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Modeling Hydro-Isostasy: Isostatic Flexure Along the Global Coastlines Due to Sea-Level Rise and Fall*

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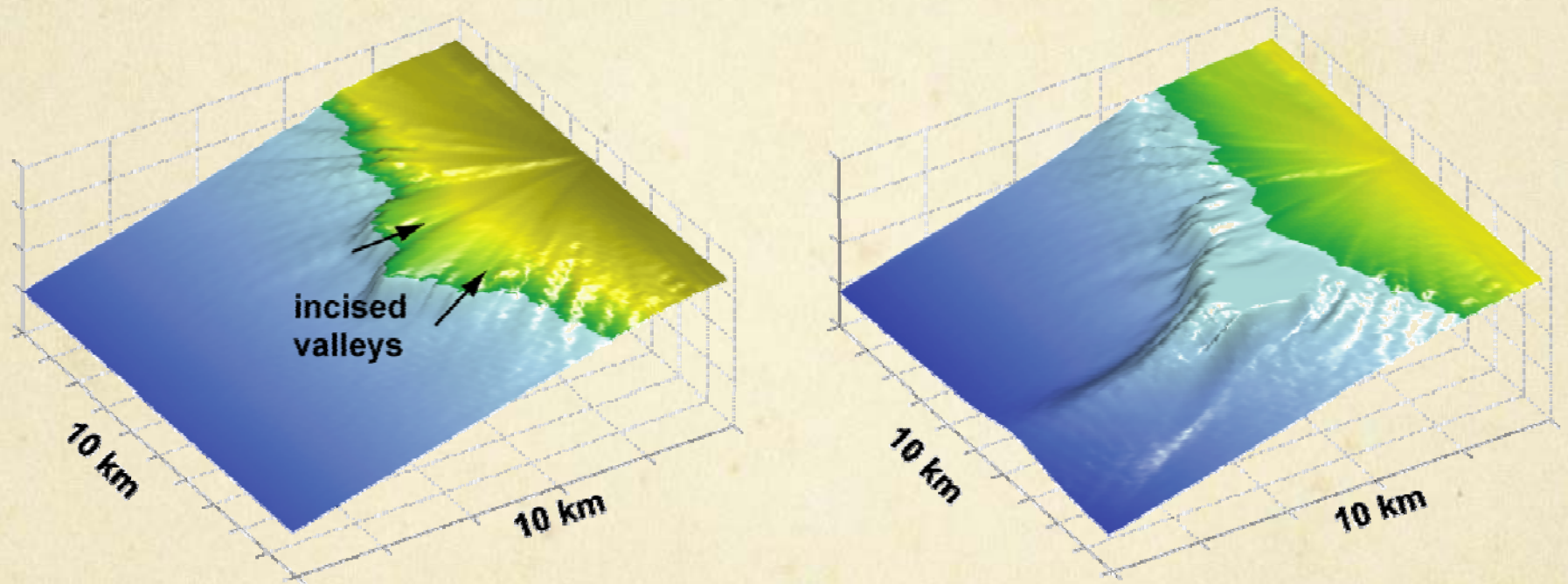
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Abstract

Ice Age sea-level cycles are on the order of 100 m, causing changes in the overlying load on continental shelves worldwide. These load changes cause the lithosphere to deflect along the Earth's coastlines. Based on a one-dimensional elastic flexure model, an analytic solution for the deflection of a linear slope, due to sea level rise and fall, is derived. This analytic solution allows a global database of deflection estimates for continental shelves, due to increases in water loading and the shape of LGM continental margins. Thus, changes in eustatic sea level are disengaged from changes in relative sea level. Variations in water loading can change the slopes of continental shelves on the order of 30%. Hydro-isostasy adds to the magnitude of a sea level rise, long after the eustatic component of the sea level rise has ended. A sea-level rise over a continental shelf will produce a wedge-shaped loading pattern that increases from the landward shoreline until it reaches its maximum at the lowstand shoreline. This asymmetric loading pattern causes a steepening of the shelf. A fall in sea level has a similar effect, but opposite in sign. The wedge-shaped unloading pattern, due to a sea-level drop, causes a decrease in shelf gradient and an increase in the total shoreline regression. Quantifying this effect is essential to reconstructing stream gradients, or accommodation estimates through a sea level cycle. While the water depth of a paleo-shoreline gives an estimate of relative sea-level change, without an estimate of the amount of deflection at this location, eustatic sea-level change remains unknown.

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Outline

- Present an analytical method for the deflection of a continental shelf due to hydro-isostasy given 1) a change in eustatic sea level rise or fall, 2) the shape of LGM continental margins, and 3) estimates of lithospheric thickness
- Apply the model at the global scale
- Discuss the implications for this model development and application

Primer

- ❖ *Hydro Isostasy is the crustal deflection related to load change from a fluctuation in sea level.*
- ❖ *The flexural response takes thousands of years (E -folding $\approx 2500y$) to complete because the viscous asthenosphere has to flow out of the way before the lithosphere can deflect.*
- ❖ *Isostatic displacements extend over a region much larger than the area directly affected by the load change due to the regional elastic lithosphere thickness.*

