

On the Characterization of Porosity and Permeability in Limestones*

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Abstract

About half of all producing oil reservoir are limestones and dolomites. While porosity and some other physical properties are routinely evaluated from logging, measuring in situ permeability is usually not workable at low cost. Furthermore, it is well admitted that permeability is a scale-dependent property, so that its measurement on core plugs cannot be directly used for assessing reservoir scale permeability. The ability to estimate the permeability of a reservoir rock from other more readily measurable parameters would be of great value to the oil industry. Empirical permeability models such as the Kozeny-Carman equations that make use of the porosity and a specific surface parameter or Katz-Thomson equations relating permeability to porosity and the electrical formation factor are usually used for sandstone reservoirs. However, such models are insufficiently accurate for limestone reservoir characterization purposes because of the complex pore and pore-network structure of these rocks. Indeed, the pore network structure, porosity and permeability of limestone reservoirs as they are observed on cores are the result of complex mass transfer mechanism during deposition, diagenesis and hydro-chemical alteration periods. The effects of the hydro-chemical alteration processes, mainly dissolution and precipitation, are probably the most difficult to assess because of large variability of the forcing mechanisms and the intrinsic mineralogical and structural heterogeneity of the rock. For the same type of limestone, distinctly different pore network morphologies and hydrodynamic properties are usually observed at less than meter scale. Moreover, it is often observed that post-depositional hydro-chemical alteration processes act as increasing irreversibly the heterogeneity due feedback effects of the flow, transport and reactions.

Acknowledging that both the heterogeneity and its scale dependence are the main issues for evaluating permeability from other measurable properties in limestones different research strategies can be conjectured. A direction to follow would be to characterize the relations between easily measurable physical properties that control permeability at core plugs scale then upscale these relations. This is a highly challenging work, the success of which being dependent on the ability to determine scale-invariant properties and relations. If the properties display a large variability, then their characterization requires stochastic models.

Introduction

The problem posed for determining pertinent models for evaluating permeability from structural and physical parameters is multifold and different investigation routes can be followed. Hereafter we discuss a promising research direction based on the use of structural characteristic parameterization using modern investigation methods such as X-ray tomography. Albeit being restricted to millimeter to few tens of centimeters size samples, X-ray tomography (XMT), coupled with standard core measurements, is probably the only technique available to investigate hydrochemical processes over more than 5 orders of magnitude of scale. Actually, using new multi-scale acquisition techniques presently under development coupled with hybrid-method modeling tools, it is probable that we may soon be able to study the properties and geometry scale-dependence of porous media over 6 to 7 orders of magnitude. The main objective here is to parameterize permeability (or other primary hydrodynamic properties such as dispersivity) models from quantities that can be readily measured from core plugs to reservoir scale. The process could be as following. From pore shape and pore distribution parameterization (for instance in the context of stochastic models) we must deduce what we call *hydrodynamical sub-properties* such as, for examples, the effective hydraulic radius, the flow path tortuosity and the connectivity, as well as their scale dependence or independence. Others variables can be defined. The same approach can be used to relate geometrical parameters to properties controlling the acoustic wave propagation or the electrical conductivity. Then, the changes of these variables according to hydro-chemical alteration processes (dissolution and precipitation) must be modeled.

Such analysis can be done by analyzing large set of core samples, but a much pertinent way of addressing the problem is to study the changes of the pore structures and (scale-dependent) properties of the same core sample undergoing controlled dissolution. Yet, this approach introduces a new controlling parameter: the reactive surface area.

Results

Figure 1 display permeability (k) versus porosity (ϕ) data obtained during three dissolution experiments (noted D1, D2 and D3) of oolitic limestone samples cored at depth 1635 m in the Middle Jurassic Mondeville formation of the Paris Basin (Luquot and Gouze, 2009). Each experiment is characterized by the dimensionless Damköhler number $Da \equiv \bar{\mathcal{G}}l/\bar{u}$, with \bar{u} the average water seepage velocity and $\bar{\mathcal{G}}$ the average dissolution velocity. The value of Da decreases from experiment D1 to experiment D3. The $k - \phi$ function can be modeled by a power law equation with a single exponent coefficient that is fully determined by the effective reactivity $\bar{\mathcal{G}}$ of the fluid percolating the sample and the flow pattern:

