_b} \cdot (p - p_e) \cdot [1 - \frac{2(1-2\nu) K_u}{3(1-\nu)} \cdot \frac{\alpha RT}{\phi_e K_b M_g} \cdot \frac{m_a p_L}{(p + p_L)(p_e + p_L)}] \tag{23}$$

From Equations (17) and (18) we obtain new expressions both for recovery pressure:

$$p_{rc} \approx \frac{2(1-2\nu)K_u}{3(1-\nu)} \cdot \frac{\alpha R T m_a}{\phi_0 K_b M_g} \cdot \frac{p_L}{(p_e + p_L)} - p_L \qquad (24)$$

and for rebound pressure:

$$p_{rc} \approx \sqrt{\frac{2(1-2\nu)K_u}{3(1-\nu)} \cdot \frac{\alpha R T m_a p_L}{\phi_0 M_g K_b}} - p_L \qquad (25)$$

Comparison with Experiment

As it was found by Robertson and Christiansen, 2007 all previously existed permeability models (Shi and Durucan, 2004; Palmer and Mansoori, 1998; Seidle and Huitt, 1995) overestimated variation of permeability vs. pressure compared to their experimental measurement.

We compare our OSG model given by Equations (17) and (18) with the same experimental data for methane permeability vs. pressure. In notation of paper (Robertson and Christiansen, 2007), parameter S_{max} can be expressed in our model as follows:

$$S_{\max} = \frac{\alpha R T m_a}{3\phi_0 M_g K_b}. \qquad (26)$$

For comparison of our model with experiment we used the following values: Poisson's ratio $\nu = 0.35$. As it was noted in (Robertson and Christiansen, 2007), Young's modulus for coal varies from 100,000 psi to 10,000,000 psi (Ojeifo et al., xxxx). The value which was used in (Robertson and Christiansen, 2007) was 200,000 psi. For experimentally measured parameters it was found (Robertson and Christiansen, 2007) that S_{max} is 0.00931 for sub-bituminous Anderson seam coal, and 0.00765 for high-volatile bituminous Gilson seam coal. Initial permeability was 1.31% and 0.804% for Anderson and Gilson cores respectively. Langmuir pressure constant was found (Robertson and Christiansen, 2007) as $p_L = 886$ psi.

Using these values we generated permeability plots shown at Figure 1 and Figure 2 based on Equations (17) and (18). Good agreement between the model and experimental data is achieved for Anderson core at Young's modulus 250,000 psi similarly to that assumed before (Robertson and Christiansen, 2007). However as we can see from Figure 2 for Gilson core a better agreement is for lower Young's moduli ~ 124,000 psi.

One of the relatively uncertain parameters is Biot's coefficient α. Because of that we plotted permeability for different Biot's coefficient at Figure 3 and Figure 4.

As we can see, sensitivity to variation Biot's coefficient is lower than to that of Young's modulus, and in general agreement between our model and experimental measurements [14] is sufficiently good.

Experimental measurement of permeability (Liu and Harpalani, 2012) performed with San Juan core, exhibited descending trend as a function of pressure. The permeability model, expressed by Equations (17) and (18), allows parametric study as function of input parameters. If we use for Langmuir parameter p_L = 700 psi, Poisson's ratio ν = 0.2, we are able to reproduce the descending trend at sufficiently high values of Young's modulus as it is shown at the Figure 5. Interestingly enough at sufficiently low Young's modulus the trend of permeability vs. pressure becomes ascending which is understandable from physical point of view.

Conclusion

- A new model of coal-bed methane (CBM) permeability is derived based on Open-System Geomechanics (OSG) approach.
- The OSG approach allows to explicitly express sorption-induced volumetric strain via Skempton coefficient and gas and solid densities.
- In turn, this allows to explicitly link sorption-induced porosity and permeability with mechanical properties of porous media such as gas and solid bulk modules, Biot's constant and initial porosity.
- Sorption-induced variation of permeability vs. pore pressure, based on OSG approach, gives significantly better agreement with experimental data (Robertson and Christiansen, 2007) compared to previous theoretical models of permeability.
- This approach allows us to perform parametric study of permeability as a function of input parameters. It is shown that depending on mechanical and sorption parameters the system may not exhibit a minimum permeability vs. pressure. The absence of permeability rebound vs. pressures is in agreement with published in the past experimental data.