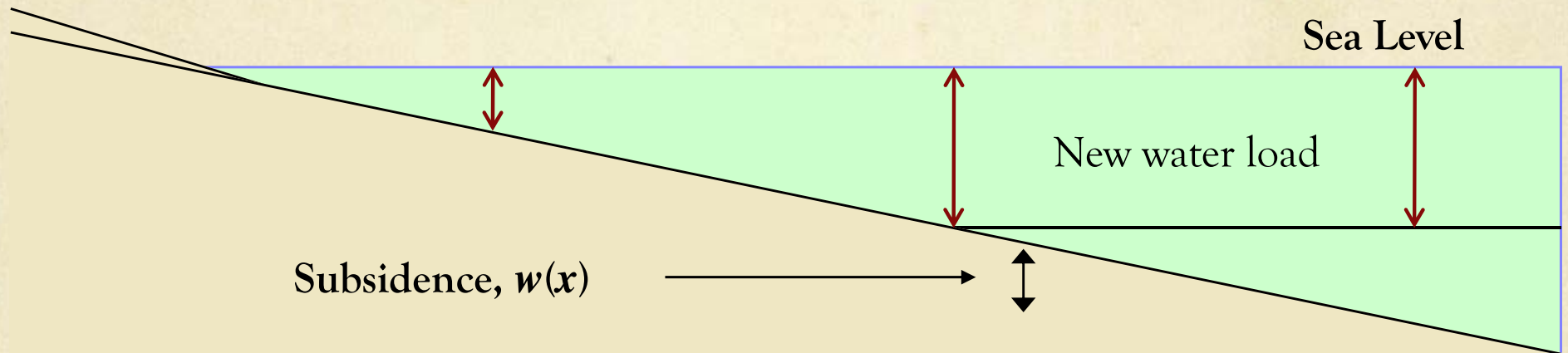


The flexure equation can be used to predict the deflection due to sea-level rise and fall

Governing Equation

$$D \frac{d^4 w}{dx^2} + N \frac{d^2 w}{dx^2} + \Delta \gamma w = q(x)$$

or $\frac{d^4 \bar{w}}{d\bar{x}^4} + 4\bar{w} = 4\bar{q}$ if $\bar{x} = \frac{x}{\alpha}$, $\bar{w} = \frac{\Delta \gamma}{q_0} w$, $\bar{q} = \frac{q}{q_0}$, $\alpha \equiv \sqrt[4]{\frac{4D}{\Delta \gamma}}$, $N = 0$



w = deflection, x = distance perpendicular to loading, D = flexural rigidity, N = interpolate force, q = load distribution, $\Delta \gamma$ = specific gravity difference of the mantle and the added (or removed) material, and q_0 is the maximum load due to some sea-level rise or fall.



An analytic solution to flexure equation exists for this wedge-shaped loading pattern

Line Load Solution

$$\bar{w}(\bar{x}) = e^{-|\bar{x}|} (\cos|\bar{x}| + \sin|\bar{x}|)$$



Distributed Water Load

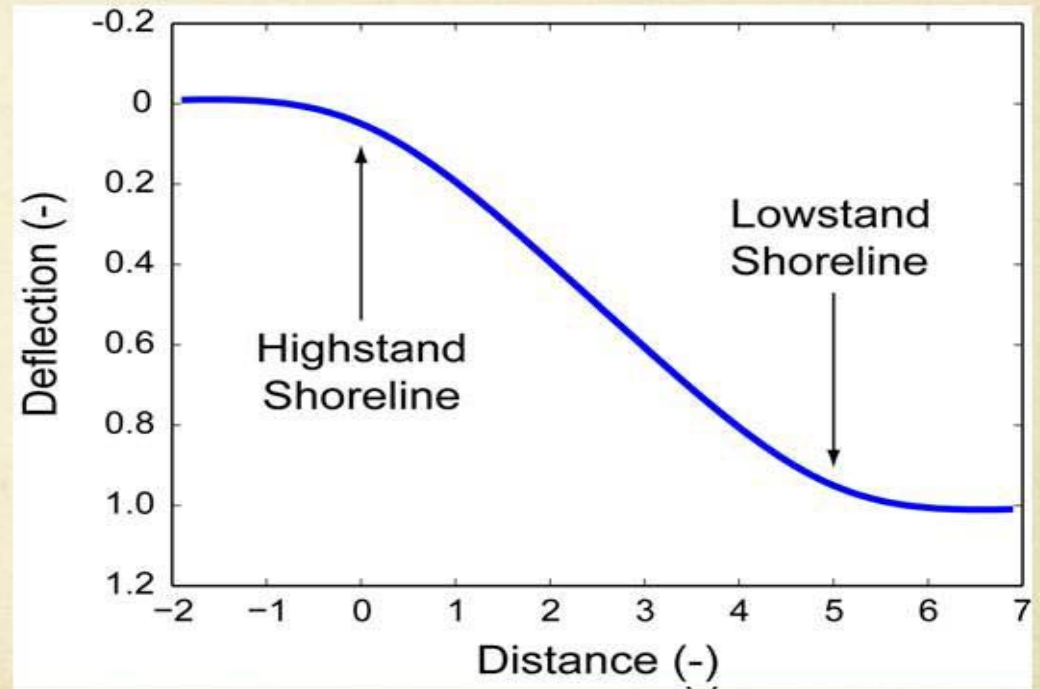
$$\bar{w}(\bar{x}) = \frac{1}{4\bar{x}_0} [F(\bar{x}) - F(\bar{x} - \bar{x}_0) + 4R_{\bar{x}_0}(\bar{x})]$$

$$F(x) \equiv e^{-|x|} (\cos|x| - \sin|x|)$$



Slope of the new shelf

Shoreline position



Notes by Presenter: where e is the half-width of the impulse function, \bar{x} is x/α or distance/flexural parameter, w = deflection

