Application of Staged Differential Effective Medium Models for the Prediction of Velocities*

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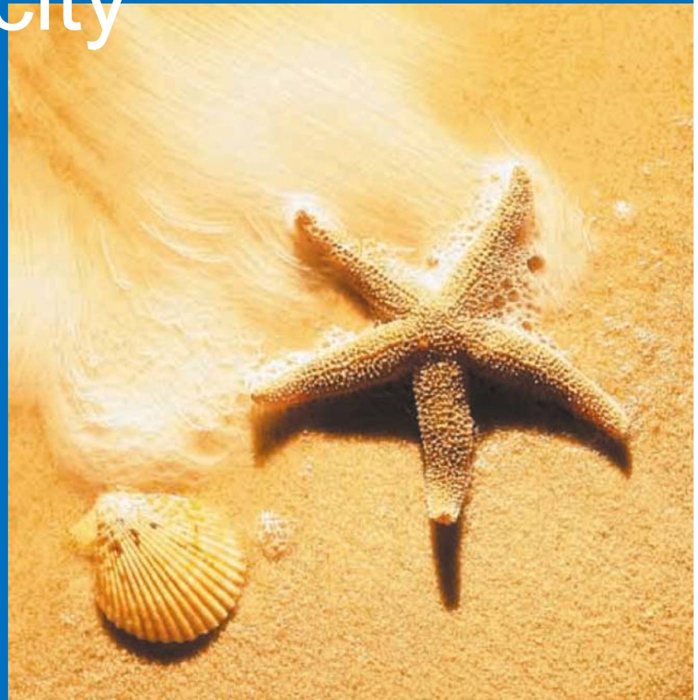
Abstract

A new staged differential effective medium model for acoustic velocities, which was recently published, allows the mineralogy and texture of rocks to be modeled. The model is based on interpolating between series and parallel additions of inclusions with an effective medium approximation for the host. It was shown to be consistent with critical porosity models and Gassman's equation. The model represents an extension of these ideas in that it allows variations in mineralogy and texture to be included. The development of this model is briefly reviewed here. What remains to be demonstrated is how the parameters in the model are derived. Several examples are shown for specific lithologies. The model is first applied to a suite of artificial sands. Sand packs were made so that individual parameters could be changed and their influence on acoustic properties measured. Key parameters are derived for variations in packing, sorting, grain size and shape, and framework mineralogy. Suites of samples were measured for increasing ductile grain and feldspar content, for sands having mean grain size of 150 and 300 microns. As feldspar content increases from 3-12 weight percent, the velocity of the sand pack increases with very minimal changes in total porosity. As feldspar content increases above 12%, continuing to add feldspar causes the velocity to decrease. Thin-sections cut from 1" diameter plugs of the artificial sands were analyzed using Shell's proprietary image analysis tool. The pattern of changes in measured acoustic properties was tied to changes in contact length and the mineralogy of the contact pairs. Adding small volumes of feldspar causes the overall contact index of the sands to increase because of the lathlike shape of the feldspar grains, although quartz:quartz contacts dominate the elastic properties. Above about 12 weight % feldspar, feldspar:feldspar, and feldspar:quartz contacts begin to dominate the grain pack and exert the primary influence on the acoustic properties. The effects of changing other properties of the sand pack (e.g., sorting) and the influence of more complex mineralogic mixtures on acoustic properties are also discussed. Finally application will be made to real samples both for clastics and carbonates. The dominant mechanisms for carbonates are the texture of the intergranular porosity and the inclusion of vuggy porosity. The field examples of the clastics follow the mechanisms discussed above for sand packs.

Application of Staged Differential Effective Medium Models for the Prediction of Velocity



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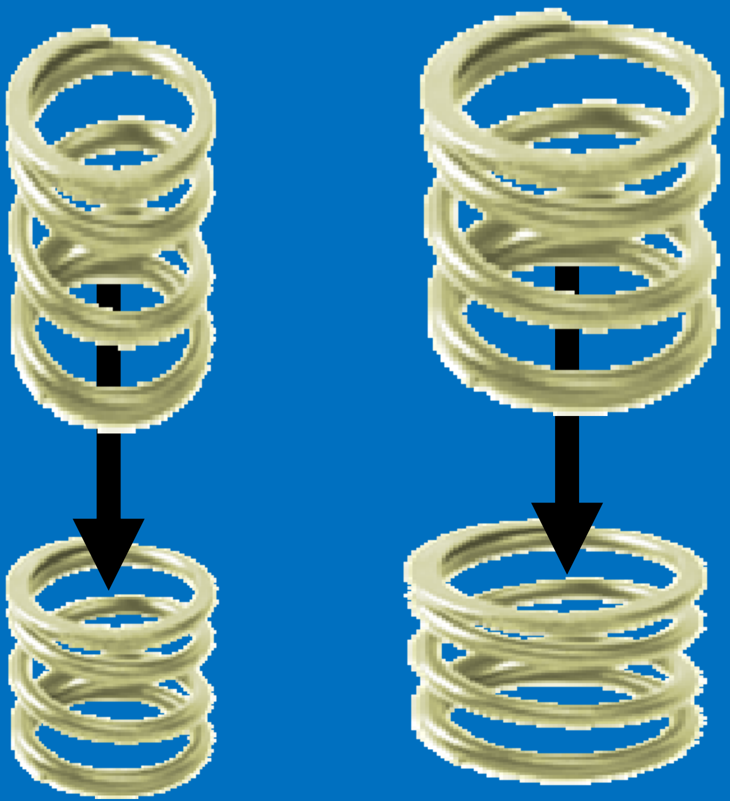
Jeff Scheibal, Hans De Jonghe

Iso-Stress and Iso-Strain Models
A Place to Start

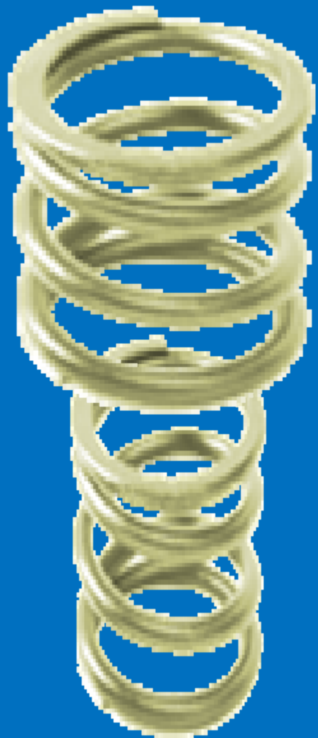
- Voight Average



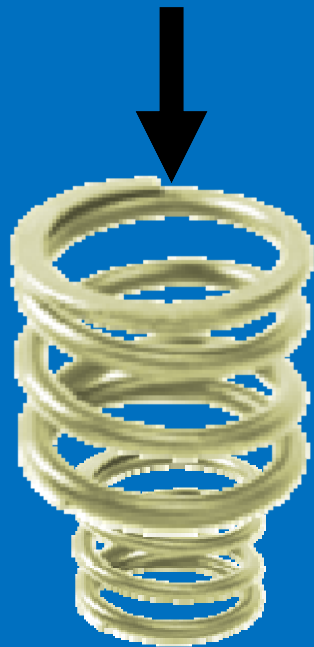
- Iso-strain Average: Gives the the ratio of the average stress to average strain when all the components are assumed to have the same strain. The stiffer spring dominates

Iso-Stress and Iso-Strain Models
A Place to Start

- Reuss Average



- Iso-stress Average: Gives the ratio of the average stress to average strain when all the components are assumed to have the same stress. The weaker spring dominates



Iso-stress and Iso-strain Models
A Place to Start

$$M_{Voight} = f_1 \cdot M_1 + \left(1 - f_1\right) \cdot M_2$$

$$M_{Reuss} = \left(\frac{f_1}{M_1} + \frac{1 - f_1}{M_2}\right)^{-1}$$

f_1 is the volume fraction of material one, M_1 and M_2 are the individual moduli.