$$k(t) = k_0(\phi(t) - \phi_c)^n, \quad (1)$$

where ϕ_c is the value of the porosity (assumed the same for the 3 samples) below which the pore network is unconnected at the scale of the sample (percolation threshold). Such formula was proposed by Martys et al. (1994) who derived a value of $n \approx 4$ for a set of various random porous media formed of overlapping and non-overlapping sphere packings. To interpret these results we now focus on the relation between the permeability, the porosity, the hydraulic radius and the tortuosity, with the help of the XMT images.

The permeability is a macroscopic parameter that measures the energy loss of the viscous fluid along the connected pore cluster in the sample. In first approximation, permeability is controlled by the effective hydraulic radius θ and the average length λ of flow paths within the connected pore cluster. Both θ and λ are viewed here as macroscopic parameters. The hydraulic radius is generally defined as $\theta = \phi/\sigma^*$, where σ^* denotes the effective specific surface. Note that for matter of consistency this relation must be rewritten $\theta = (\phi - \phi_c)/\sigma^*$, so that $\theta = 0$ when $\phi = \phi_c$. For simple pore geometries (e.g. sphere packings) $\sigma^* \approx \sigma$ with σ the total (geometric) specific surface. Conversely, defining the tortuosity $\tau = \lambda/L$ (i.e. $\tau \geq 1$), its value is often related to porosity by the empirical relationship $\tau = B_\tau(\phi - \phi_c)^{-\beta}$, where B_τ is a dimensionless geometrical parameters. The origin of this relation based initially from the Archie's law is discussed in Dullien (1992), for examples. Consequently, the permeability versus porosity law (equation 1) is now:

$$k(t) = k_0(\phi(t) - \phi_c)^\alpha \left(\frac{B_\tau}{\tau} \right), \quad (2)$$

where $\alpha = n - \beta$ and the constant $k_0 = k^{(0)}/(\phi^{(0)} - \phi_c)^n$ is easily determined from [Figure 1](#) with $\phi_c = 5.9\%$ evaluated experimentally. For heterogeneous porous media the effective hydraulic radius is a complex function of the porosity and pore space structure. Yet, assuming that permeability depends only on θ and τ , and that θ and τ are independent parameters, equation (eq. 2) tells us that $\theta(t)$ scales with $(\phi(t) - \phi_c)^\alpha$:

$$\theta = B_\theta(\phi - \phi_c)^\alpha, \quad (3)$$

where B_θ is the scaling constant. Recalling the definition of θ , $\theta = (\phi - \phi_c)/\sigma^*$, and using the power law scaling between the reactive surface area and porosity given by Luquot and Gouze (2009), $\bar{\sigma}_r(t) = B_\sigma \phi(t)^{-w}$, it follows that $\sigma^* \propto (\phi - \phi_c)^{-\alpha+1}$. The values of α and β are weighting coefficients denoting the differential importance of the effective hydraulic radius and the tortuosity for controlling the $k(\phi)$ relationship:

$$\log\left(\frac{k}{k_0}\right) = \log\left(B \frac{\theta}{\tau}\right) = \alpha \log(\phi - \phi_c) + \beta \log(\phi - \phi_c) \quad (4)$$

with $B = B_\tau/B_\theta$ a geometrical coefficient (independent of time) that characterizes the rock type. The tortuosity τ before and after dissolution is obtained by computing the diffusion of randomly distributed particles in the rock pores where the connected pore cluster is obtained from the segmented XMT at $t^{(0)}$ and $t^{(f)}$. The initial value of the tortuosity measured for sub-volumes of the sample is homogeneous along the sample. Unsurprisingly, for this highly heterogeneous low-porosity medium the sample-averaged value is high; $\bar{\tau}^{(0)} \approx 4.8$. At the end of the percolation experiments, tortuosity decreases for each experiment. The exponent β is now calculated for $t = t^{(f)}$ by solving $\bar{\tau} = B_\tau(\phi - \phi_c)^{-\beta}$ with the condition $\tau = 1$ for $\phi = 1$:

$$\beta = \log(\tau^{(f)}) / \left[\log(1 - \phi_c) - \log(\phi^{(f)} - \phi_c) \right] \quad (5)$$