Acknowledgement

Authors are thankful to Shell International E & P for permission to publish this paper and to Dr. M. Khodaverdian (Shell) for useful discussions.

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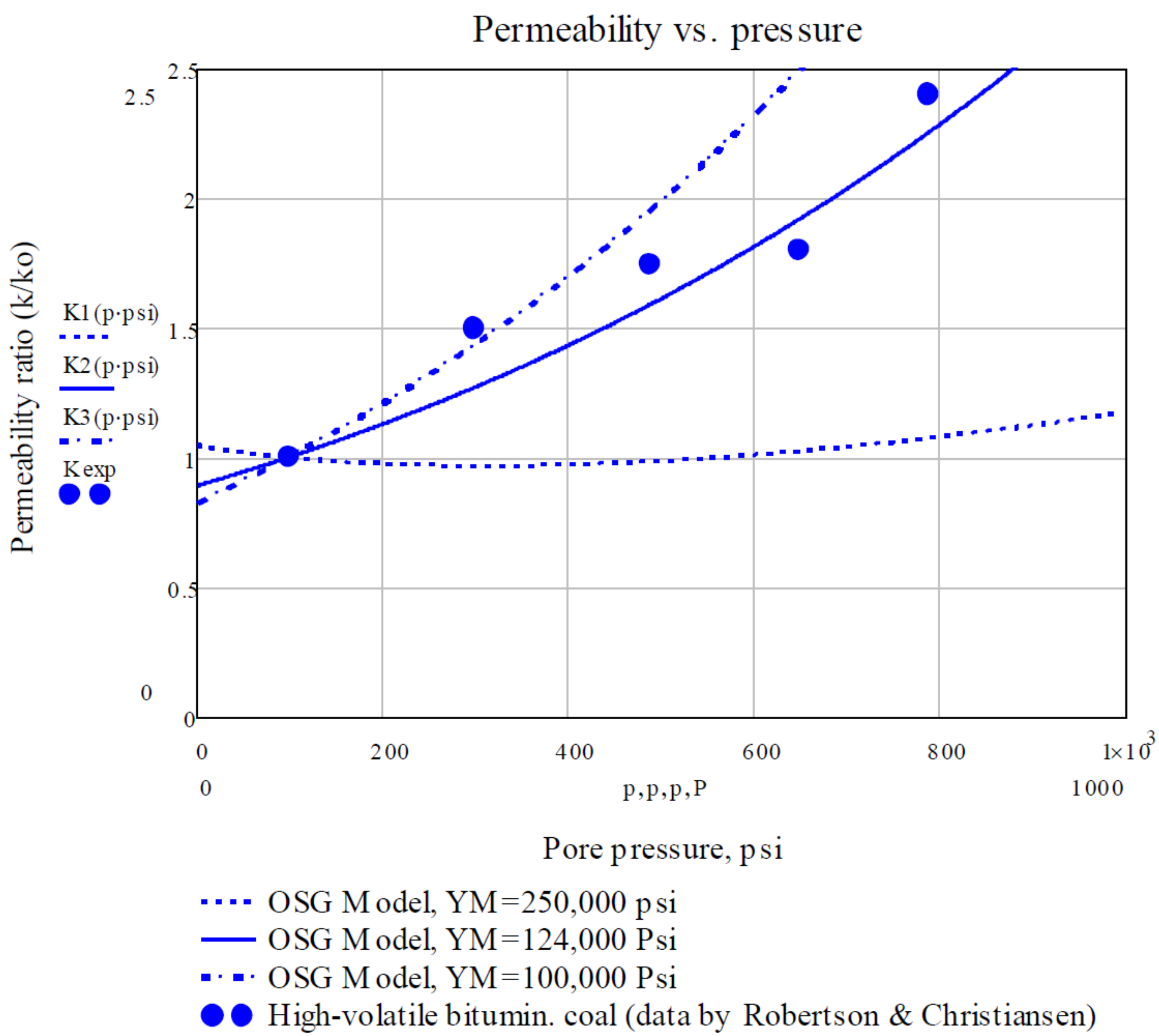

Figure 1. Comparison of Open-System Geomechanics (OSG) Model prediction (Biot's coefficient $\alpha = 0.9$) for coal permeability with experimental data for high-volatile bituminous coal from Gilson seam, Utah (Robertson and Christiansen, 2007).