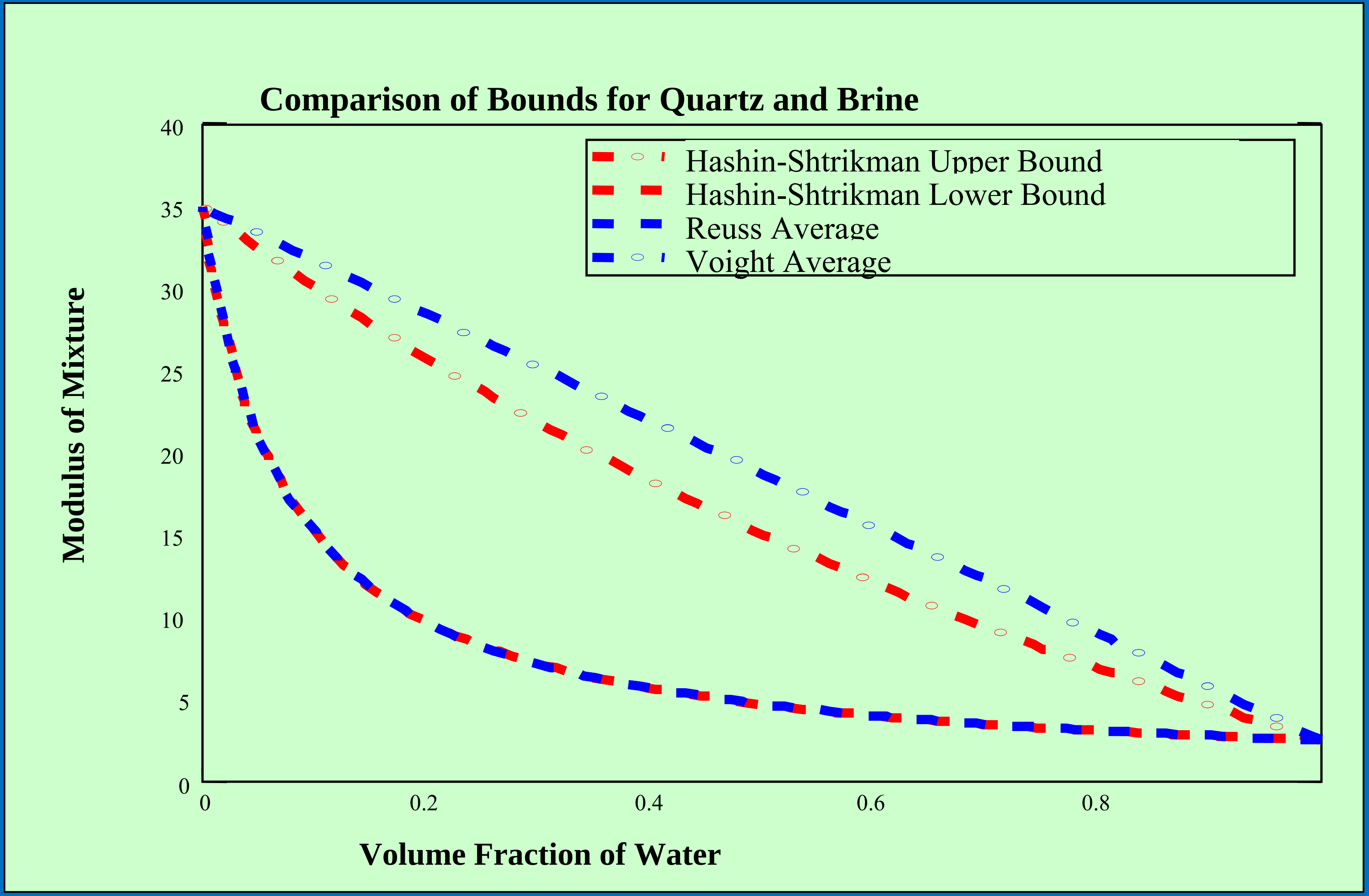
For 3D Models These are Bounds on the Moduli

- **Voight average represents an upper bound**
- **Reuss average represents a lower bound**
- **Even narrower bounds may be set: For an isotropic material the Hashin-Shtrikman bounds are**

$$K^{HS\pm} = K_1 + \frac{f_2}{(K_2 - K_1)^{-1} + f_1\left(K_1 + \frac{4}{3}\cdot\mu_1\right)^{-1}} \qquad \mu^{HS\pm} = \mu_1 + \frac{f_2}{(\mu_2 - \mu_1)^{-1} + \frac{2\cdot f_1}{5\cdot\mu_1}\left(\frac{K_1 + 2\cdot\mu_1}{K_1 + \frac{4}{3}\cdot\mu_1}\right)}$$

K_1 and K_2 are the bulk moduli of material one and two; μ_1 and μ_2 are the shear moduli. Upper and lower bounds are found by interchanging which material is termed 1 and 2

The Bounds May be Broad for Mixtures of Materials with Widely Different Properties



How About a Model That Allows a Continuous Interpolation?

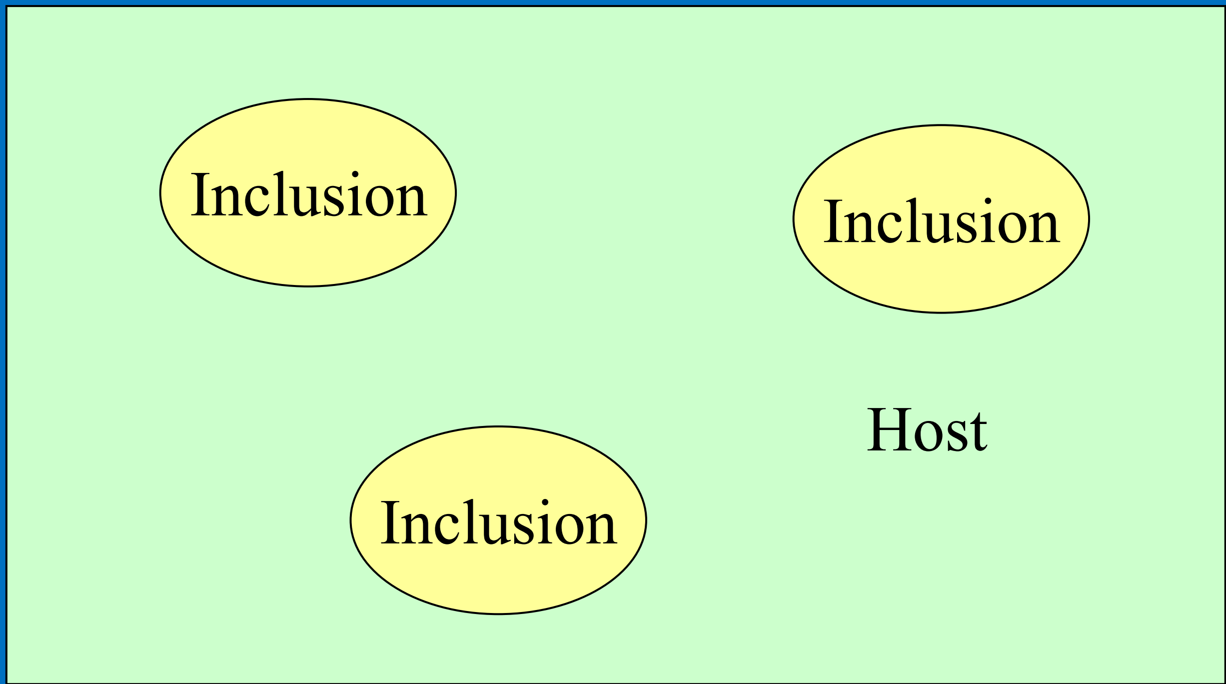
Use the following mixing law:

$$K = K_V \cdot L + K_R \cdot (1 - L)$$

Where L is a parameter which continuously varies between zero and one: Between the iso-strain and the iso-stress case.

L=0 is the Iso-Stress Average **L=1 is the Iso-Strain Average**

Develop a Differential Effective Medium Model From This Mixing Law



Form a dilute mixture assuming the inclusions only see the average properties of the host and neglecting interaction terms.

Form a Differential Effective Medium Model From This Mixing Law

- Formulate a difference equation for the change in properties due to the addition of small volumes of inclusions. Use this "effective medium" as the host. (M is the modulus of the mixture, M_h the modulus of the host, M_i and f_i are the modulus and volume fraction of the inclusion respectively)

$$\frac{M - M_h}{M_h} = f_i \cdot \left[\left(\frac{M_i - M_h}{M_h} \right) L - (1 - L) \left(\frac{M_h - M_i}{M_i} \right) \right]$$

- Use this equation to formulate a differential equation. (Φ is the porosity)

$$\frac{dM}{\left(\frac{M_i - M}{M_i} \right) \left[LM_i + (1 - L) \cdot M \right]} = \frac{-d\Phi}{\Phi}$$

Compare to the Hanai-Bruggeman Equation

- Hanai-Bruggeman

$$\frac{\Phi}{\Phi o} = \left(\frac{M_i - M}{M_i - M_h}\right)\left(\frac{M_h}{M}\right)^L$$

- DEM modulus model

$$\frac{\Phi}{\Phi o} = \left(\frac{M_i - M}{M_i - M_h}\right)\left[\frac{(1 - L)M_h + L \cdot M_i}{(1 - L)M + L \cdot M_i}\right]$$

Where Φ is the final porosity, Φ_o is the initial porosity, M_i is the Modulus of the inclusion, M_h is the modulus of the host, M is the modulus of the mixture.

In the limiting cases, L=0 or L=1, they both simplify to parallel or series model

Iso-Stress and Iso-Strain Models are Preserved for the Combination of the Host and Impurity

- For L=0 (iso-stress)

$$\frac{1}{M_f} = \frac{1}{M_i} \cdot \left(\frac{\Phi_o - \Phi}{\Phi_o} \right) + \frac{1}{M_h} \cdot \frac{\Phi}{\Phi_o}$$

