Reservoir Continuity Assessment with Mass Moments of Inertia*

Olena Babak[1] and Clayton V. Deutsch[2]

Search and Discovery Article #40659 (2011)
Posted January 11, 2011

*Adapted from oral presentation at AAPG International Conference and Exhibition, Calgary, Alberta, Canada, September 12-15, 2010

[1]University of Alberta, Edmonton, AB, Canada (olena.babak@total.com)
[2]University of Alberta, Edmonton, AB, Canada

Abstract

The size and continuity of non-net reservoir intervals is of great practical importance in petroleum engineering. Well placement and prediction of fluid flow responses are affected. The thickness of non-net intervals provides some indication of lateral extent and connectivity; however, there is significant uncertainty in the size of these intervals. There is a need for a geologically-sound and repeatable approach to quantify the relationship between the length and thickness of non-net reservoir intervals.

This paper develops one such approach based on a novel idea of incorporating mass moments of inertia in calculating the size of non-net intervals. The idea is as follows. The irregularly shaped non-net intervals are replaced by a non-net ellipsoid that has equivalent moment of inertia tensor. Then, calculation of the thickness and length of given non-net interval is done indirectly but in straightforward manner: the thickness of a non-net interval is calculated as a vertical radius of the ellipsoid and the length of the non-net interval is calculated as the average of the two ellipsoid radii in the horizontal plane.

The proposed methodology is illustrated with several examples. All examples are based on Sequential Indicator Simulation (SIS) of binary codes, that is, net (0) and shale (1), with different proportions of net/non-net facies and different indicator variogram models of non-net.

Results are visually compared and summarized as probability curves for length as a function of thickness. These curves can be used to predict the length or give a probability interval for the length of non-net intervals for a given observed thickness. This information is important for the performance assessment of SAGD.

Reference

Hassanpour, R.M., O. Leuangthong, and C.V. Deutsch, 2008, Calculation of Permeability Tensors for Unstructured Grid Blocks: SPE 2008-187, 13 p.

RESERVOIR CONTINUITY ASSESSMENT WITH MASS MOMENTS OF INERTIA

Olena Babak and Clayton V. Deutsch

TOTAL

OUTLINE

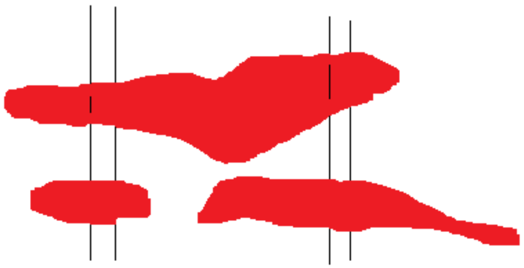
- Introduction: Analysis of Size and Connectivity of Non-Net Facies Intervals
- Moment of Inertia
- Calculation of Thickness and Length/Width of Non-Net Intervals
- Examples
- Results
- Conclusions

TOTAL

INTRODUCTION: ANALYSIS OF SIZE AND CONNECTIVITY OF NON-NET FACIES INTERVALS

- The question of non-net size and connectivity determination is of great practical importance in petroleum engineering.
  - Can be used for optimal well placement and prediction of fluid flow responses in latter stages of reservoir modeling.

- There is a great degree of uncertainty in the size of non-net intervals
  - The data from wells along with an indicator variogram of non-net facies provides only a hint on their extent and connectivity.



- There is a need for a sound replicable method able to quantify the relationship between the length and thickness of non-net facies and able to predict the length of non-net interval based on its thickness if needed.

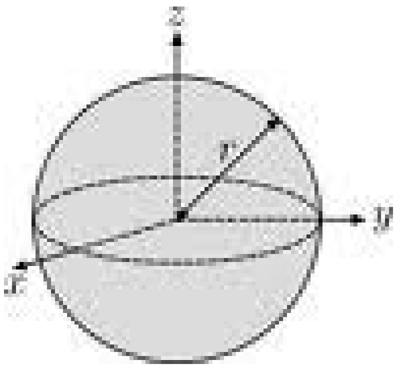
TOTAL

OUTLINE

- Introduction: Analysis of Size and Connectivity of Non-Net Facies Intervals
- Moment of Inertia
- Calculation of Thickness and Length/Width of Non-Net Intervals
- Examples
- Results
- Conclusions

MOMENT OF INERTIA

- Moment of inertia, a.k.a mass moment of inertia or the angular mass of a body, is the rotational analog of mass. It is the inertia of a rigid rotating body with respect to its rotation:



http://www.google.ca/imgres?imgurl=http://upload.wikimedia.org/wikipedia/commons/thumb/1/19/Moment_of_inertia_solid_sphere.svg/170px-Moment_of_inertia_solid_sphere.svg.png&imgrefurl=http://en.wikipedia.org/wiki/List_of_moments_of_inertia&usg=__GWoAa5CDJMEuCjzVzMON5QAr5dw=&h=156&w=170&sz=10&hl=en&start=34&itbs=1&tbnid=_4cxJEy9GEmbVM:&tbnh=91&tbnw=99&prev=/images%3Fq%3Dmoment%2Bof%2Binertia%2Bellipsoid%26start%3D20%26hl%3Den%26sa%3DN%26gbv%3D2%26ndsp%3D20%26tbs%3Disch:1

$$I = \int r^2 dm$$

where *m* is the mass and *r* is the perpendicular distance of the point mass to the axis of rotation.

- Moment of inertia has two forms:
  - scalar form which is used when the axis of rotation is known; and
  - the tensor form which summarizes all moment of inertia for different axes of rotation with one quantity.