Calculation shows that β is higher for the experiment D1 than for D2 and D3, or in others words that the tortuosity change associated with porosity change is higher for D1 than for D2 and D3. Yet, the asymptotic $k(\phi)$ relationship (i.e. the n coefficient) is controlled by both the hydraulic radius changes (which rate $\partial\theta/\partial\phi$ is controlled by α ; $\partial\theta/\partial\phi = \alpha B_\theta(\phi - \phi_c)^{\alpha-1}$) and the reorganisation of the flow paths that is modeled by the tortuosity change rate $-\partial\tau/\partial\phi$ controlled by β ($\partial\tau/\partial\phi = -\beta B_\tau(\phi - \phi_c)^{\beta-1}$), with $n = \alpha + \beta$. The results displayed in [Figure 2](#)

show that for experiment D1, where heterogeneous dissolution is observed during all the experiments, the changes in permeability are due to both changes in the hydraulic radius and tortuosity, while for experiment D3, where dissolution is homogeneous, the increase of permeability is only due to the decrease of the tortuosity of the porous media. However, for experiment D2, $\alpha > \beta$, and thus we conclude that the change in the effective hydraulic radius during the dissolution is the dominant process for D2. For this experiment, the $k(\phi)$ relationship is controlled by the radial dissolution of the wormhole-like porosity clusters that occupy only a portion of the sample and resulting in high permeability channels, which are induced by minor changes in porosity.

Conclusions

Here, we investigate the relationships between the reactive surface area (σ), the permeability (k) and the porosity (ϕ) in the case of dissolution, using the experimental data published by Luquot and Gouze (2009). First, we derive a heuristic model for calculating the sample-scale averaged rate of change of the reactive surface as a function of the porosity:

$$\partial \bar{\sigma}_r(t) / \partial \bar{\phi}(t) = -\beta_\sigma w \phi^{-w-1}. \quad (6)$$

Second, we investigated what parameters control the distinctly different permeability-porosity relationships characterizing different dissolution regimes:

$$\partial \bar{k}(t) / \partial \bar{\phi}(t) = k_0 n (\phi(t) - \phi_c)^{n-1}, \quad (7)$$

where w and n (fitted on the experimental data) characterize the dissolution regime and can be related to the Damköhler number, while β_σ and k_0 characterize the rock under consideration. We used 3D high-resolution XMT data, which were acquired before and after the dissolution experiments, and new processing protocols to compute the reactive surface area and the parameters that control permeability. Furthermore, XMT data can be used to determine the relative importance of tortuosity and pore enlargement that rules the $k - \phi$ law for the different regimes of dissolution. Results emphasize that the intermediate dissolution regime (moderate positive value of Da where multiple thin unramified wormholes develop, gives the higher value of n (i.e. the optimal regime to increase permeability with small changes in porosity) due to a large increase in the mean effective hydraulic radius in the sample. Conversely, it is shown that the increase of permeability is only due to the spatially homogeneous decrease of the tortuosity for homogeneous dissolution.

As a further remark, it is worth noticing that the approach presented here, not only explains the origin of the distinctly different values of the n exponent depending on the dissolution regime, but gives the foundations for relating this expression to those developed by several authors on the basis of the pioneering works of Kozeny and Carman, who derived the functional equation $k = \phi / 2 \sigma^2 \tau$ assuming that the porous structures behave as a bundles of cylindrical smoothly tortuous capillaries such as clean high porosity sandstones. Actually, defining $\alpha = iw + j$ and introducing equation 1 in equation 2 and integrating over ϕ gives $k = (k'_0 \phi^j) / (\tau \sigma^i)$, where $k'_0 = k_0 B_\sigma^i B_\tau$ is a rock-dependent scaling coefficient. For instance, a similar relation, called the PARIS-equation, $k = (k_0 \phi) / (\tau \sigma^m)$, was derived by Pape et al. (1982). In this model, which appears to be useful to model sandstones (Pape et al., 2000), the value of m ($2 < m < 3$) is controlled by the fractal dimension of the fluid-rock interface. This model is characterised by a strong control of the specific surface changes on the permeability changes.