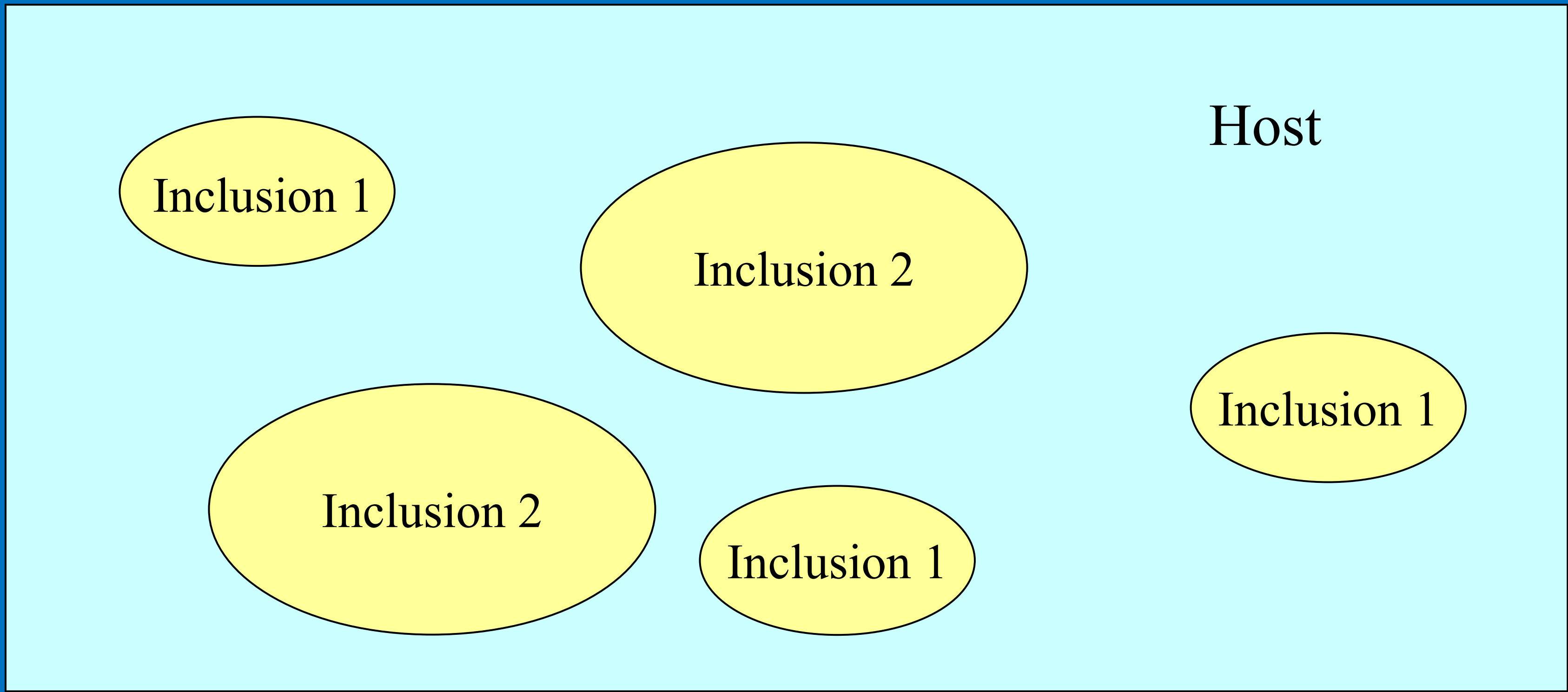
- For L=1 (iso-strain)

$$M_f = M_i \cdot \left(\frac{\Phi_o - \Phi}{\Phi_o} \right) + M_h \cdot \frac{\Phi}{\Phi_o}$$

DEM Modulus Model

- Preserves the Voight and Reuss averages at the limits L=0 and L=1.
- Allows continuous interpolation between iso-strain and iso-stress averages.
- Allow multiple lithology changes at separate integration steps. Ordering of the integration is dependent on length scales present in the sample (implicit in the derivation).

The Inclusions Are Imbedded in a Host With Average Properties of the Mixture

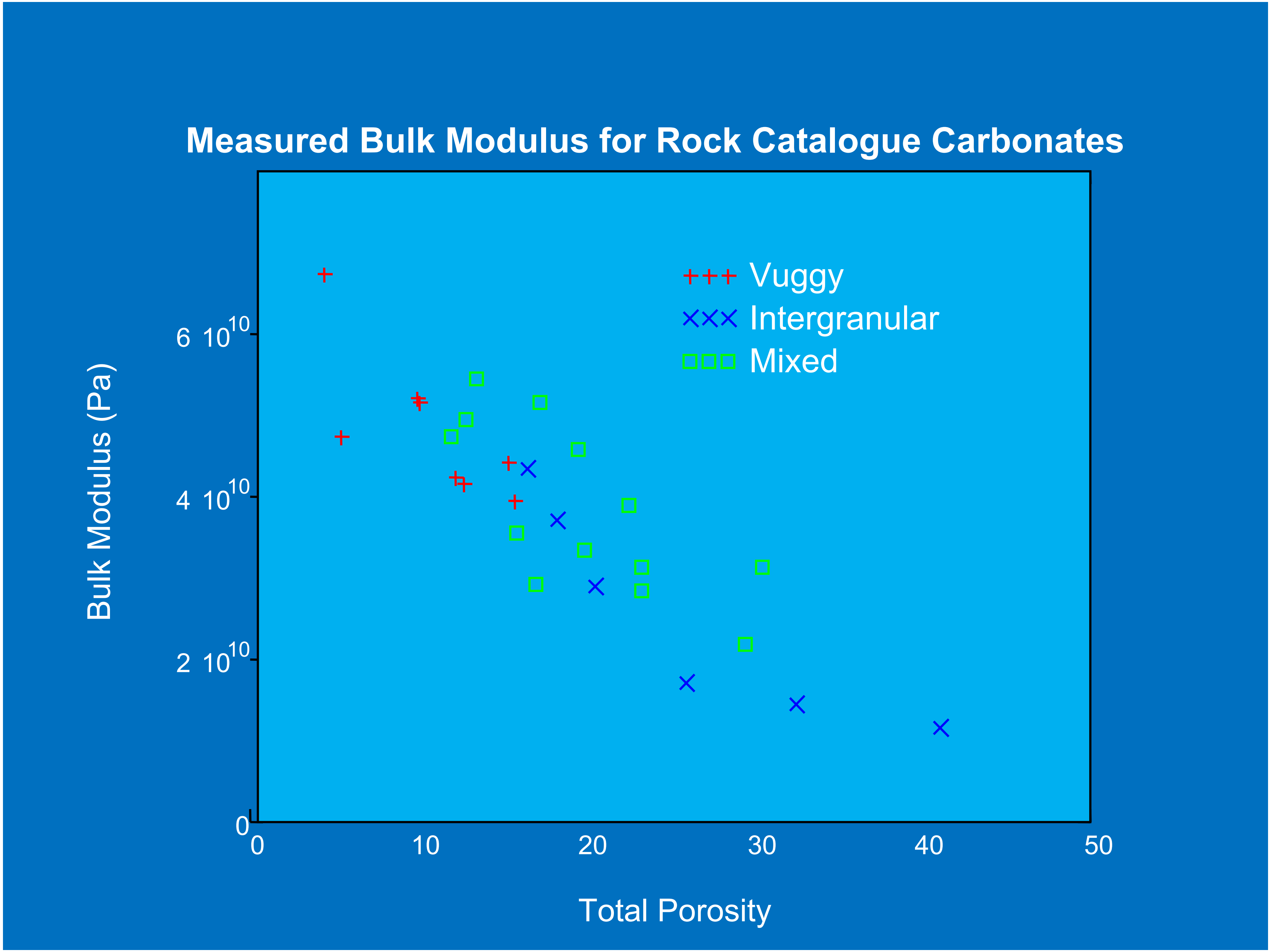


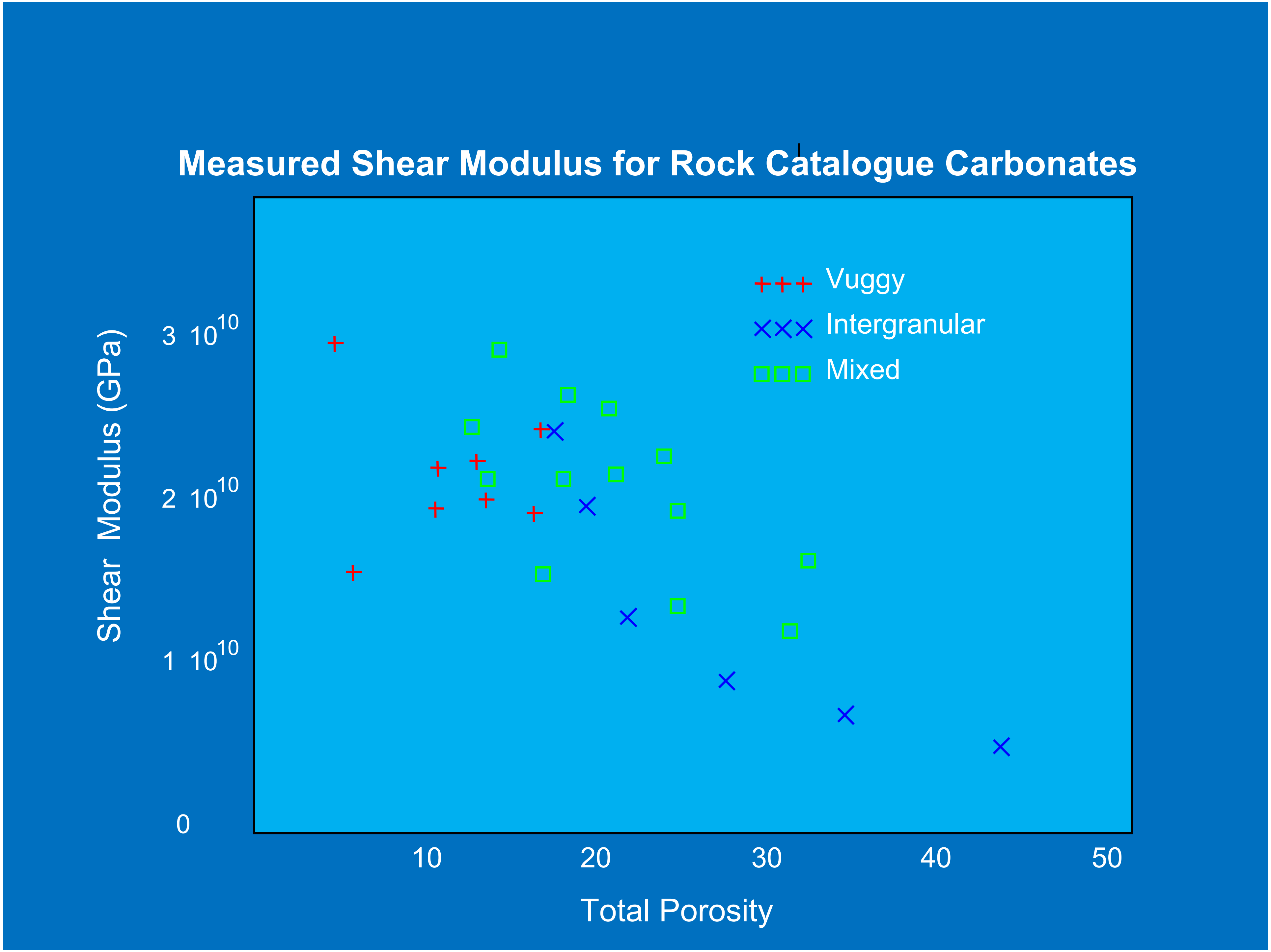
Size is implicit in the order: longer length scales are added after smaller. This is implied by the effective medium approximation