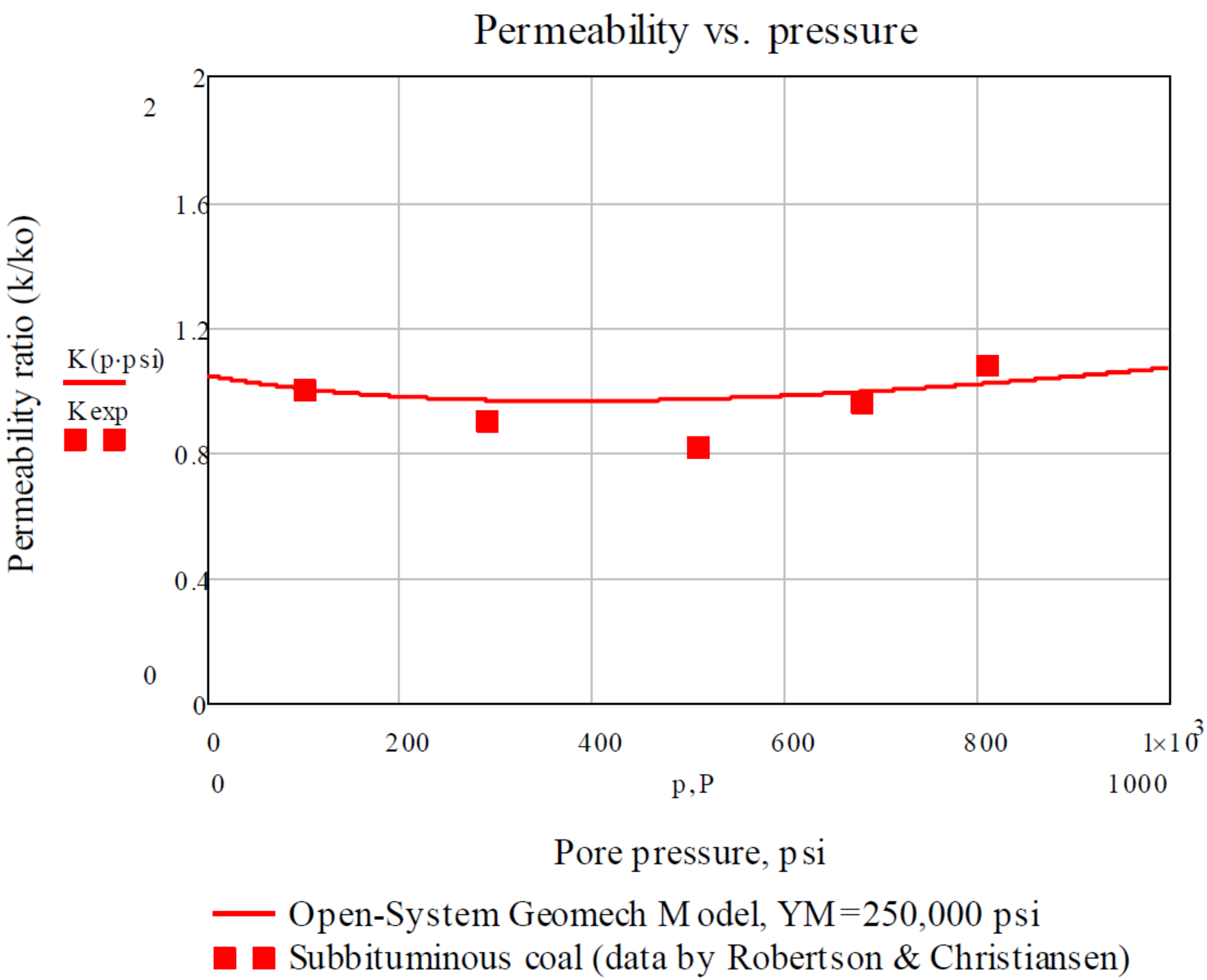

Figure 2. Comparison of Open-System Geomechanics (OSG) Model prediction (Biot's coefficient $\alpha = 0.9$) for coal permeability with experimental data for sub-bituminous coal from Anderson seam, Wyoming (Robertson and Christiansen, 2007).

