*Hölderian Regularity-Based Seismic Attributes: A Case Study from Algerian Field Data

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Abstract

The Hölderian regularity is a strong characteristic of real signals and its strength, measured by the Hölder exponent (H), indicates the degree of differentiability of the analyzed signal at a given point. In actual data, a singularity corresponds to a point where an abrupt change or pulse is observed around it. The analysis of this local H parameter allows to study its temporal and spatial evolution, and to extract meaningful information related to the nature of the studied physical process.

This study presents Hölderian regularity-based seismic attributes computed from seismic data. Here, the local regularity is quantified at each point on the seismic section using different estimators. The local regularity trace is calculated independently for each seismic trace, and then the regularity attribute slice is built by collecting all the resulting individual regularity traces.

Applications on seismic data from Algeria collected from different regions, show that the derived attributes may help identify subtle hydrocarbon traps that cannot be recognized using conventional attributes.

Keywords: Hölderian regularity, Hölder exponent, seismic data

Extended Abstract

A seismic attribute is any characteristic of seismic data that allows to visually improve or measure features of interpretation interest. A good seismic attribute is either directly sensitive to the desired geologic feature or reservoir property of interest, or allows us to define the structural or depositional environment and thereby enables us to infer some fea-tures or properties of interest (Chopra and Marfurt 2007). First introduced in the early 1970s, seismic attributes are nowadays receiving an increasing interest in reservoir characterization (Chopra and Marfurt 2007, Yoon and Farfour, 2012; Farfour et al., 2015, 2018; Greenhalgh and Irving, 2016; Farfour 2020; Gaci 2018; Gaci and Farfour, 2021).

The Hölderian regularity is a significant mathematical characteristic of real signals. Its strength is measured by the Hölder exponent (H), which indicates the degree of differentiability of the analyzed signal at a given point. In actual data, a singularity corresponds to a point where sudden change or pulse is observed around it. The analysis of this local H parameter helps us investigate its temporal and spatial evolution, and leads to a significant information describing the nature of the studied physical process. Mallat and Hwang (1992) demonstrated that the Hölder exponent can be used to distinguish smooth from abrupt discontinuities.

The local regularity analysis has been successfully applied in geophysics (Fedi et al., 2005; Gaci et al., 2010, 2011b; Gaci and Zaourar, 2010, 2011a,b,c; Gaci and Nicolis, 2021; Amoura et al., 2019; Boulassel et al., 2021). Here, it is used to suggest Hölderian regularity-based seismic attributes computed from seismic data using different algorithms.

Estimation of the local Hölderian regularity

For a stochastic process X whose trajectories are continuous but nowhere differentiable, the local Hölderian regularity exponent is given by: (Peltier and Lévy-Véhel, 1994)

$$\alpha_X(t_0) = \sup\left\{\alpha, \limsup_{h\to 0} \frac{|X(t_0+h)-X(t_0)|}{|h|^\alpha} = 0\right\}. \quad (1)$$

This parameter measures the local regularity strength of the process ant any time t_0. Geometrically speaking, that implies that the increments $X(t) - X(t_0)$ in the vicinity of t_0 are comprised by a Hölderian envelopes defined by$|X(t) - X(t_0)|^{\alpha_X(t_0)}$. The higher $\alpha_X(t_0)$ value indicates that the process is smoother at t_0, and conversely.

In the following, algorithms have been suggested to estimate local regularity from seismic data.

a. Peltier and Lévy-Véhel (PLV) algorithm

The local Hölder function *H*(*t*) at the point $t = i/(n-1)$, can be estimated by: (Peltier and Lévy-Véhel, 1995)

$$\hat{H}(i) = -\frac{\log\left[\sqrt{\pi/2}\,S_{k,n}(i)\right]}{\log(n-1)}, \quad (2)$$

based on the local growth of the increment process $S_{k,n}(i)$, given by:

$$S_{k,n}(i) = \frac{m}{n-1}\sum_{j\in\left[i-\frac{k}{2},\ i+\frac{k}{2}\right]} |X(j+1) - X(j)|,\ 1 < k < n, \quad (3)$$

where *n* is the total number of signal samples, *k* is a fixed window size, and m is the largest integer not higher than *n*/*k*.

b. Ayache and Lévy-Véhel (ALV) algorithm

Ayache and Lévy-Véhel (2004) proposed a parametrical method to evaluate the local Hölder exponent of a signal by introducing its generalized quadratic variations (GQV).