DEM Modulus Model

- **Preserves the Voight and Reuss averages at the limits L=0 and L=1.**
- **Allows continuous interpolation between iso-strain and iso-stress averages.**
- **Allows multiple lithology changes at separate integration steps. Ordering of the integration is dependent on length scales present in the sample (implicit in the derivation).**
- **Naturally includes critical concentration models.**
- **Fluid substitution is consistent with Gassmann.**

Application of the Model to Predict Velocities


Measured Bulk Modulus for Rock Catalogue Carbonates
Vuggy
Intergranular
Mixed
Bulk Modulus (Pa)
6 10^10
4 10^10
2 10^10
0
0
10
20
30
40
50
Total Porosity


Measured Shear Modulus for Rock Catalogue Carbonates
Vuggy
Intergranular
Mixed
Shear Modulus (GPa)
3 10^10
2 10^10
1 10^10
0
10
20
30
40
50
Total Porosity

The SDEM model for the modulus is:

$$M = M_h \frac{1+\left(\frac{\Phi_h-\Phi}{\Phi_h}\right)\cdot\left(\frac{M_i-M_h}{M_h}\right)\cdot L}{1-\left(\frac{\Phi_h-\Phi}{\Phi_h}\right)\cdot\left(\frac{M_i-M_h}{M_i}\right)\cdot(1-L)}$$

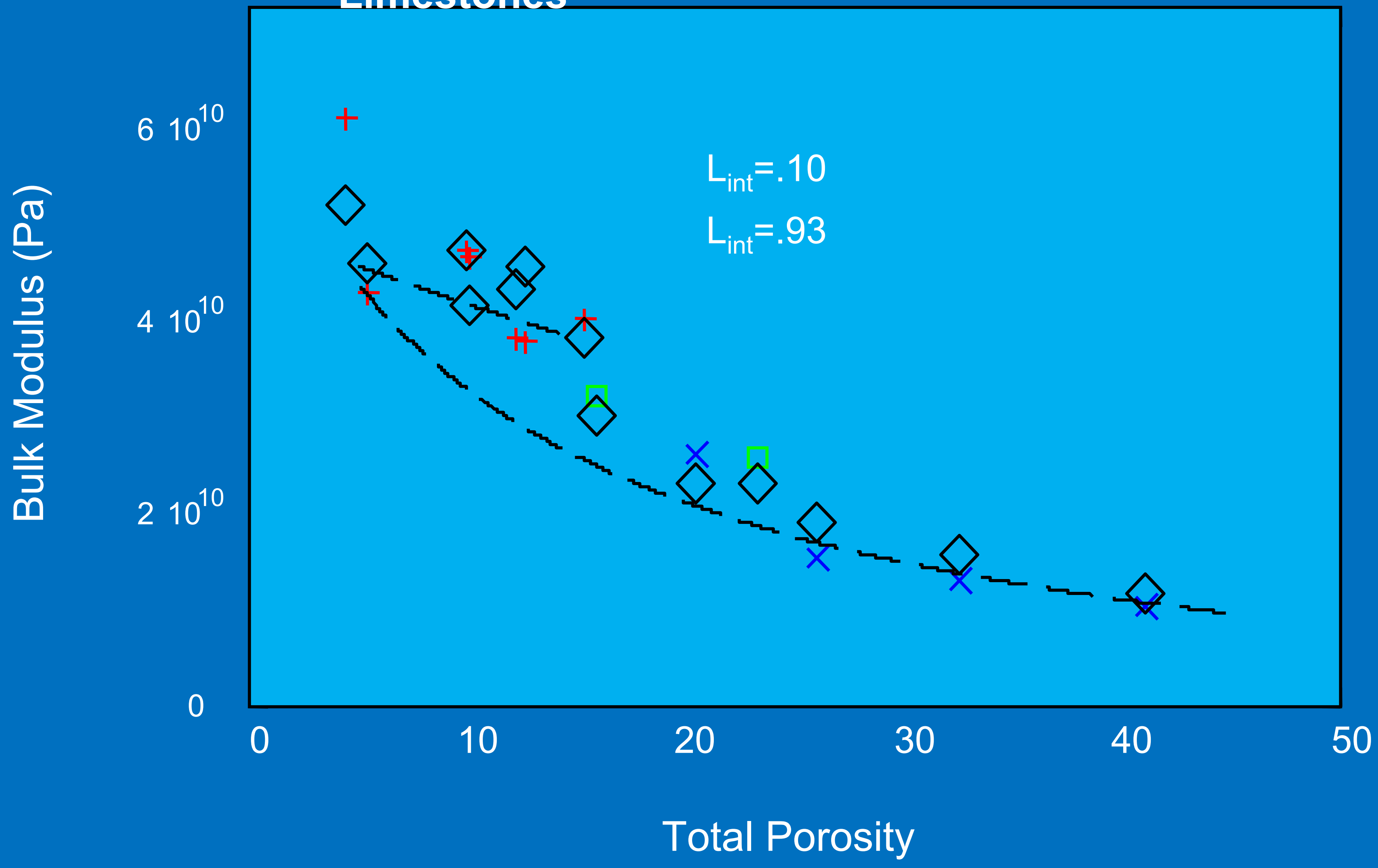
Solving for the porosity:

$$\Phi = \Phi_h\left(\frac{M_i-M}{M_i-M_h}\right)\left[\frac{(1-L)\cdot M_h+L\cdot M_i}{(1-L)\cdot M+L\cdot M_i}\right]$$

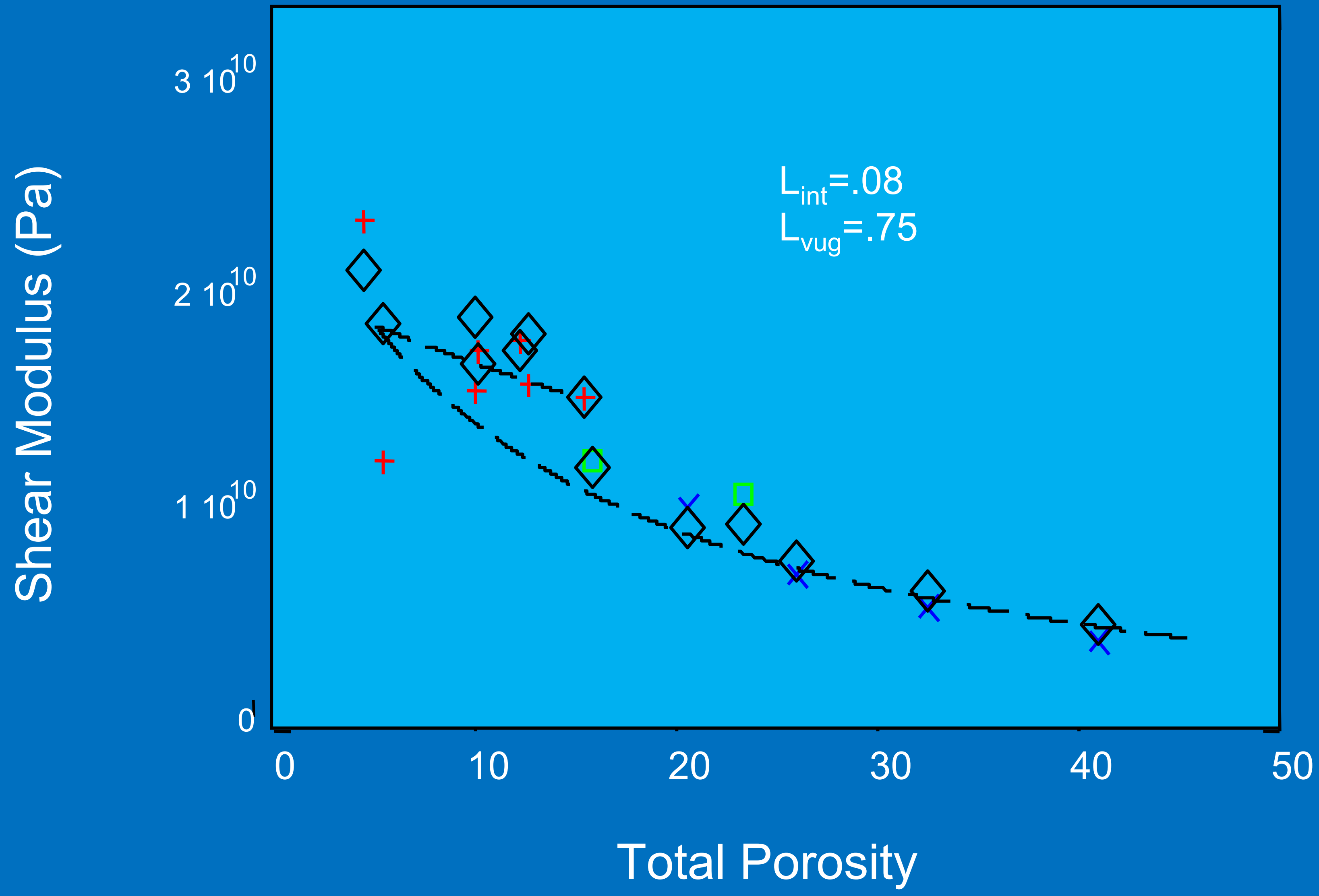
Solving for the L:

$$L = \frac{\frac{M}{M-M_i}-\frac{M_h}{M_h-M_i}\frac{\Phi_h}{\Phi}}{1-\frac{\Phi_h}{\Phi}}$$