Conversely, for low porosity carbonates it is not surprising that the porosity variation plays a more important role ($j > 1$) in ruling the $k(\phi)$ law.

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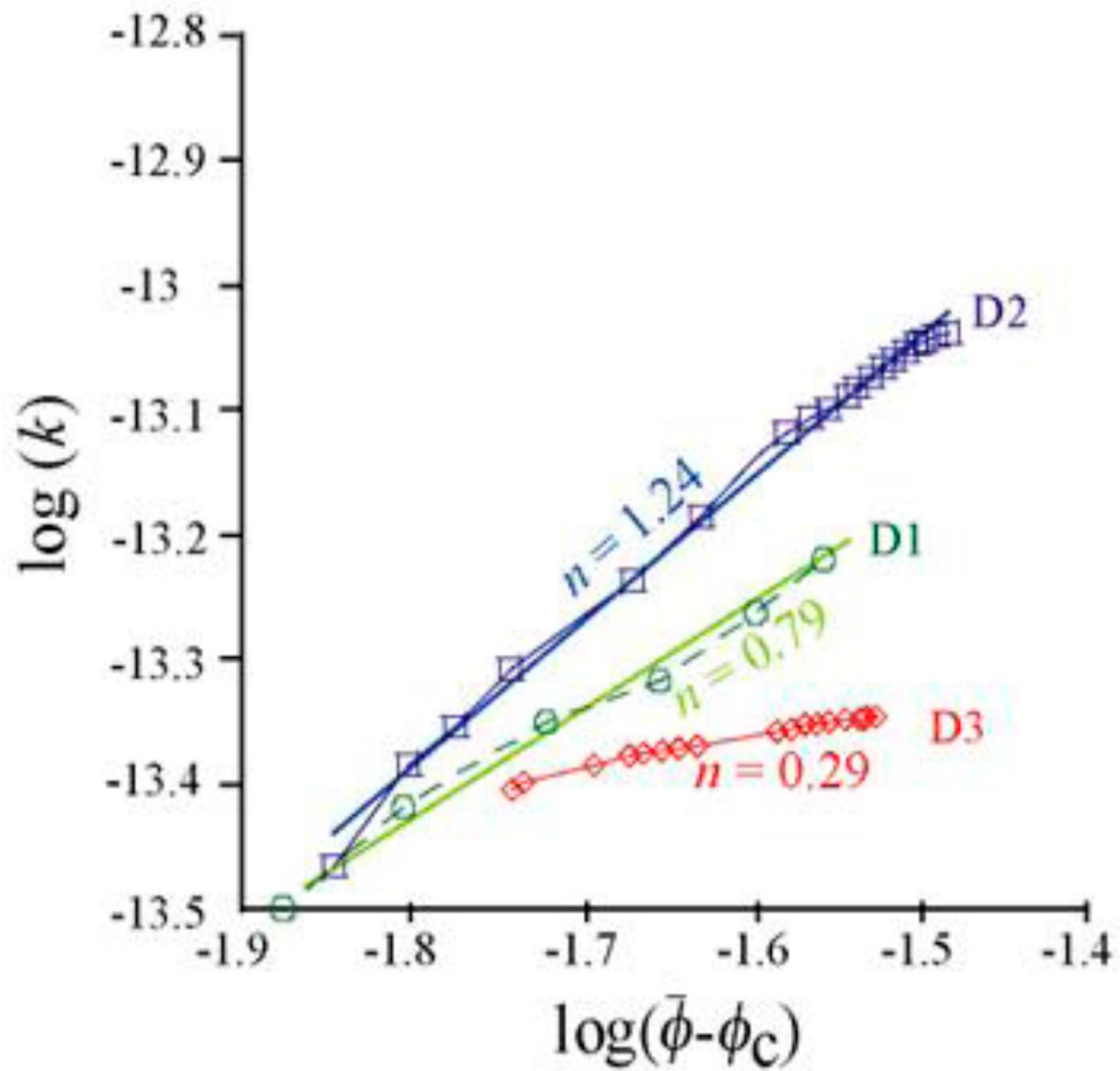


Figure 1. Sample-scale averaged porosity versus permeability for experiments D1, D2 and D3.