TOTAL

MOMENT OF INERTIA TENSOR

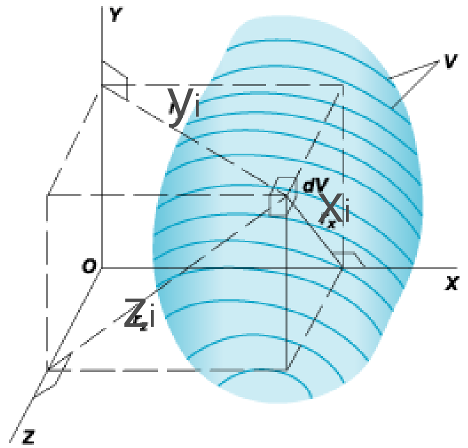
For a rigid body consisting of N point masses m_i the moment of inertia tensor are defined as follows:

$$\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

$$I_{xx} = \sum_{i=1}^{N} m_i \left(y_i^2 + z_i^2 \right) \quad , \quad I_{xy} = I_{yx} = -\sum_{i=1}^{N} m_i x_i y_i$$

$$I_{yy} = \sum_{i=1}^{N} m_i \left(x_i^2 + z_i^2 \right) \quad , \quad I_{xz} = I_{zx} = -\sum_{i=1}^{N} m_i x_i z_i$$

$$I_{zz} = \sum_{i=1}^{N} m_i \left(x_i^2 + y_i^2 \right) \quad , \quad I_{yz} = I_{yz} = -\sum_{i=1}^{N} m_i y_i z_i$$



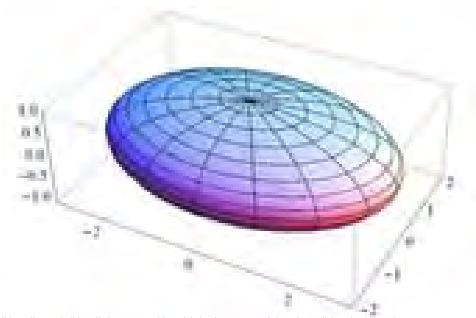
http://www.google.ca/imgres?imgurl=http://content.answers.com/main/content/img/McGrawHill/Encyclopedia/images/CE432000FG0010.gif&imgrefurl=http://www.answers.com/topic/moment-of-inertia&usg=__E1Xoh0k1fOIMKdmCi5vrtm1copk=&h=251&w=254&sz=12&hl=en&start=9&itbs=1&tbnid=X6g_PFWnDQQ5NM:&tbnh=110&tbnw=111&prev=/images%3Fq%3Dmoment%2Bof%2Binertia%2Bellipsoid%26hl%3Den%26gbv%3D2%26tbs%3Disch:1

where x_i, y_i and z_i are the distances of point i from the coordinate axes. Here the physical meaning of I_{xx} is the moment of inertia around the x-axis when the objects are rotated around the x-axis and I_{xy} is the moment of inertia around the y-axis when the objects are rotated around the x-axis.

* Hassanpour and Deutsch, 2008.

MOMENT OF INERTIA TENSOR FOR AN ELLIPSOIDE

- When a rigid body is an ellipsoid, then its moments of inertia tensor is diagonal.



http://www.google.ca/imgres?imgurl=http://upload.wikimedia.org/wikipedia/commons/thumb/1/19/Moment_of_inertia_solid_sphere.svg/170px-Moment_of_inertia_solid_sphere.svg.png&imgrefurl=http://en.wikipedia.org/wiki/List_of_moments_of_inertia&usg=__GWoAa5CDJMEuCjzVzMON5QAr5dw=&h=156&w=170&sz=10&hl=en&start=34&itbs=1&tbnid=_4cxJEy9GEmbVM:&tbnh=91&tbnw=99&prev=/images%3Fq%3Dmoment%2Bof%2Binertia%2Bellipsoid%26start%3D20%26hl%3Den%26sa%3DN%26gbv%3D2%26ndsp%3D20%26tbs%3Disch:1

- The moment of inertia of an ellipsoid around its major, medium and minor radius is calculated as:

$$I_a = \frac{1}{5} M \left(r_b^2 + r_c^2 \right)$$

$$I_b = \frac{1}{5} M \left(r_a^2 + r_c^2 \right)$$

$$I_c = \frac{1}{5} M \left(r_a^2 + r_b^2 \right)$$

where I_a, I_b, I_c are moment of inertia around major r_a, medium r_b, and minor r_c radius, respectively and M is the ellipsoid mass.

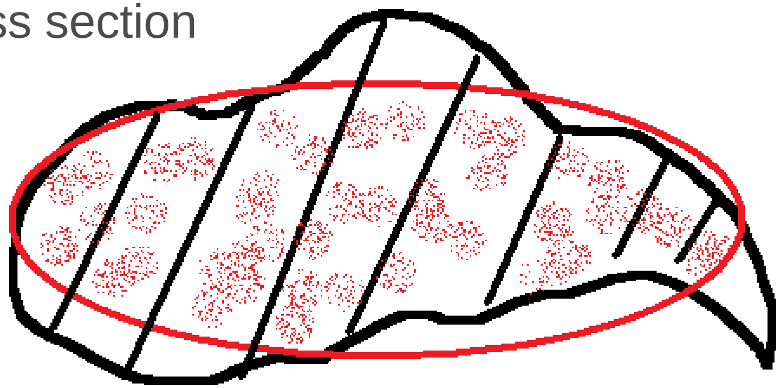
OUTLINE

- Introduction: Analysis of Size and Connectivity of Non-Net Facies Intervals
- Moment of Inertia
- Calculation of Thickness and Length/Width of Non-Net Intervals
- Examples
- Results
- Conclusions

CALCULATION OF THICKNESS AND LENGTH(/WIDTH) OF NON-NET INTERVALS

- The approach for calculating non-net size using the mass moments of inertia:
  - Each non-net interval within a rock-type model can have an arbitrary asymmetric shape and is characterized by its own moment of inertia tensor.
    - Hard to parametrize and calculate length and thickness
  - We replace the asymmetrically shaped non-net interval with a non-net ellipsoid which has ‘equivalent’ moment of inertia tensor.
    - That is, an ellipsoid such that its moment of inertia tensor has eigenvalues of the original moment of inertia tensor on the diagonal.