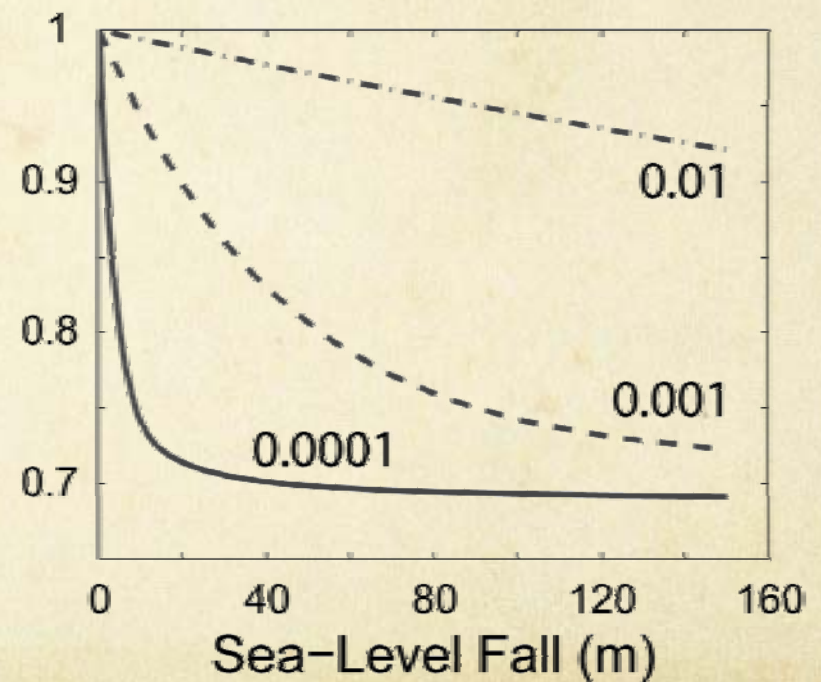
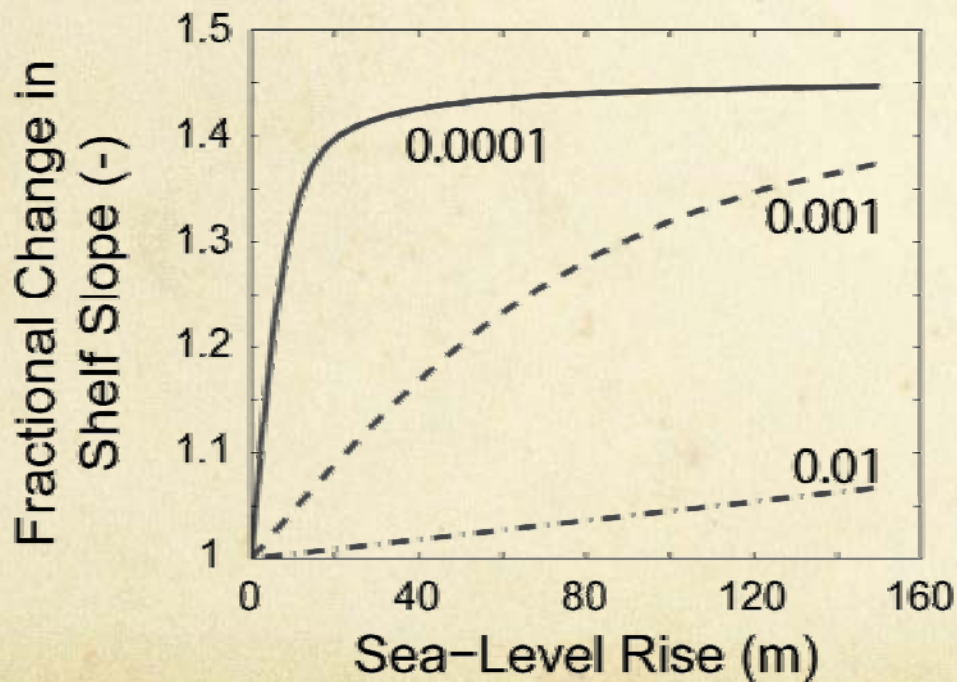
The change in shelf gradient is a function of initial gradient and sea-level change

Sea-Level Rise

$$\frac{\bar{S}_1}{\bar{S}_0} = 1 + \sigma_r + \frac{\sigma_r}{2\bar{x}_e} [F(\bar{x}_e) - 1]$$