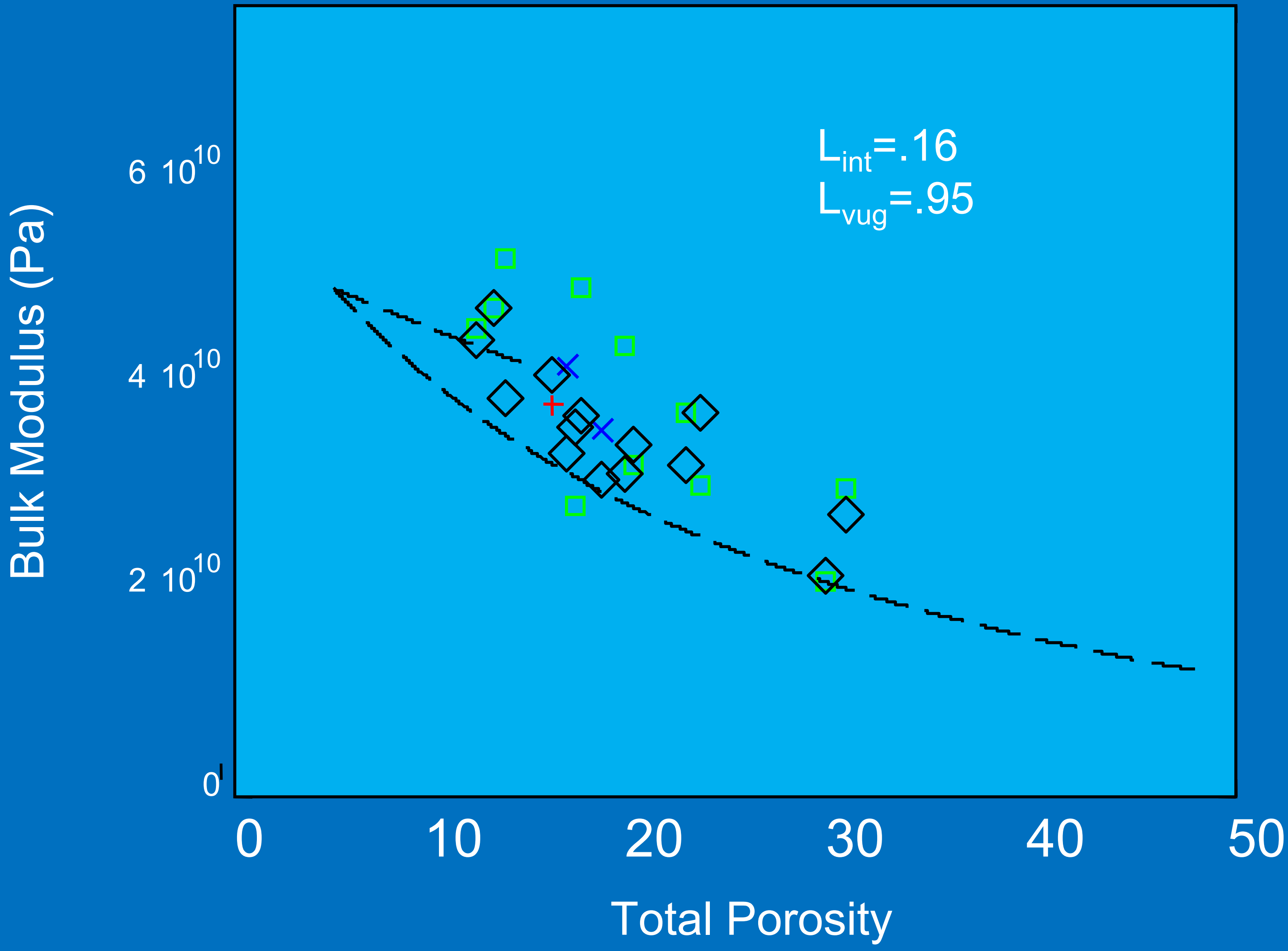
Fit for the Bulk Modulus for Rock Catalogue Limestones

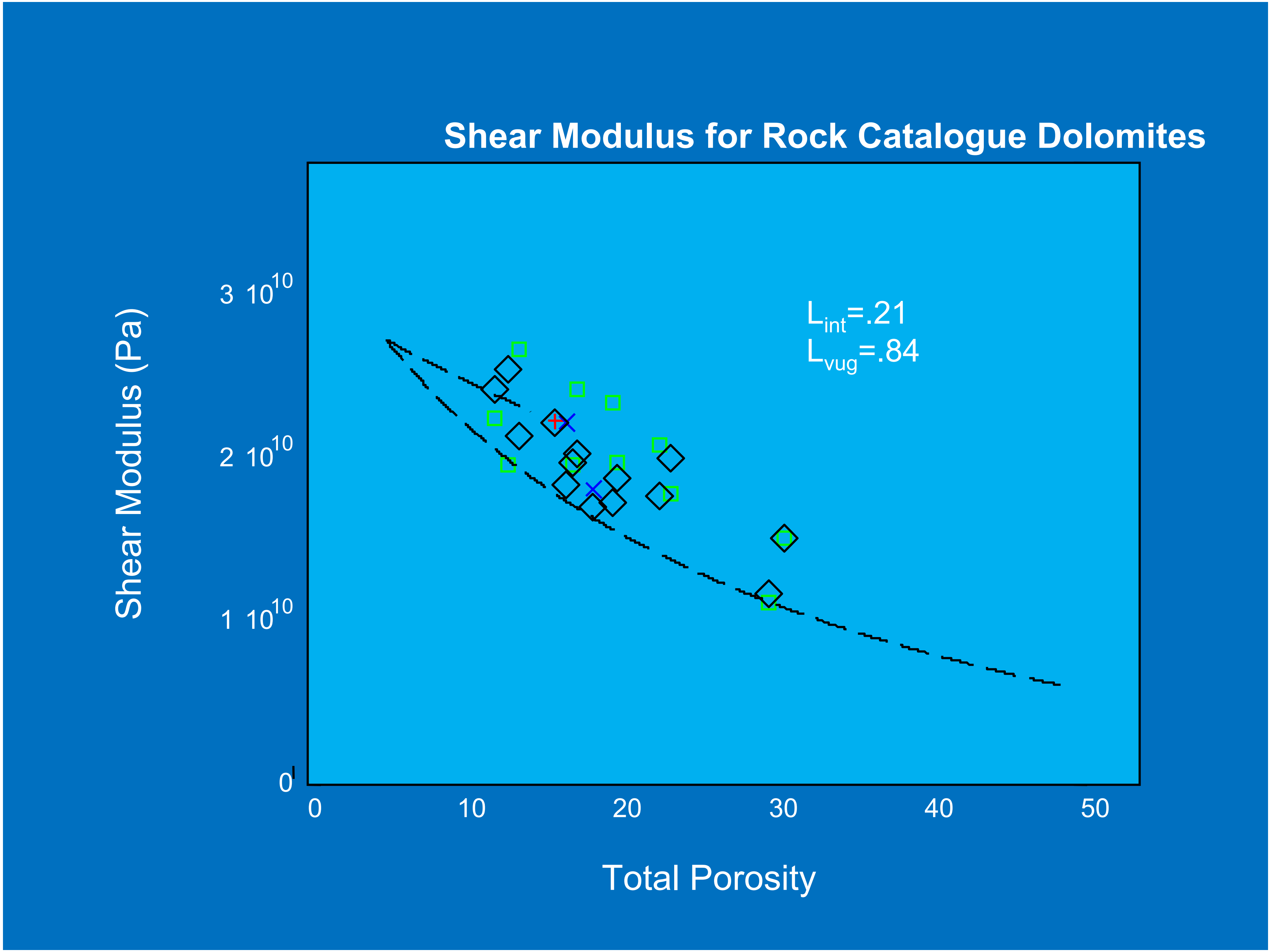


Example Fit for the Shear Modulus for Rock Catalogue Limestones



Example Fit For the Bulk Modulus for Rock Catalogue Dolomites




Shear Modulus for Rock Catalogue Dolomites
L_{int}=.21
L_{vug}=.84
Shear Modulus (Pa)
3 10^{10}
2 10^{10}
1 10^{10}
0
0
10
20
30
40
50
Total Porosity

Comparison of Fits for L values for the Carbonate Data Sets

Bulk Modulus Dolomites	Shear Modulus Dolomites	Bulk Modulus Limestones	Shear Modulus Limestones	L
.16	.21	.10	.08	Intergranular / Intercrystalline
.95	.84	.93	.75	Vugs