For a trajectory of the process $\{X(t)\}_{t\in[0,1]}$, discretized at the points $\frac{p}{N}, p \in 0, \dots, N-1$ with $N \geq 1$, the GQV are given by:

$$\tilde{V}_N(t) = \sum_{p\in\tilde{v}_N(t)} \left(X\left(\frac{p}{N^\delta}\right) - 2X\left(\frac{p+1}{N^\delta}\right) + X\left(\frac{p+2}{N^\delta}\right)\right)^2 \qquad (4)$$

where $\tilde{v}_N(t) = \left\{p \in \mathrm{N}; 0 \leq p \leq N-2 \text{ and } \left|t - \frac{p}{N^\delta}\right| \leq N^{-\gamma}\right\}$ can be regarded as ‘’the vicinity’’ of t.

It is proven that under some conditions imposed on the constants, γ and δ, the following relation is fulfilled.

$$\lim_{n\to\infty} \frac{1}{2\delta}\left((1-\gamma) - \frac{\log \tilde{V}_N(t)}{\log N}\right) = H(t) \qquad (5)$$

Therefore, the local Hölder exponent can be measured by:

$$H(t) = \frac{1}{2\delta}\left((1-\gamma) - \frac{\log \tilde{V}_N(t)}{\log N}\right) \qquad (6)$$

c. Bianchi algorithm

For a process $\{X(i)\}_{i=1,\ N}$, of length N and a window of length δ, the estimator suggested by Bianchi (2005) is based on a set of "moving-window" estimators of *H(t)* established on the k-th absolute moment of a Gaussian random variable of mean zero and given variance V_H . It is given by:

$$H^k_{\delta,N}(t) = \frac{\log\left(2^{k/2}\Gamma\left(\frac{k+1}{2}\right)V_H^{k/2}\right) - \log\left(\frac{\sqrt{\pi}}{\delta}\sum_{j=z-\delta}^{z-1}\left|X_{j+1,N} - X_{j,N}\right|^k\right)}{k\log(N-1)} \qquad (7)$$

$$\text{for } j = t-\delta, \dots, t-1; t = \delta+1, \dots, N; k \geq 1 \qquad (8)$$

Methods

The efficiency of the suggested local regularity-based seismic attribute is demonstrated on preserved seismic data recorded in some Algerian basins.

The seismic line was recorded in southeastern Algeria (Figure 1) intersects by the well W1, that has encountered a gas discovery at the targeted reservoir at a 2.3s two way time (TWT) and corresponding to CDP No 235.

The PLV, ALV and Bianchi algorithms have been used to compute seismic regularity-based attributes from the considered seismic line. The results are presented in Figures 2 to 4.

Regarding the first seismic line, as it can be noticed, all the estimators lead to a local regularity attribute slices showing low-amplitude values of the Hölder exponent (H) for the producing formation at the well W1, and some bright spots, indicated by circles (in dashed lines): A-K. They might be associated with a probable presence of gas in the marked locations.

As it can be seen in Figure 3, the local regularity attribute slice, obtained using PLV, highlights the anomalies A–K, and shows that the spots B and E are associated with the lowest H values. The spots A, B, C, D and J might be originated from their locations on the producing formation (TWT: 1.9–2.3 s), the anomaly F can be attributed to a possible accumulation of gas due to its closeness to well W2, while the spots E, G, H and I might correpond to a new producing formation. As regards for the local Hölder exponent (H) attribute section calculated using ALV algorithm (Figure 4), the anomalies show up less clearly except for the spots (B, D, F and I), that present reltively the lowest H values, while on the attribute section derived using Bianchi algorithm presented in Figure 5, more spots (B-D, and F-K) are visible.

Conclusions

Hölderian regularity attributes have been computed from seismic data using PLV, ALV and Bianchi algorithms. These attributes allow to confirm the gas discovery at the the targeted reservoir by the lowest local regularity values or the lowest Hölder exponents.