Sea-Level Fall

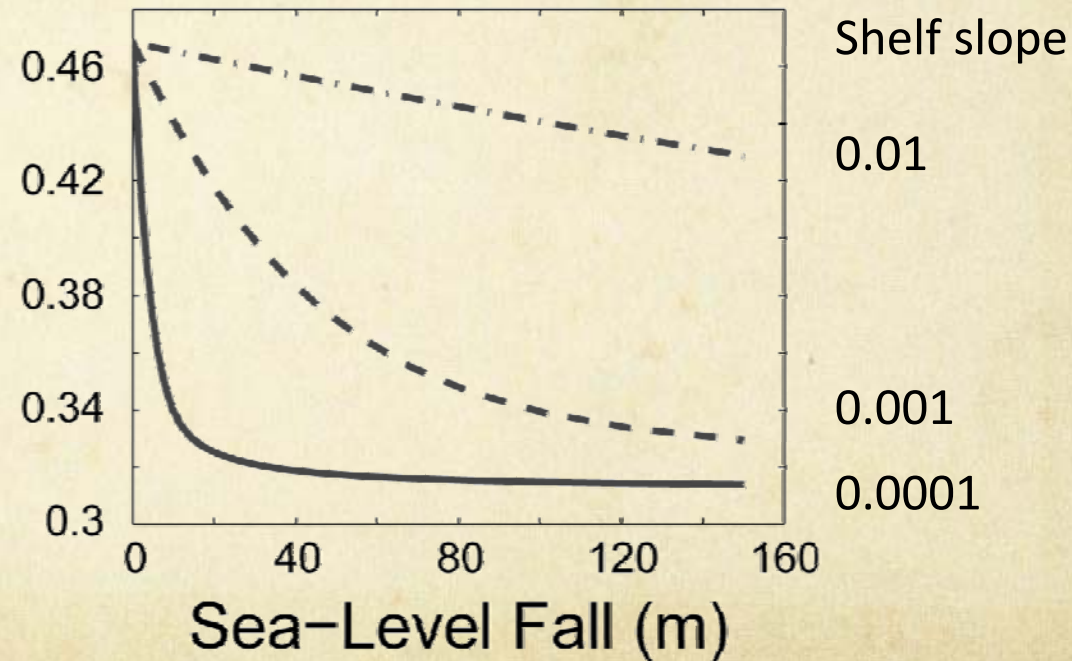
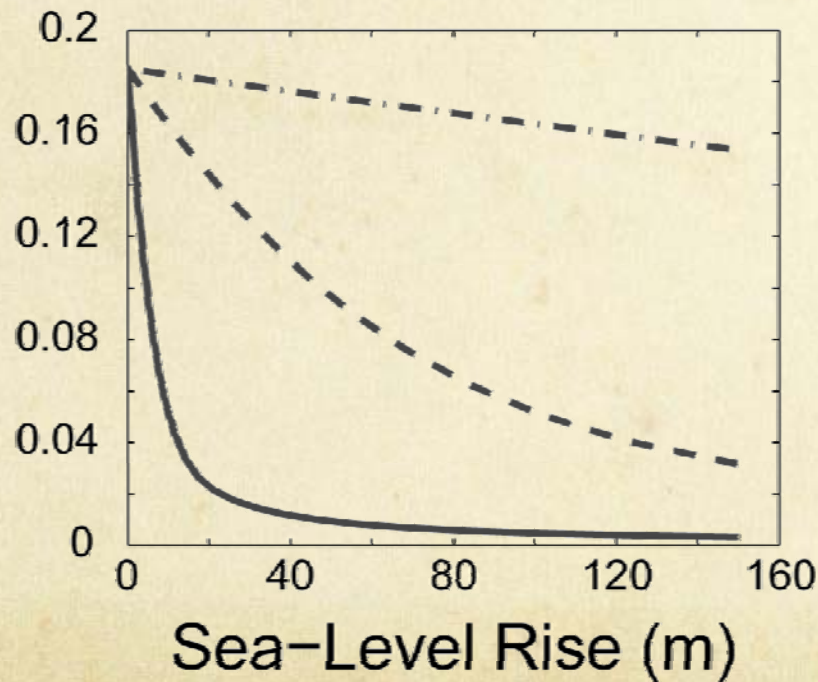
$$\frac{\bar{S}_1}{\bar{S}_0} = 1 - \frac{\sigma_f}{2\bar{x}_0} [F(\bar{x}_0) - 1 + 2\bar{x}_0]$$



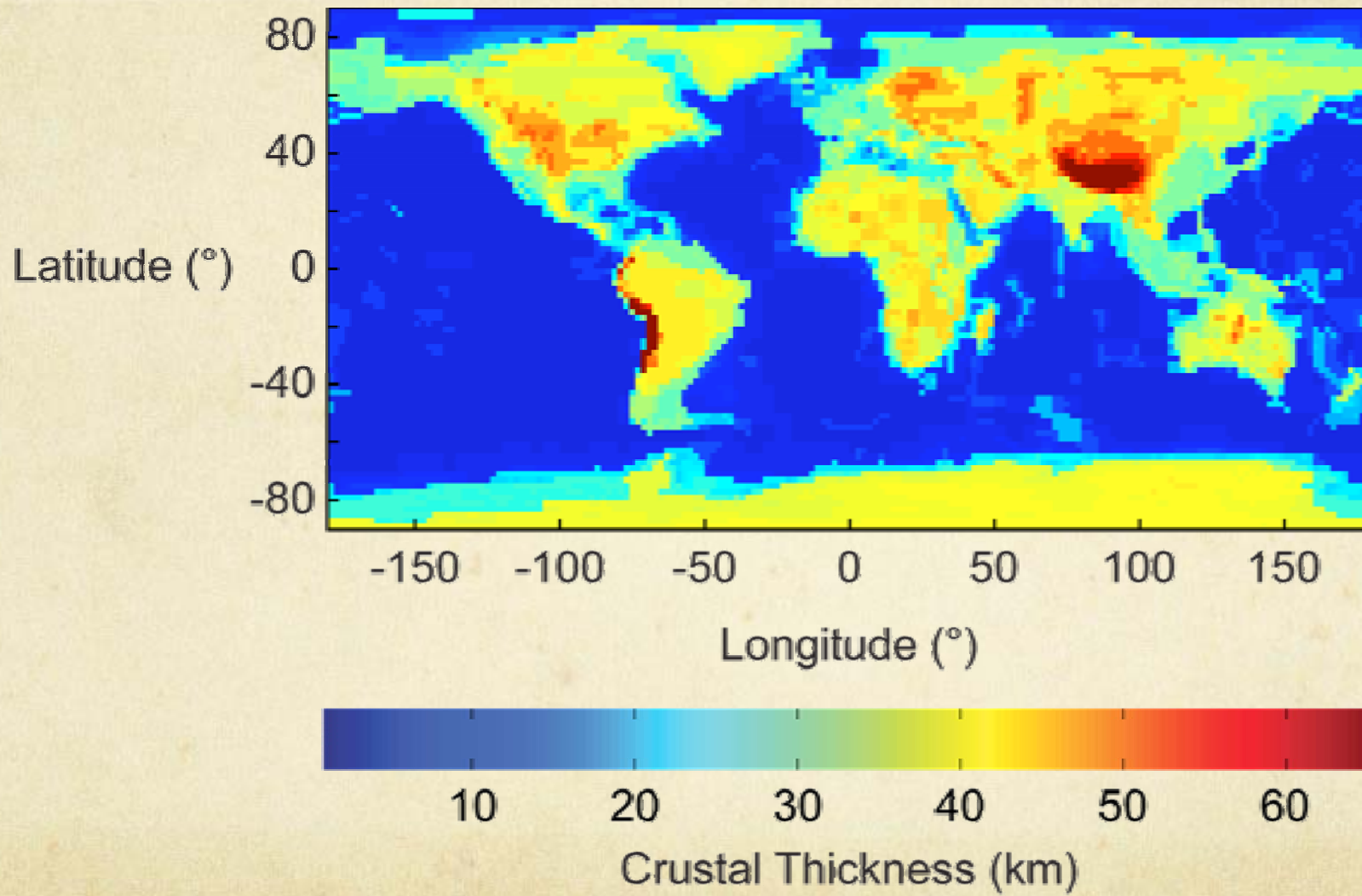
Water loading increases both shoreline transgression and regression