Schematic cross section



TOTAL

CALCULATION OF THICKNESS AND LENGTH(/WIDTH) OF NON-NET INTERVALS

- Then calculation of the thickness and length of given non-net interval is done indirectly but in straightforward manner:
  - The thickness of a non-net interval is calculated as a vertical radius of the ellipsoid; and
  - The length (or width) of the non-net interval is calculated as the average of the two ellipsoid radii in the horizontal plain.

- Note that the moment of inertia approach works well only if enough data are available. In the case of small non-net interval, additional partitioning of non-net object may be required.

OUTLINE

- Introduction: Analysis of Size and Connectivity of Non-Net Facies Intervals
- Moment of Inertia
- Calculation of Thickness and Length/Width of Non-Net Intervals
- Examples
- Results
- Conclusions

EXAMPLES

- Several examples based on unconditional Sequential Indicator Simulation (SIS) of binary codes (net (0) and non-net (1)) are considered:

$$1)\quad p_{net}=0.8;\quad p_{non-net}=0.2;\quad \gamma(h)=0.2+0.8Sph_{\substack{a_{areal}=1000\\a_{vertical}=40}}(h);$$

$$2)\quad p_{net}=0.8;\quad p_{non-net}=0.2;\quad \gamma(h)=0.2+0.8Sph_{\substack{a_{areal}=400\\a_{vertical}=10}}(h);$$

$$3)\quad p_{net}=0.8;\quad p_{non-net}=0.2;\quad \gamma(h)=0.2+0.8Sph_{\substack{a_{areal}=200\\a_{vertical}=5}}(h);$$

$$4)\quad p_{net}=0.5;\quad p_{non-net}=0.5;\quad \gamma(h)=0.2+0.8Sph_{\substack{a_{areal}=1000\\a_{vertical}=40}}(h);$$

$$5)\quad p_{net}=0.5;\quad p_{non-net}=0.5;\quad \gamma(h)=0.2+0.8Sph_{\substack{a_{areal}=400\\a_{vertical}=10}}(h);$$

$$6)\quad p_{net}=0.5;\quad p_{non-net}=0.5;\quad \gamma(h)=0.2+0.8Sph_{\substack{a_{areal}=200\\a_{vertical}=5}}(h);$$

- In each simulation study a volume of 100 by 100 by 100 cells of size 10 by 10 by 1 cubic distance units is populated. Reproduction of the target proportions is carefully checked.

DETAILED ANALYSIS OF NON-NET FACIES INTERVALS

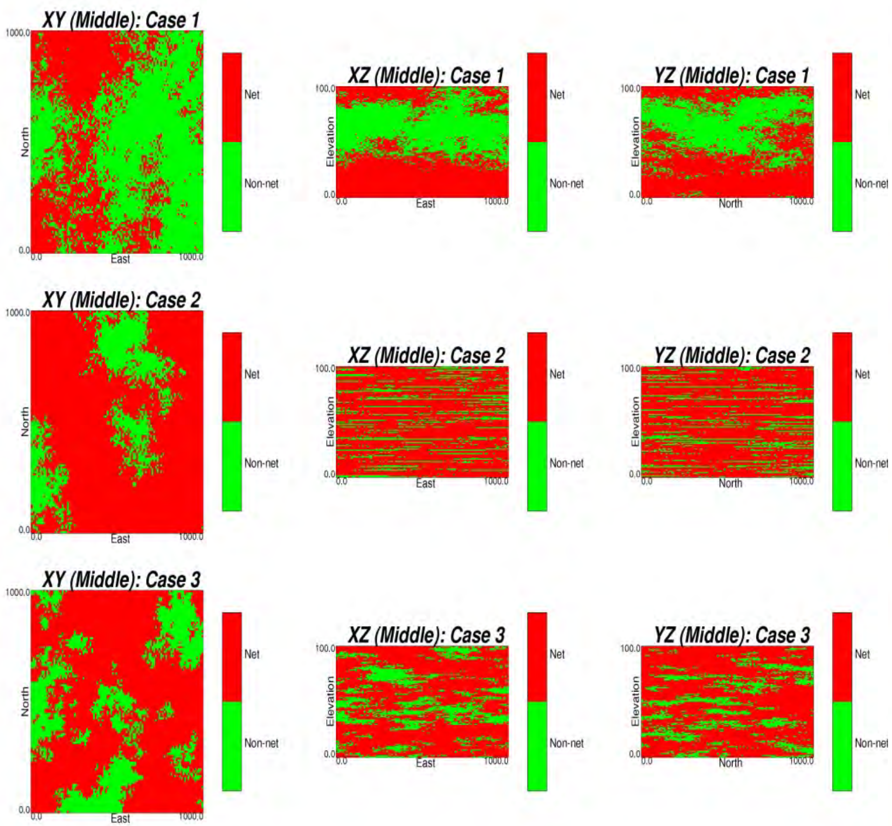
- To analyze extent and thickness of non-net thickness intervals, the following approach is used:
  - 1. Models are build using SIS.
  - 2. To avoid pixelation inherent to SIS and make clouds of non-net more continuous, a light image cleaning is done.
  - 3. Then, all non-net geo-objects are found. Connected regions are defined by face-connected blocks. Each connected region of non-net is defined as a separate non-net geo-object.
  - 4. Thickness and length(/width) of non-net geo-objects is calculated using moments of inertia approach.
  - 5. Results are assembled as graphs of length(/width) vs. thickness of non-net and histogram of the ratio of length(/width) to thickness of non-net.