Check the Consistency with Resistivity Data

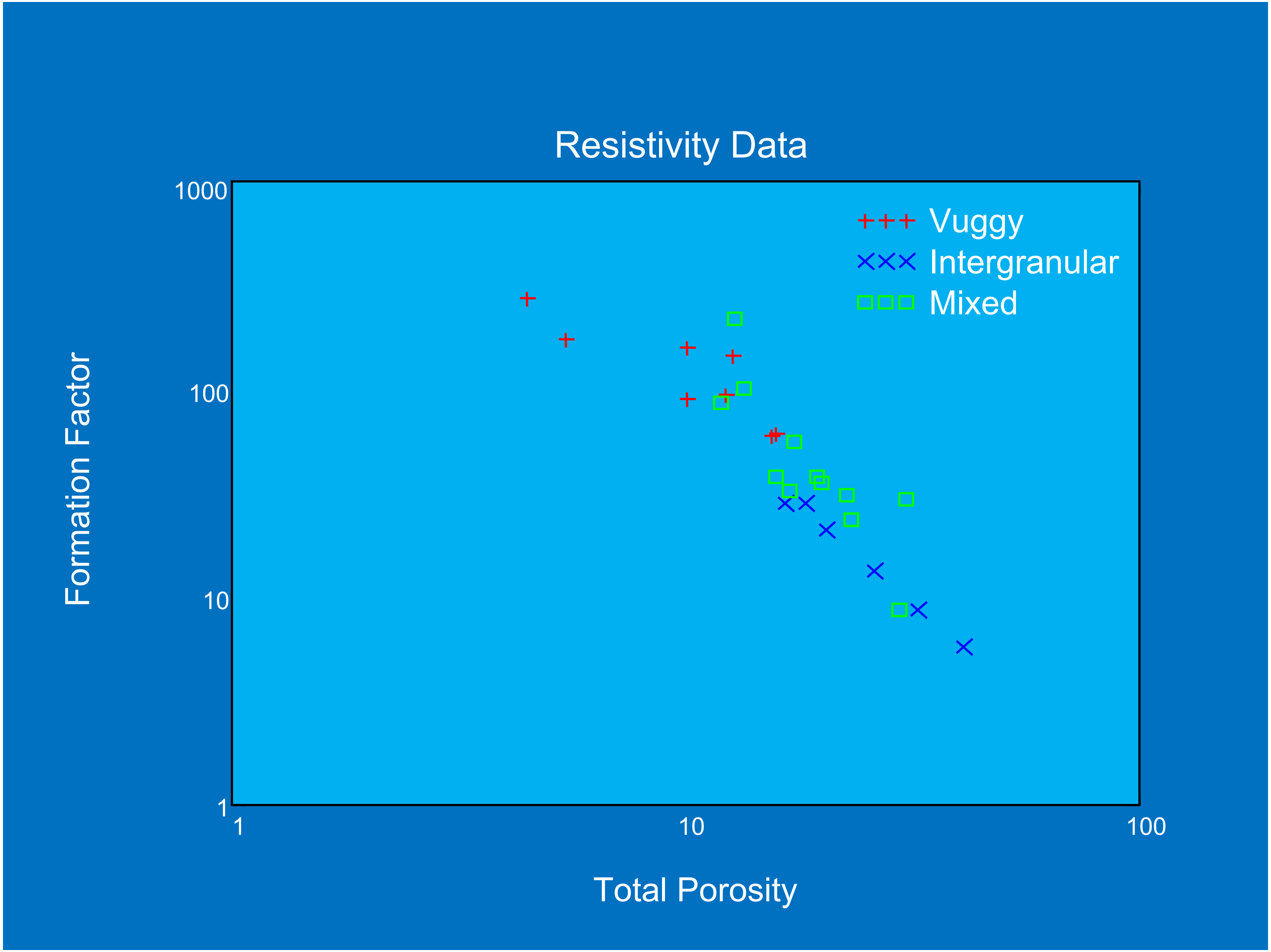
Effective medium models for the resistivity give:

$$F = \left(\frac{1}{\varphi_{\mathrm{int}}}\right)^{\lambda_{\mathrm{int}}} \cdot \left(\frac{\varphi_{\mathrm{int}}}{\varphi_{total}}\right)^{\lambda_{vug}}$$

Solving for the intergranular porosity gives:

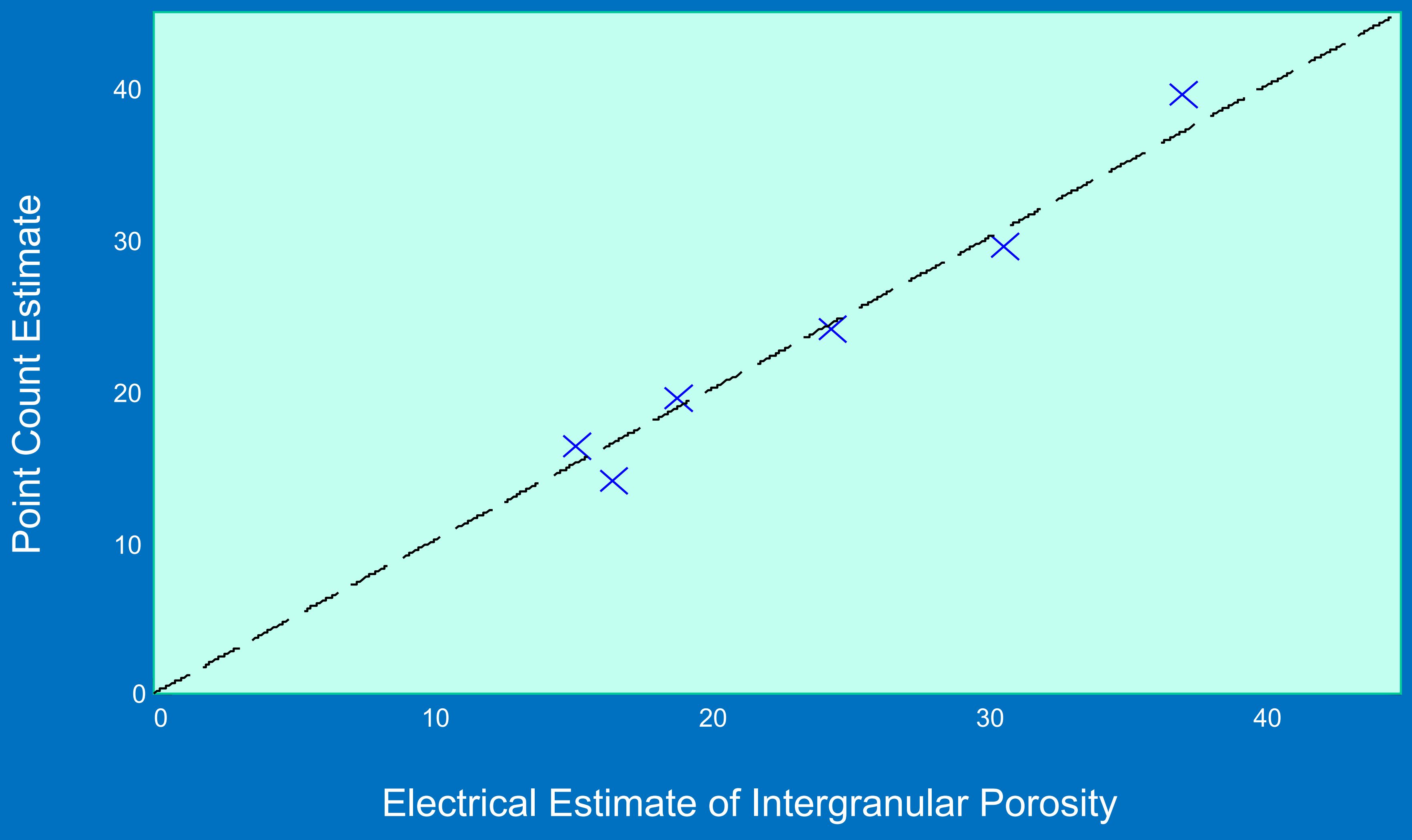
$$\varphi_{\mathrm{int}} = 10^{\frac{\left(\log_{10}(\varphi_{total}) + \lambda_{vug} \cdot \log_{10}(\varphi_{total})\right)}{\lambda_{\mathrm{int}} - \lambda_{vug}}}$$