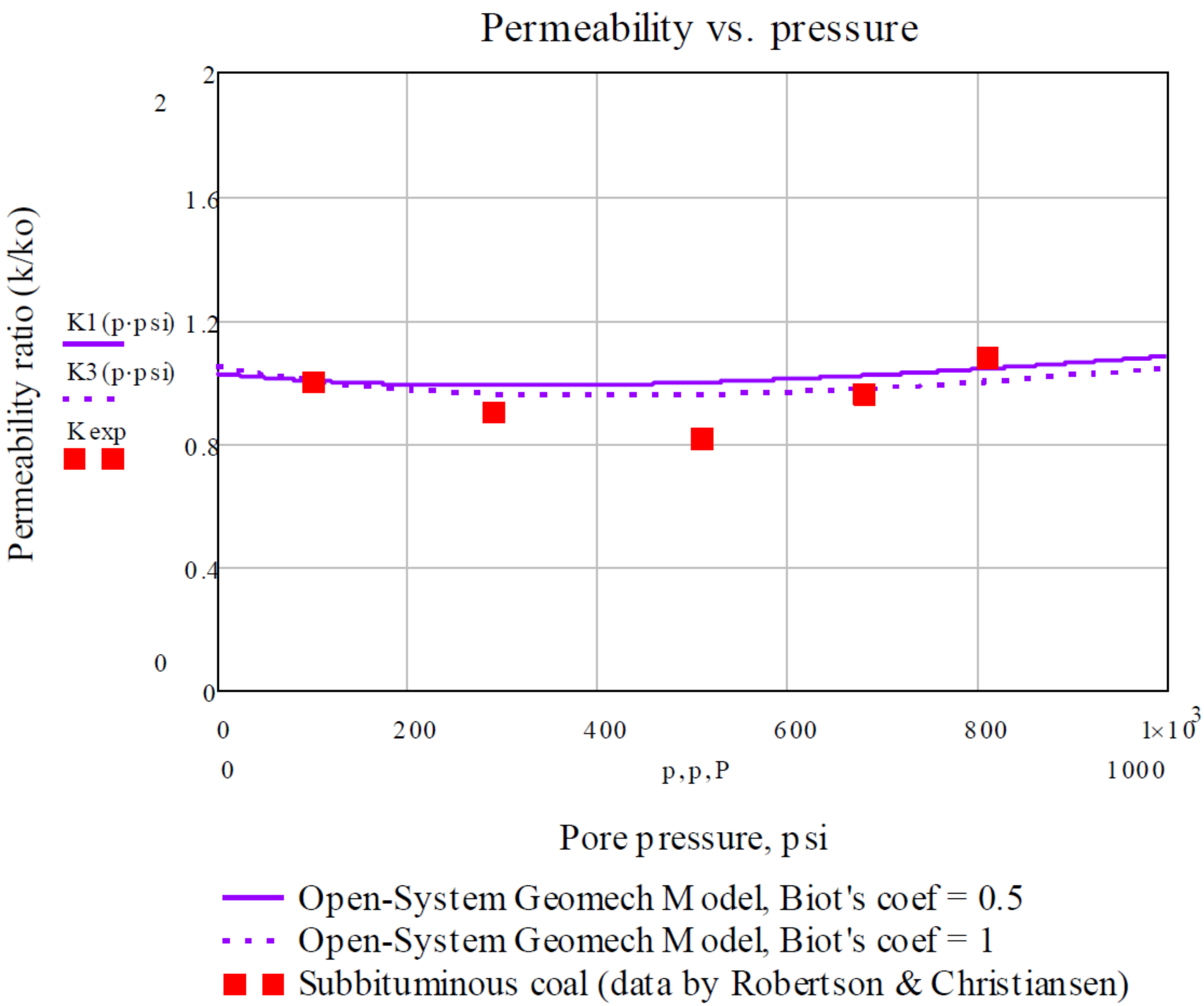

Figure 3. Comparison of Open-System Geomechanics (OSG) Model prediction for coal permeability at different Biot’s coefficients (Young’s modulus = 124,000 psi) with experimental data for high-volatile bituminous coal from Gilson seam, Utah (Robertson and Christiansen, 2007).