OUTLINE

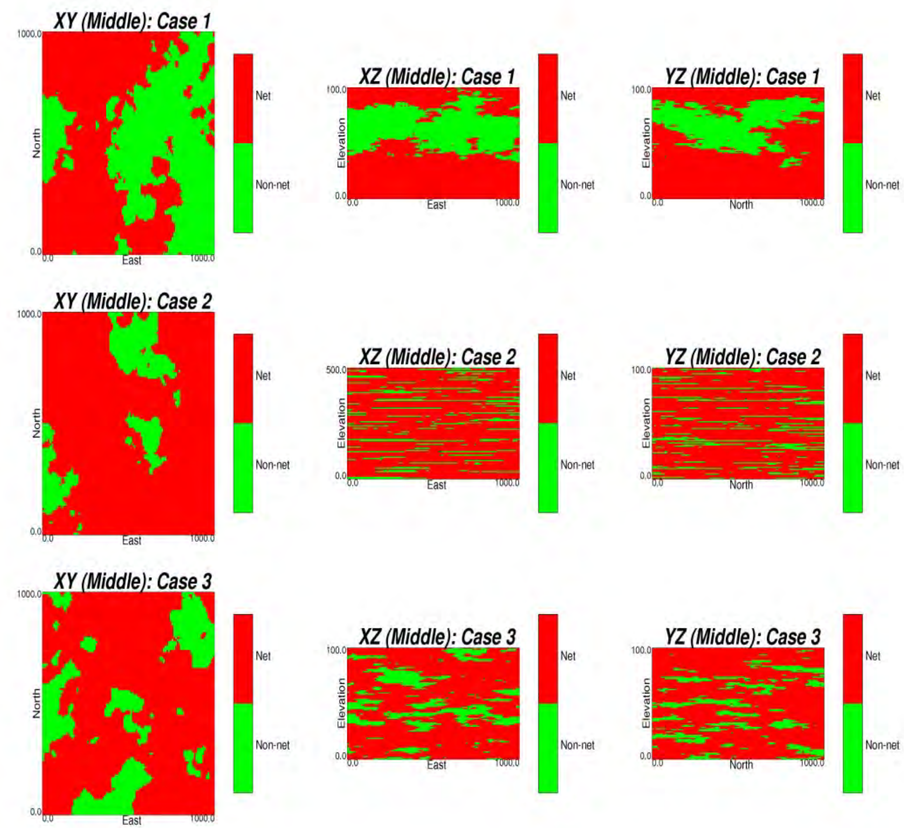
- Introduction: Analysis of Size and Connectivity of Non-Net Facies Intervals
- Moment of Inertia
- Calculation of Thickness and Length/Width of Non-Net Intervals
- Examples
- Results
- Conclusions