Deflections due to a change in water loading will act to augment either the normal shoreline transgression or regression.

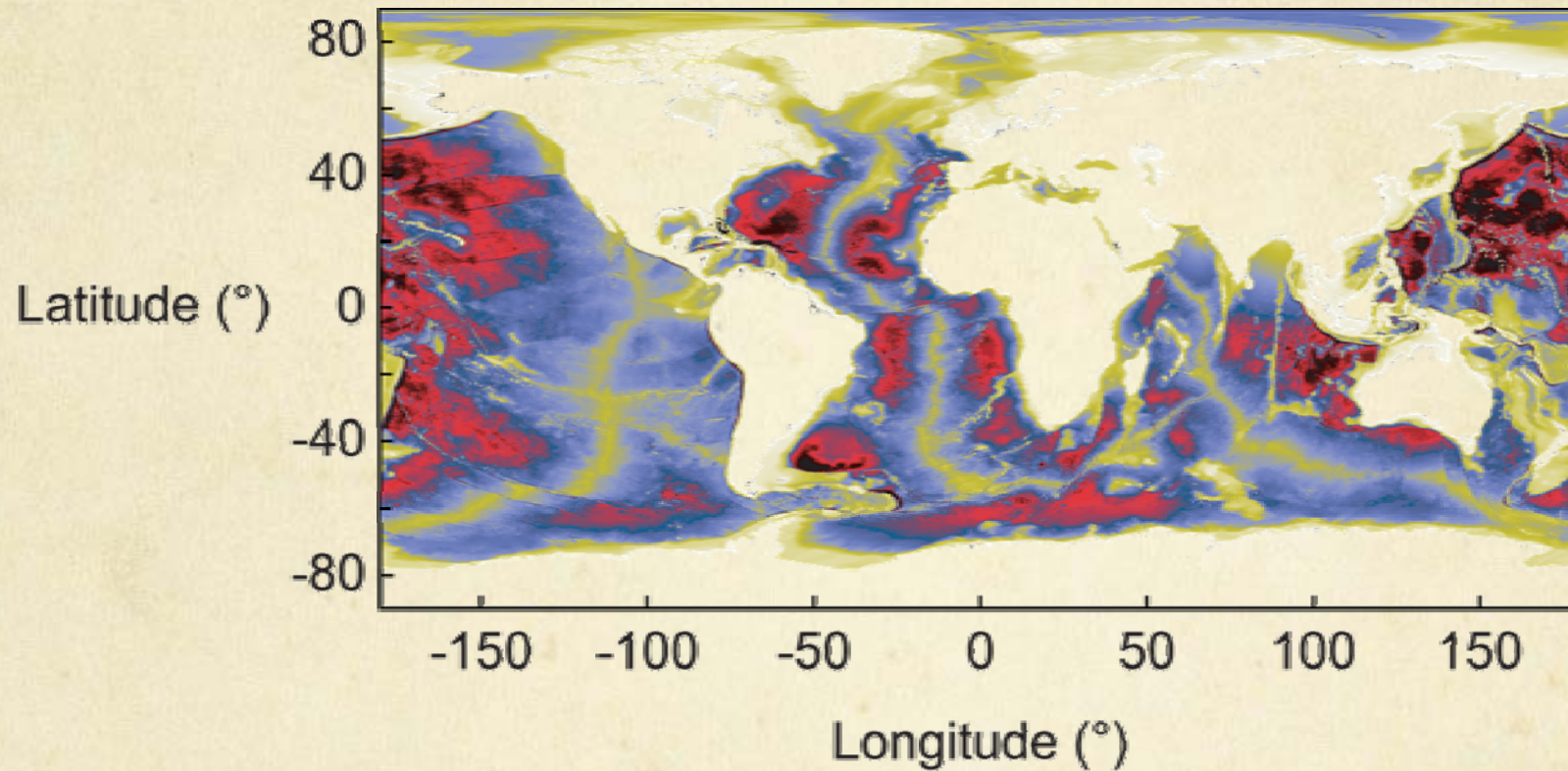
Transgression/Regression Increase (-)