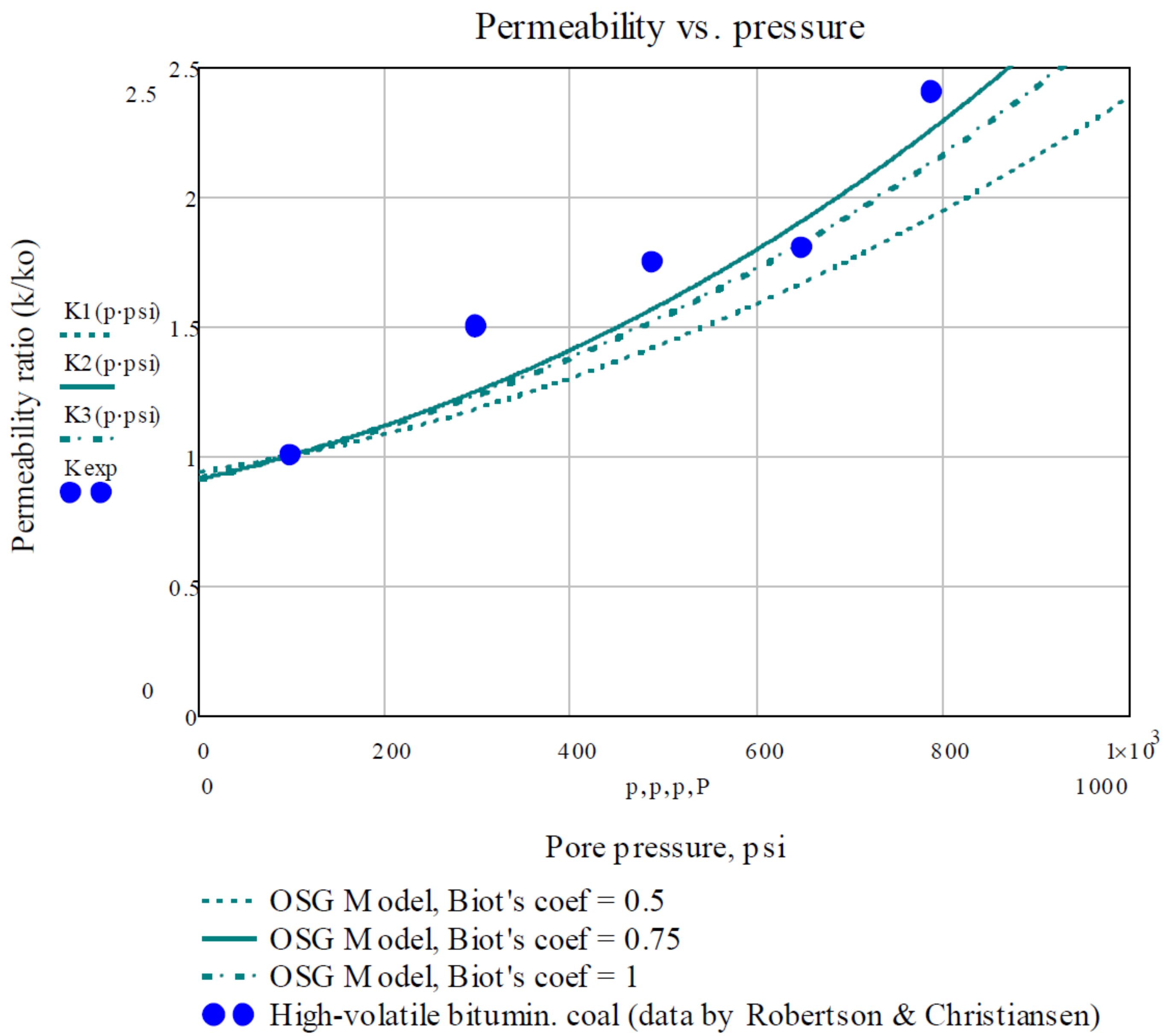

Figure 4. Comparison of Open-System Geomechanics (OSG) Model prediction for coal permeability at different Biot's coefficients (Young's modulus = 250,000 psi) with experimental data for sub-bituminous coal from Anderson seam, Wyoming (Robertson and Christiansen, 2007).