RESULTS FOR CASES 1-3

SIS Realization

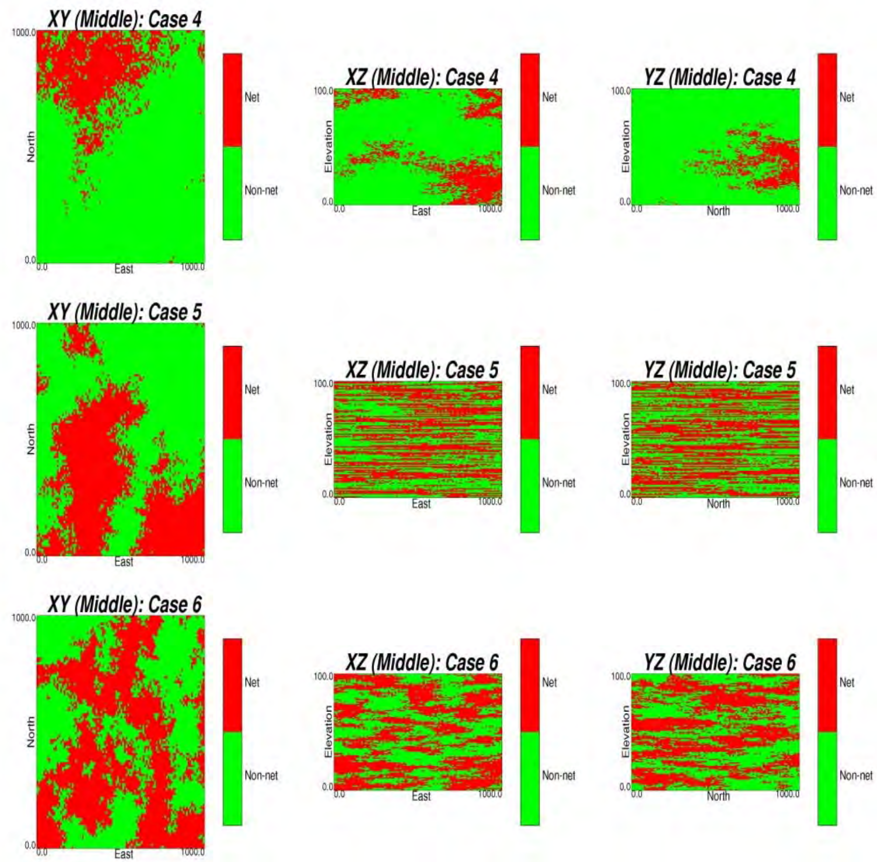


Cleaned SIS Realization

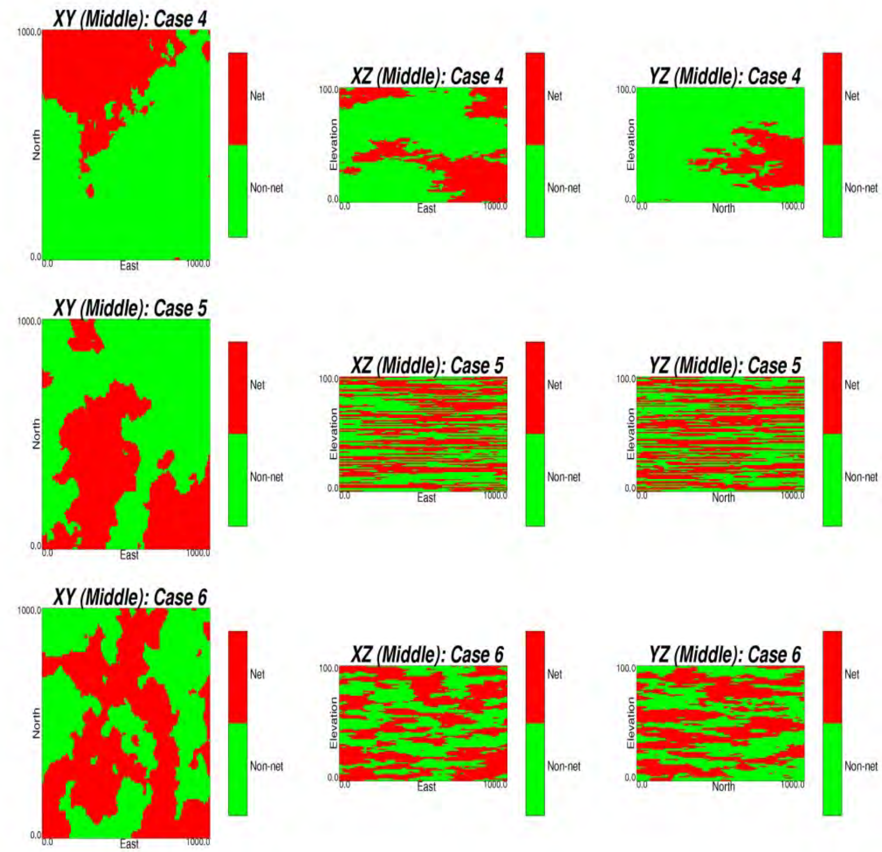


RESULTS FOR CASES 4-6

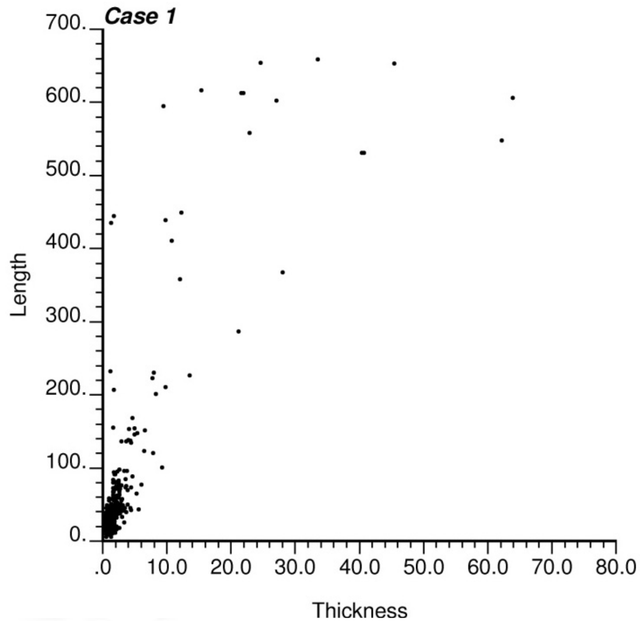
SIS Realization

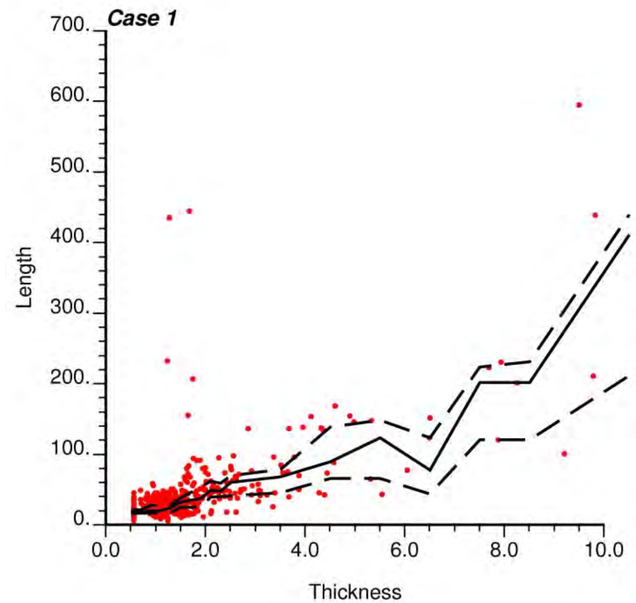


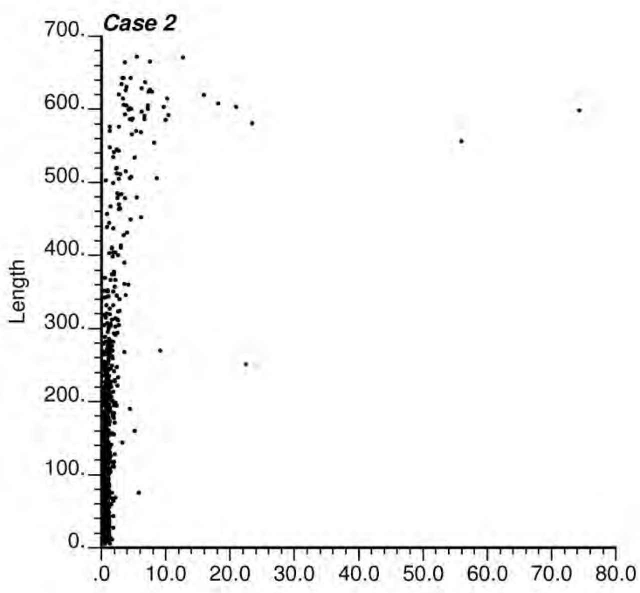
Cleaned SIS Realization

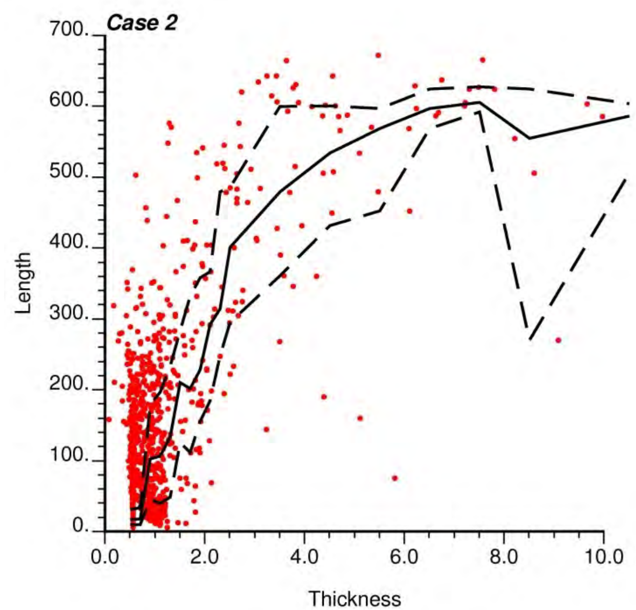


RESULTS FOR CASES 1-2