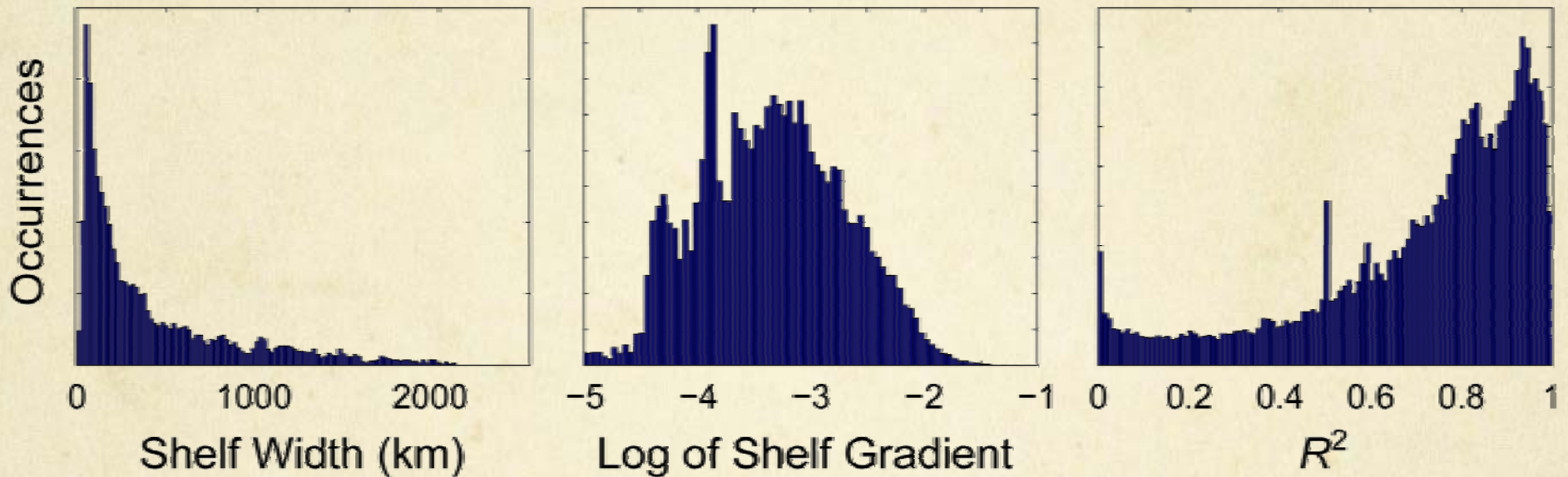
Crust 2.0 is a global crustal model at $2^\circ \times 2^\circ$ resolution



General Bathymetric Chart of the Oceans provides global bathymetry at 2' × 2' resolution



Measure shelf width and gradient for Earth's shelves using GEBCO data



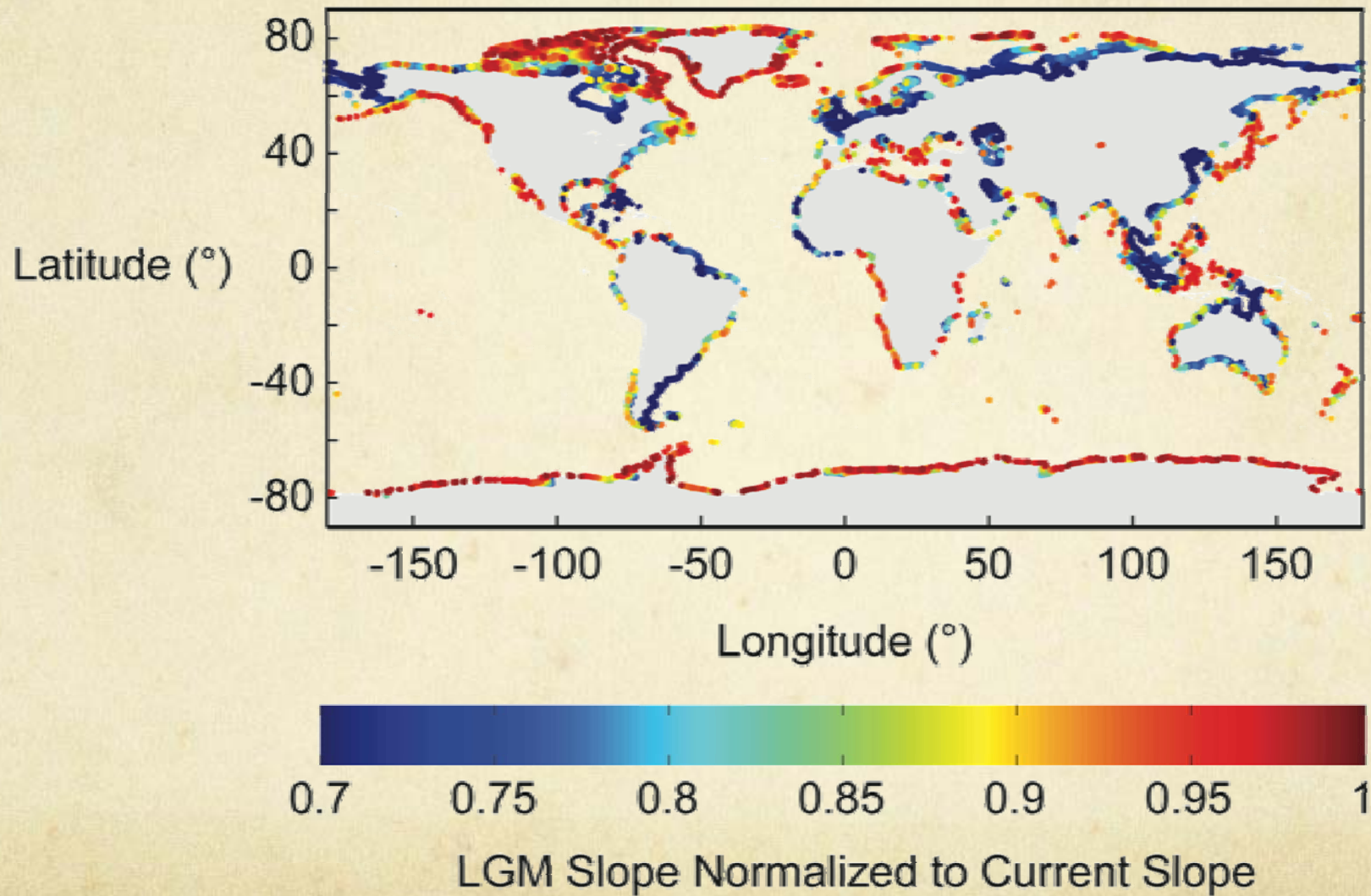
Shelf widths form an exponential density function