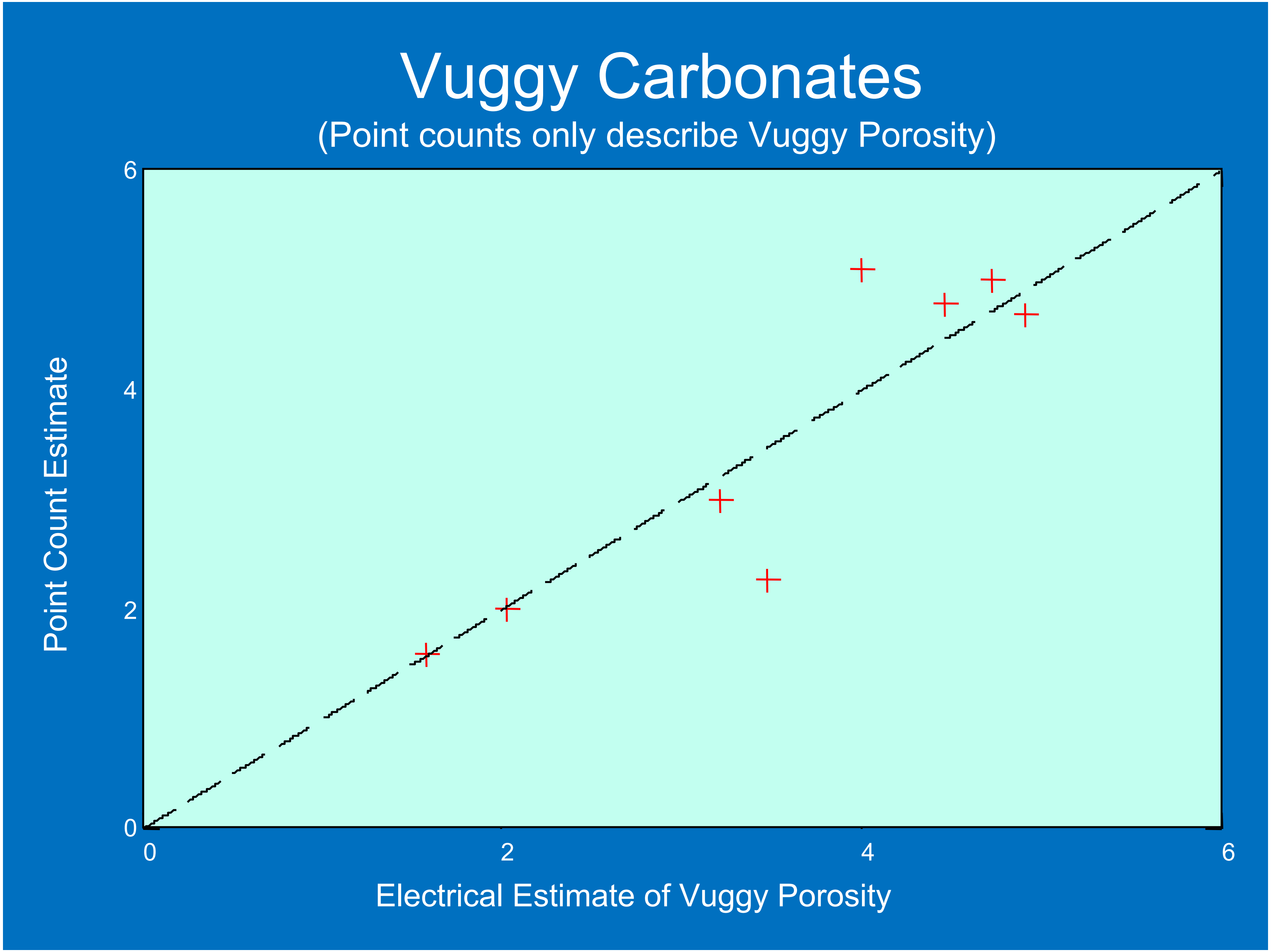
Where φ_{int} is the connected intergranular porosity, φ_{total} is the total porosity, F is the formation factor and λ_{int} and λ_{vug} are exponents similar to the exponent in Archie's equation


Resistivity Data
Vuggy
Intergranular
Mixed
Formation Factor
1000
100
10
1
1
10
100
Total Porosity

Intergranular Carbonates

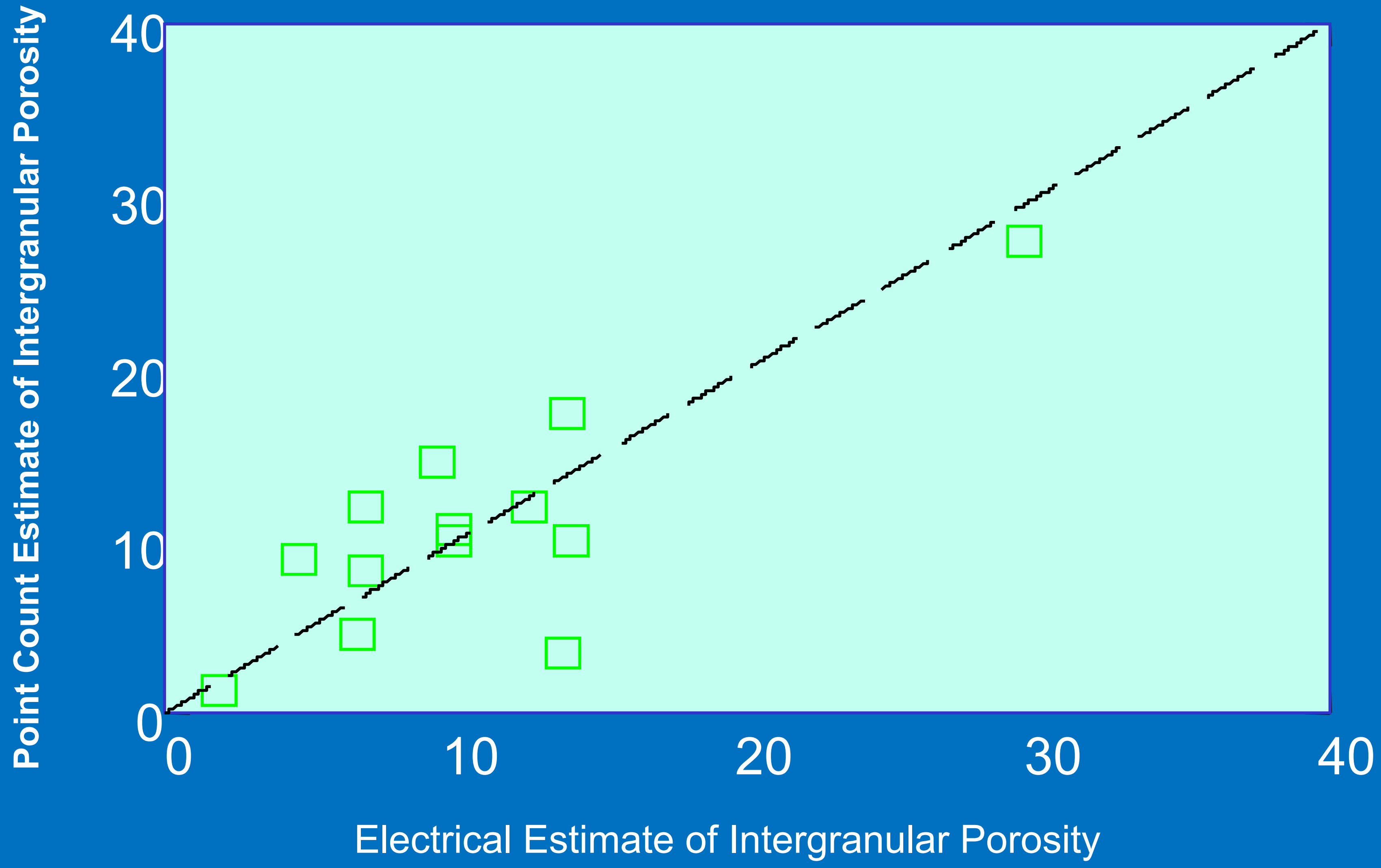
(Point Count only Describes Intergranular Porosity)