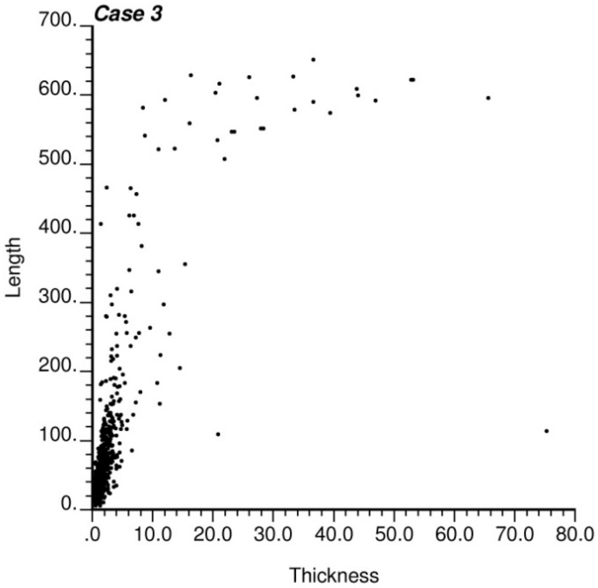


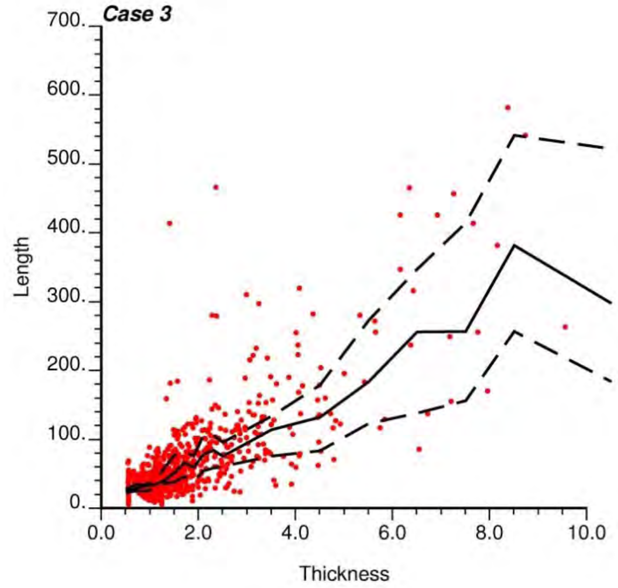


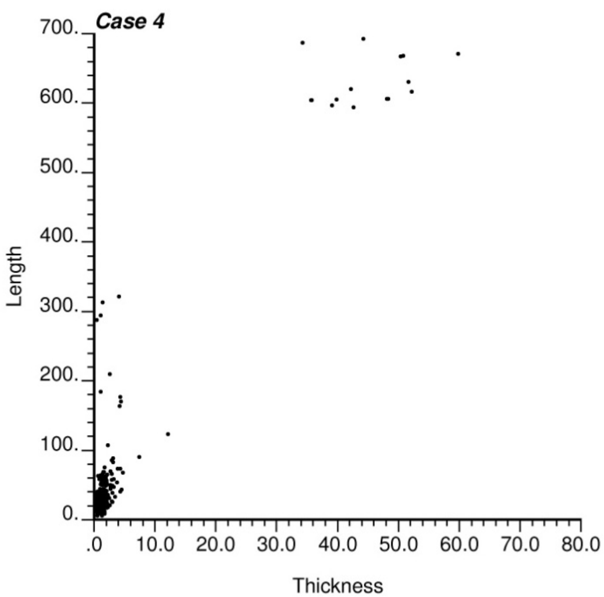


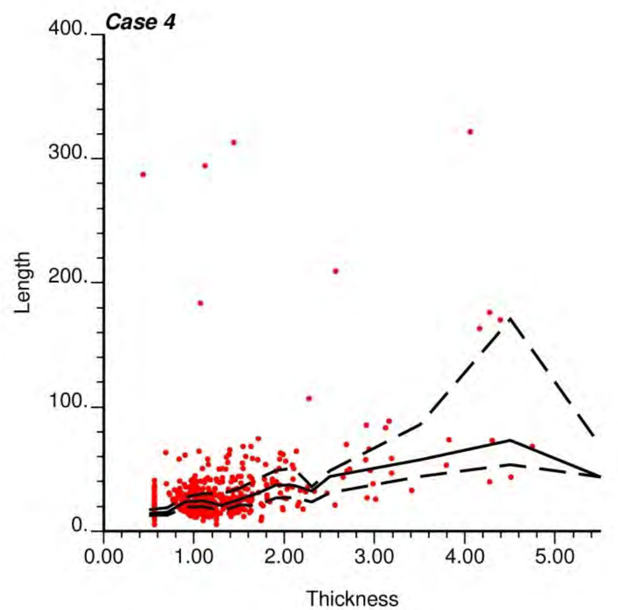


RESULTS FOR CASES 3-4



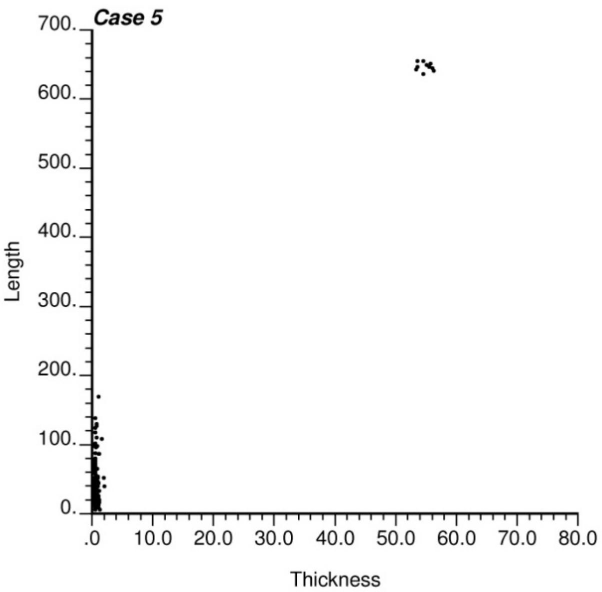


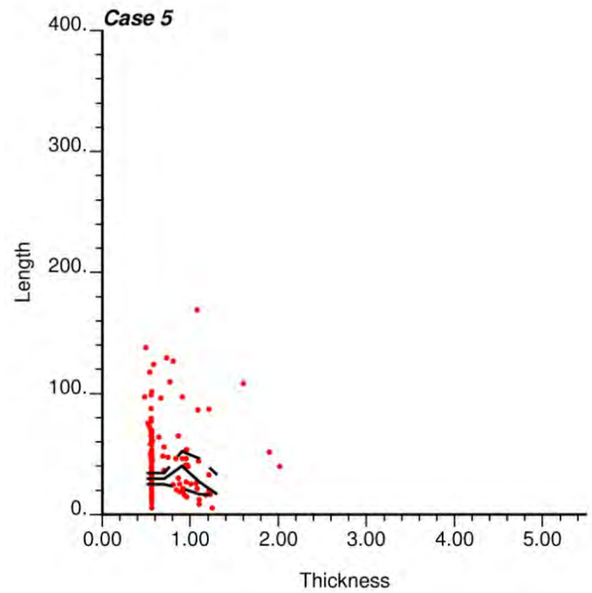


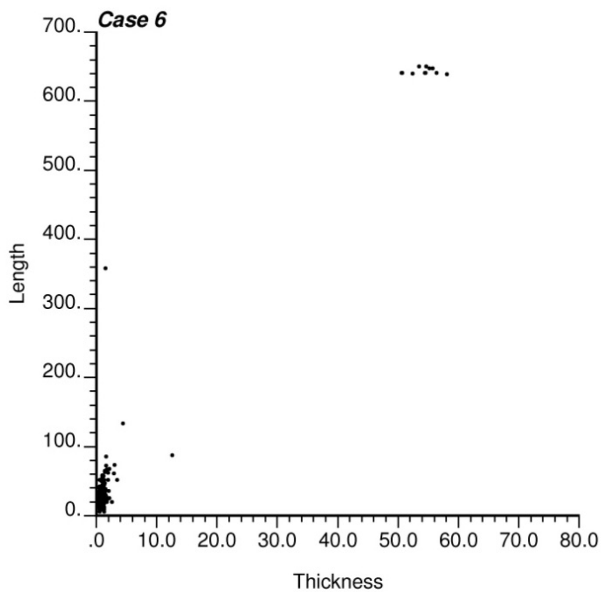


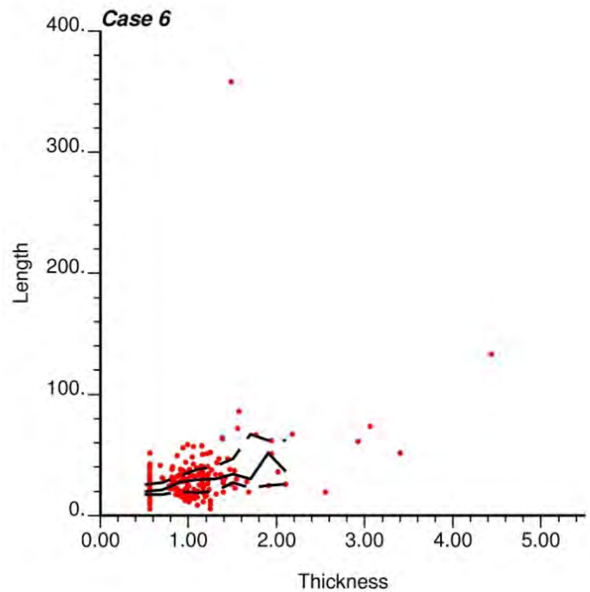
TOTAL

RESULTS FOR CASES 5-6

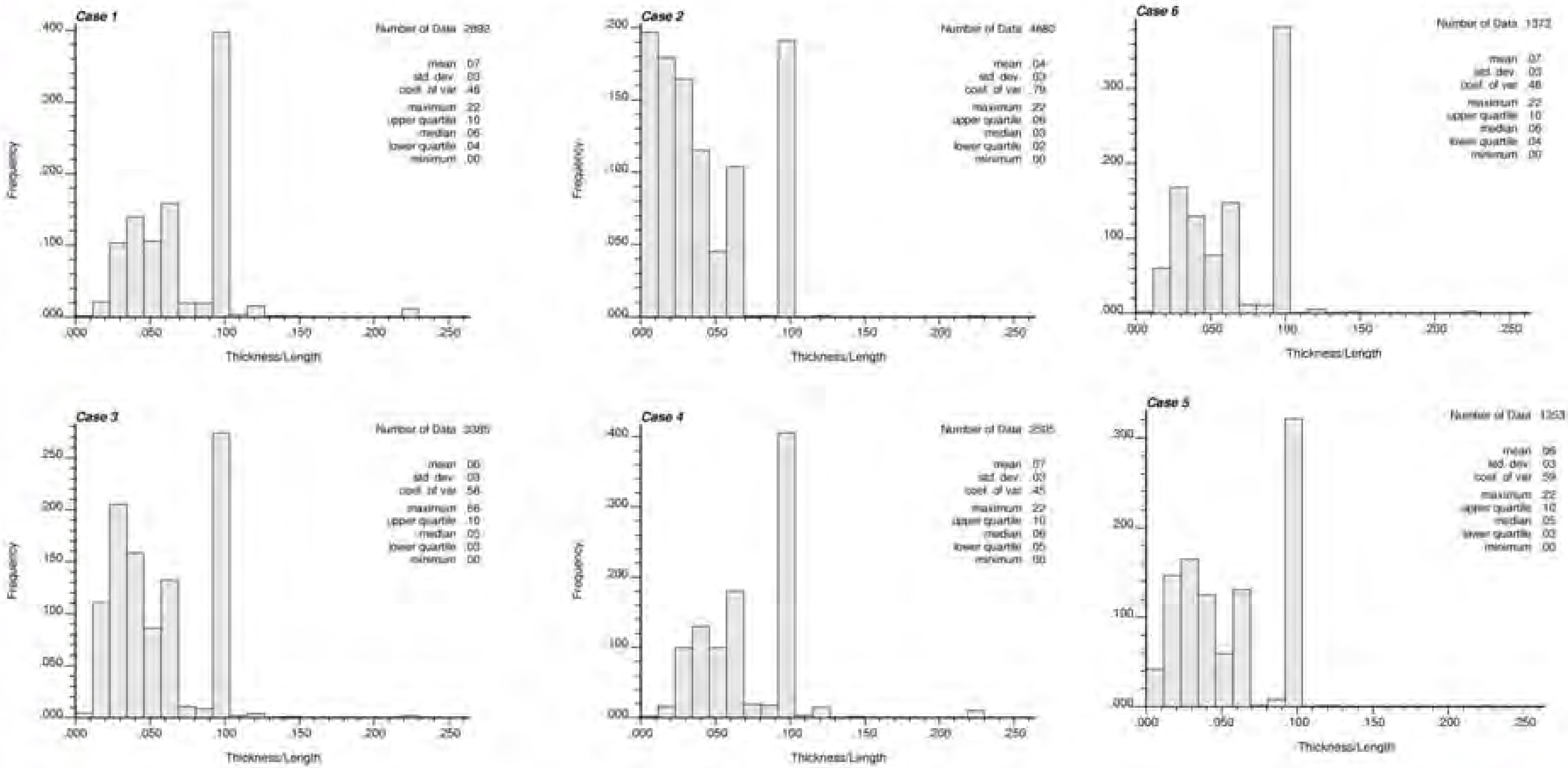








HISTOGRAM OF THE RATIO OF THICKNESS TO LENGTH OF NON-NET INTERVALS OBTAINED IN EACH OF THE SIX SIMULATION STUDIES



OUTLINE

- Introduction: Analysis of Size and Connectivity of Non-Net Facies Intervals
- Moment of Inertia
- Calculation of Thickness and Length/Width of Non-Net Intervals
- Examples
- Results
- Conclusions

TOTAL

CONCLUSIONS

- A new approach for calculating the size and connectivity of non-net intervals was proposed.

- The developed approach uses the mass moments of inertia calculation to predict the length of non-net interval based on it's thickness and to quantify the relationship between the length and thickness of non-net facies intervals.

- The proposed approach was illustrated using six different simulation studies. Each study is characterized by different proportion and connectivity of non-net facies.