Shelf gradients are \approx lognormal with mean of -3.3

Over half the shelves have an $R^2 > 0.8$ to a linear regression.



Largest changes are with shelf gradient along passive margins when applied to *shelves with a $R^2 > 0.8$ to a linear regression*



If continental shelves are less steep during lowered sea-level, then the fluvial bedload would be smaller and finer grained. Bagnold (1966) calculated bedload Q_b as:

$$Q_b = \frac{\rho_s}{\rho_s - \rho} \frac{\rho g Q S e_b}{\tan \phi} \quad \text{when } u \geq u_{cr}$$

ρ_s and ρ : sediment and water density respectively,

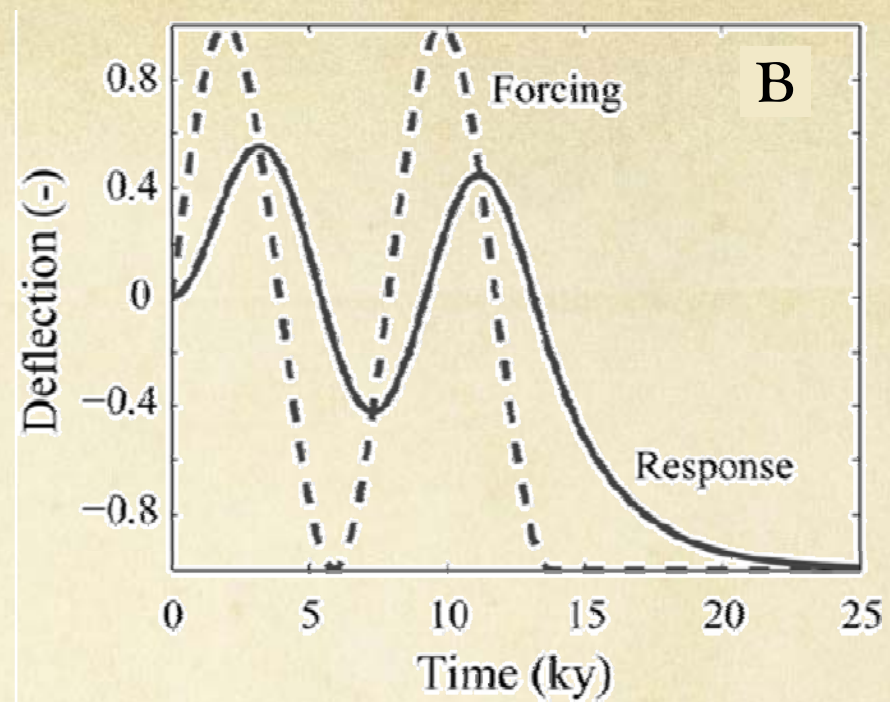
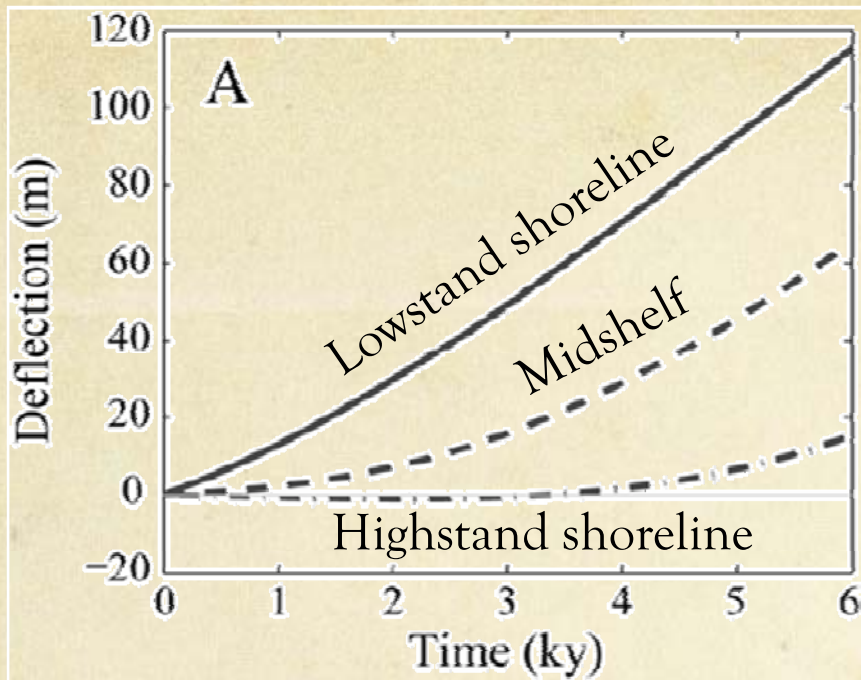
g : gravity, S : surface gradient, e_b : bedload efficiency,

ϕ : limiting angle of repose of sediment grains lying on a river bed,

u_{cr} : critical stream velocity below which no bedload transport occurs.