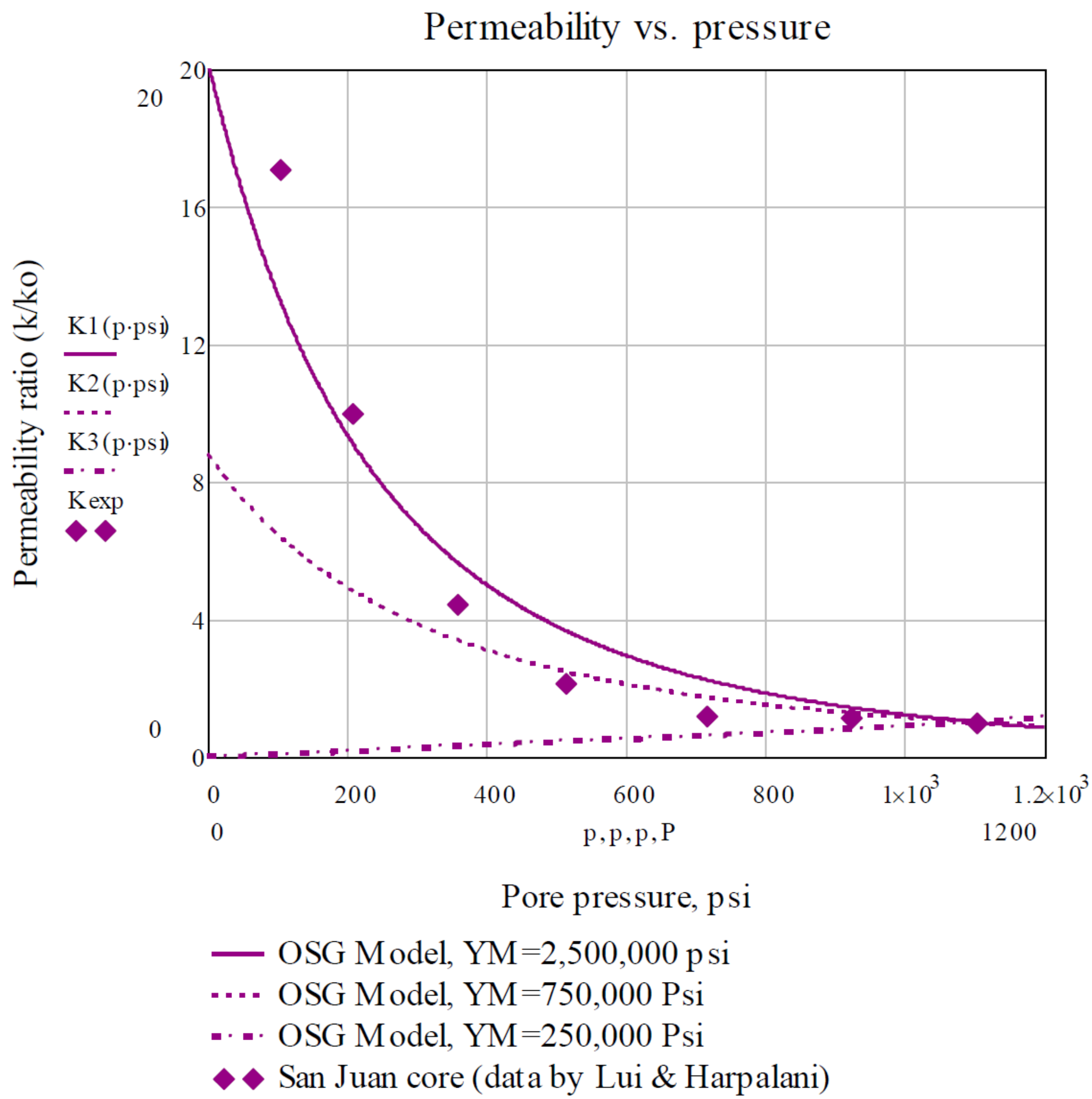

Figure 5. Comparison of Open-System Geomechanics (OSG) Model prediction for coal permeability with experimental data for San-Juan core (Liu and Harpalani, 2012).