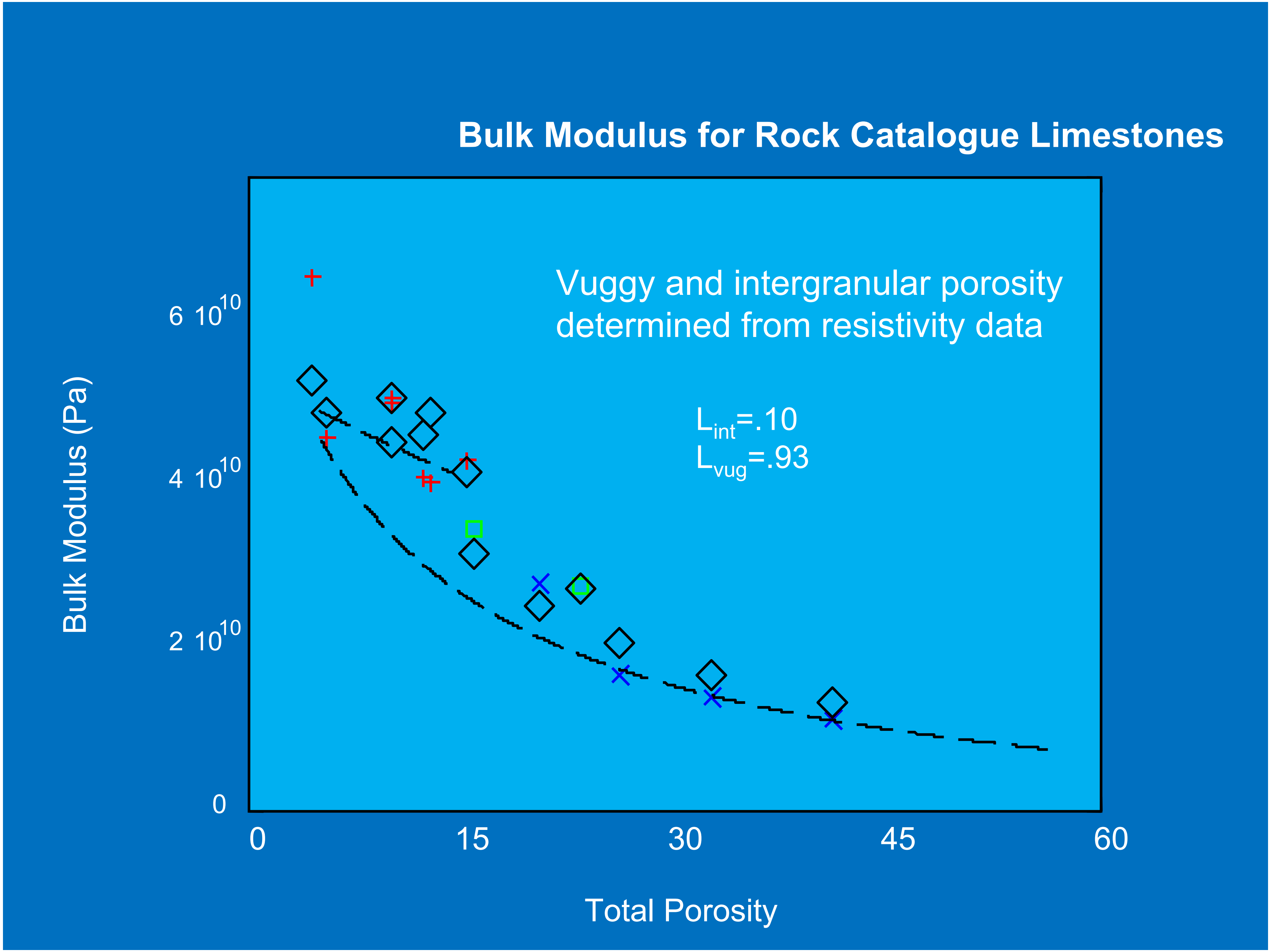


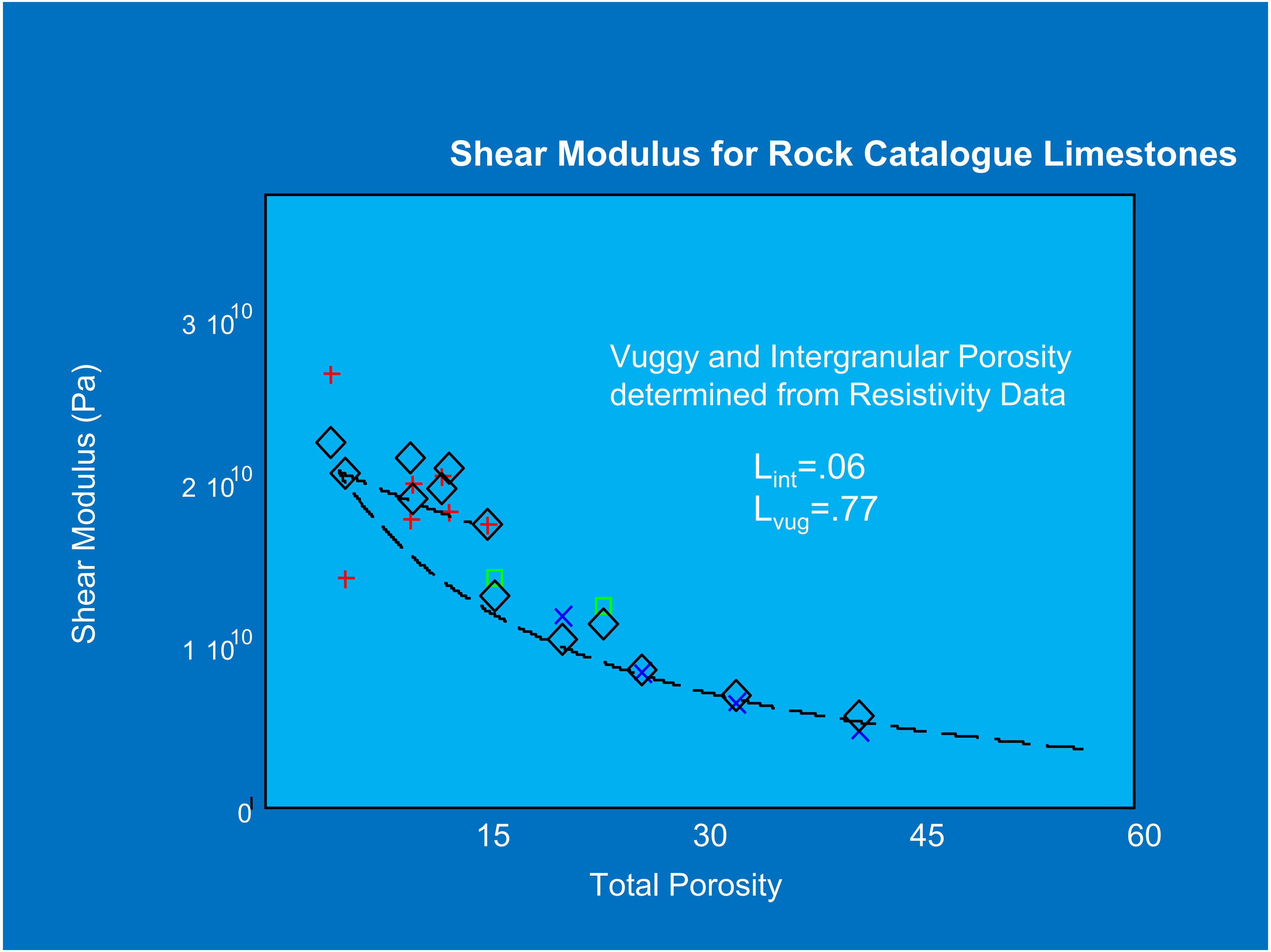

Vuggy Carbonates
(Point counts only describe Vuggy Porosity)
Point Count Estimate
0
2
4
6
Electrical Estimate of Vuggy Porosity
0
2
4
6

Mixed Carbonates

(Point Counts Identify Significant amount of Vuggy and Intergranular Porosity)




Bulk Modulus for Rock Catalogue Limestones
Vuggy and intergranular porosity
determined from resistivity data
L_{int}=.10
L_{vug}=.93
Bulk Modulus (Pa)
6 10^{10}
4 10^{10}
2 10^{10}
0
0
15
30
45
60
Total Porosity

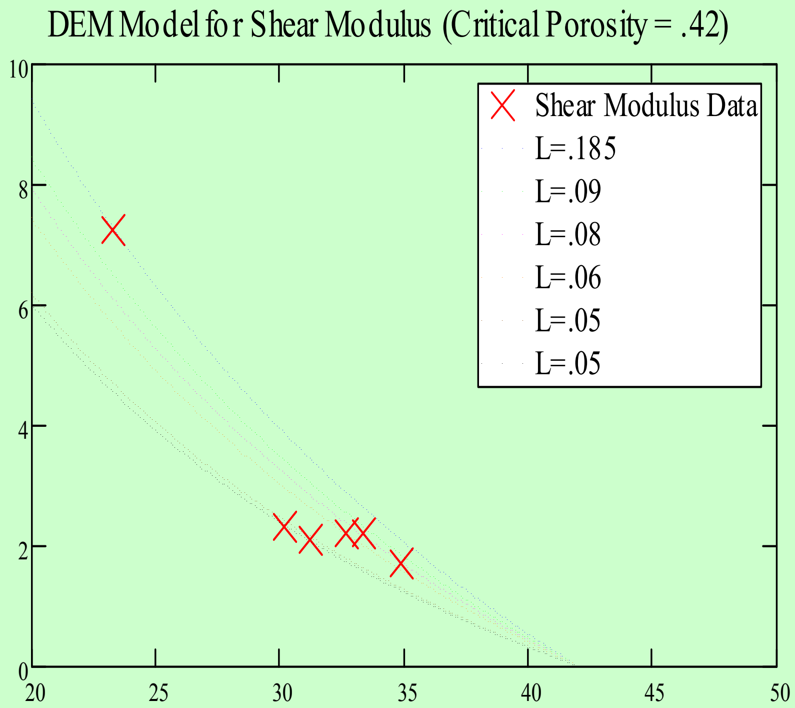


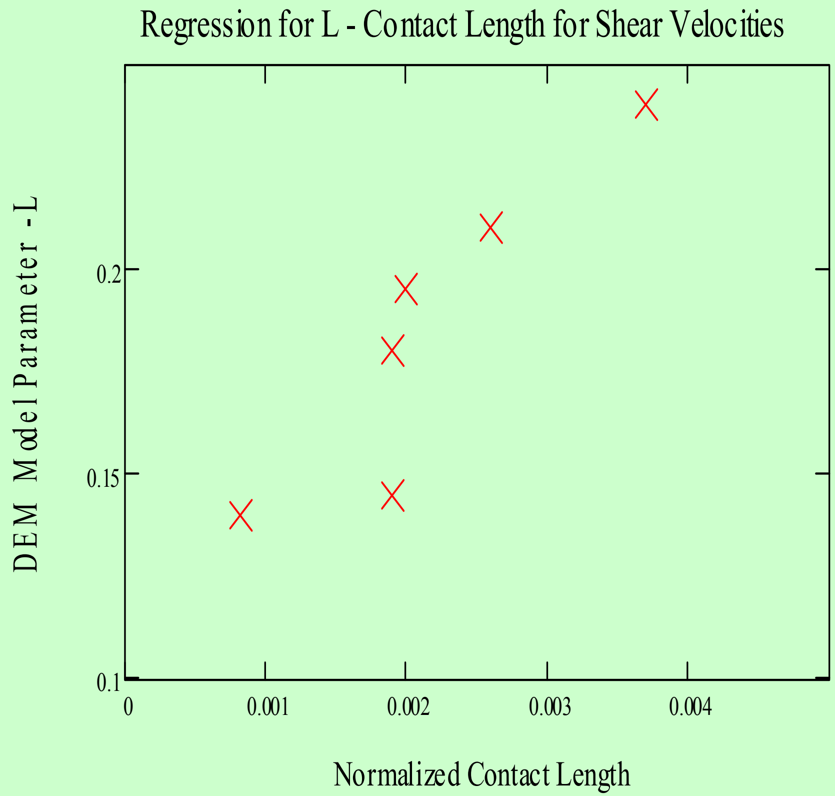
Shear Modulus for Rock Catalogue Limestones
Vuggy and Intergranular Porosity
determined from Resistivity Data
L_{int}=.06
L_{vug}=.77
3 10^{10}
2 10^{10}
1 10^{10}
0
Shear Modulus (Pa)
15
30
45
60
Total Porosity

Check the Consistency with Resistivity Data

- The electrical data does not improve the fits; the petrology data works as well!
- Some other explanation is required for the variation in the data

Example Fits for Shear Modulus For Unconsolidated Sands





Things To Do

- Develop techniques to model connected pore systems
  - Grain contacts
  - Fractures
  - Multiple mineralogies
- Apply to shales
- Identify controls on anisotropy