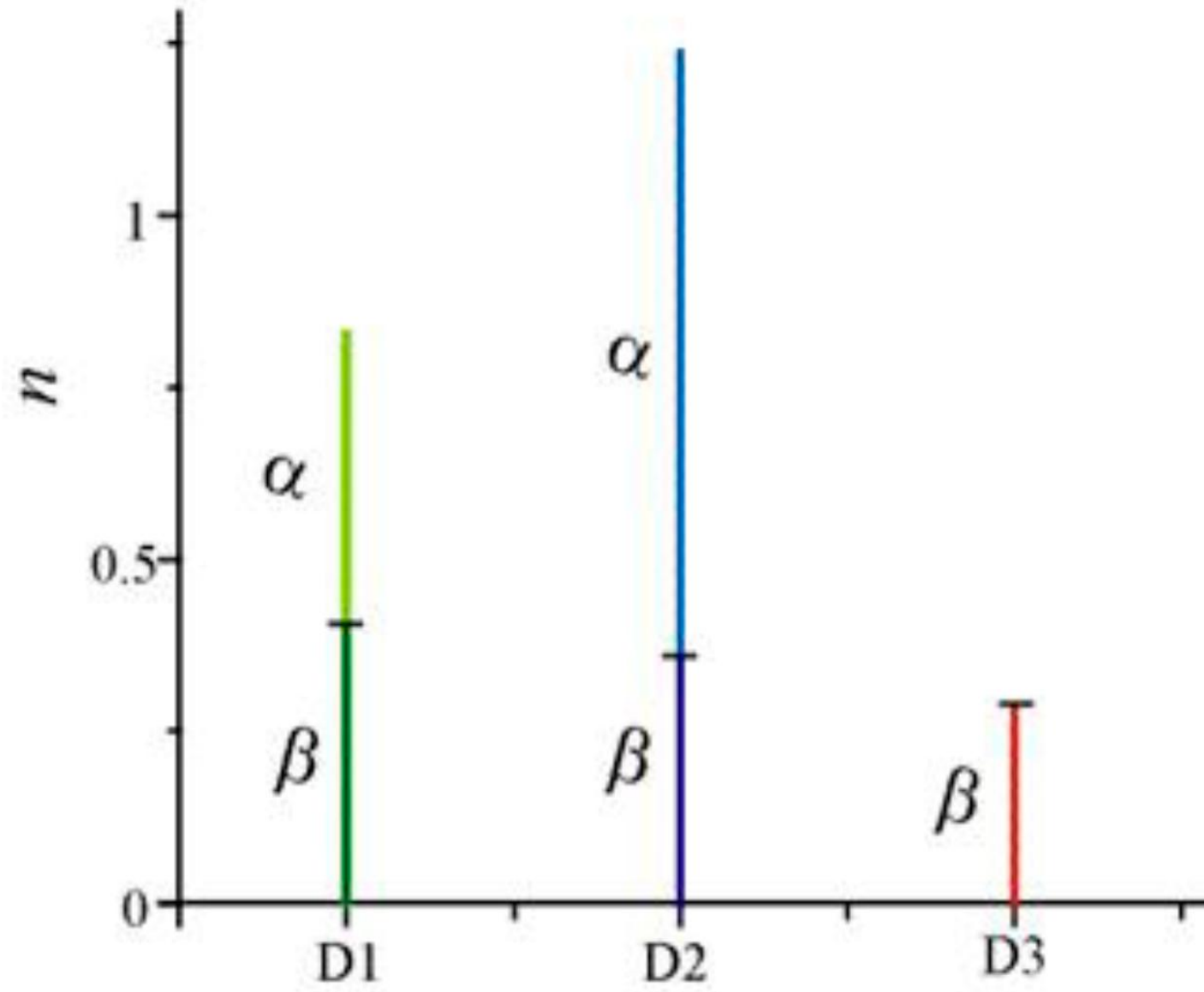


Figure 2. Values of exponent α (indicator of hydraulic radius increase) and β (indicator of tortuosity increase), with $n = \alpha + \beta$, for experiments D1, D2 and D3.