A 30% decrease in surface slope would have a 30% decrease in bedload



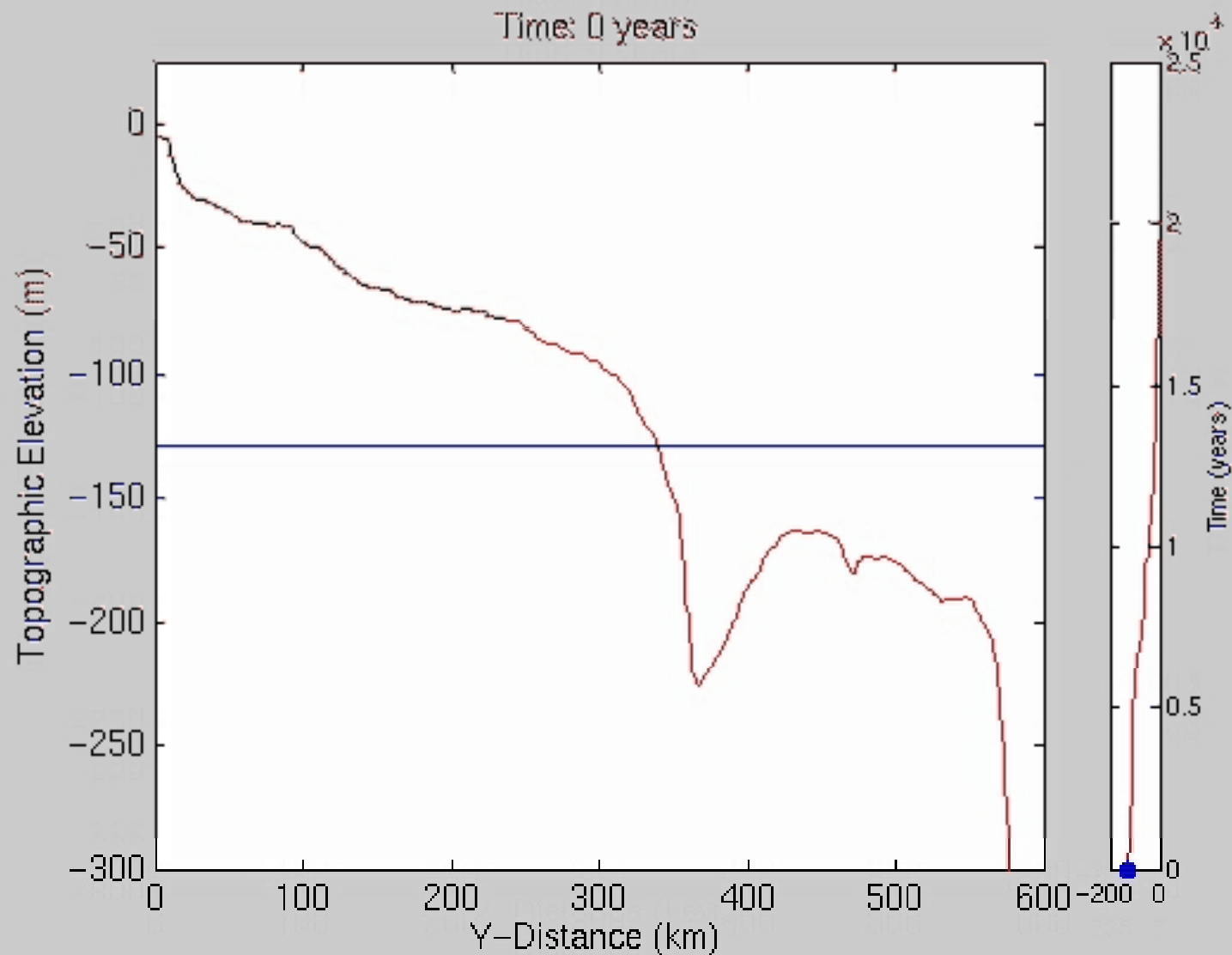


Deflection is a function of sea-level curve and lithospheric relaxation time.

(A) Deflection history at three points along a shelf given constant sea level rise. The lowstand shoreline subsides nearly linearly from the onset.

(B) Deflection (solid line) lags the forcing (dotted line) and is attenuated.





Numerical solution to the water load related to sea level rise in the Adriatic since the Last Glacial Maximum (21 Kyr) plus 10 Kyr into the future. Time step is 100 yr.



Conclusions

- Relative Sea Level (RSL) contributions related to hydro-isostasy can be disengaged from eustatic sea level fluctuations.
- Variations in water loading can change the slopes of continental shelves on the order of 30%.
- Hydro-isostasy adds to the magnitude of a sea level rise, long after the eustatic component of the sea level rise has ended.
- The wedge-shaped water loading during sea level rise causes a steepening of the shelf, and contributes to coastal retreat.
- The unloading pattern, due to a sea-level drop, causes a decrease in shelf gradient and an increase in the total shoreline regression.
- Hydro-isostasy is important for reconstructing stream gradients, or determining accommodation space through a sea level cycle.
- While the water depth of a paleo-shoreline gives an estimate of relative sea-level change, without an estimate of the amount of deflection at this location, eustatic sea-level change remains unknown.