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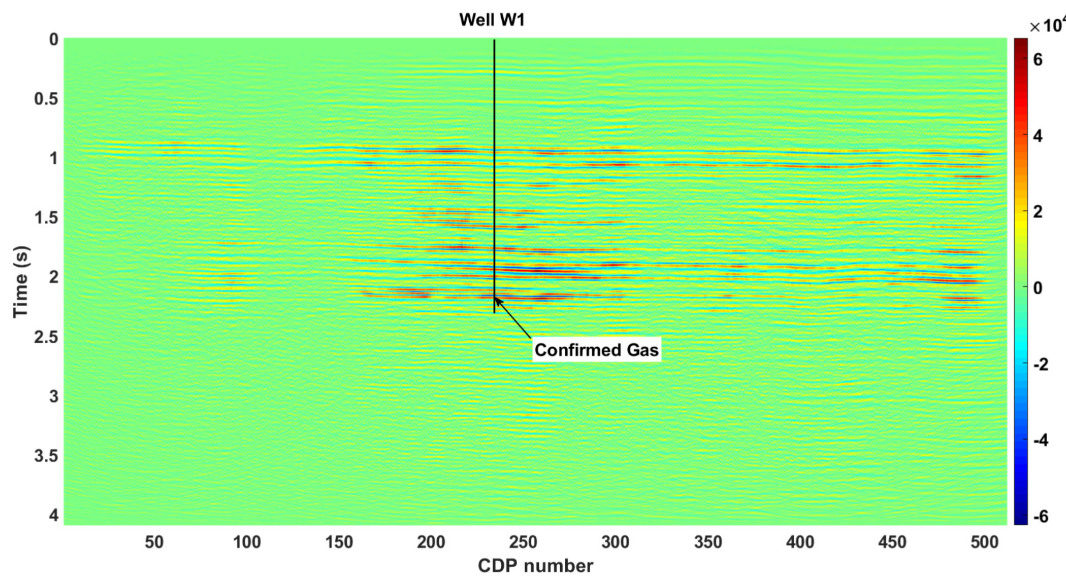

Figure 1. Seismic line acquired from southeastern Algeria, intersected with an exploration well W1 (at CDP 235). A gas discovery is at the principal reservoir at TWT= 2.3 s.

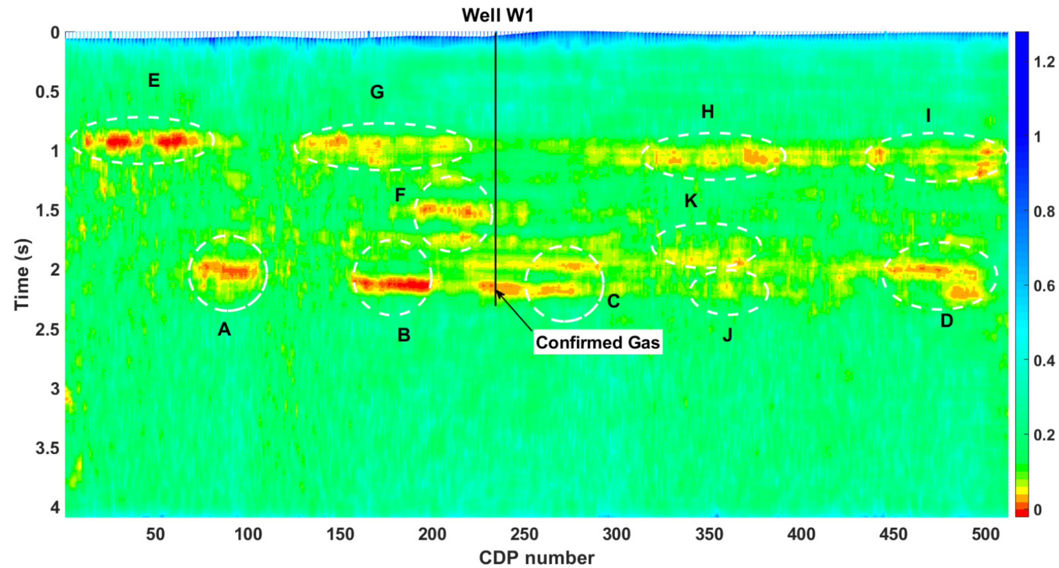

Figure 3. Seismic regularity attribute computed from the seismic line 1 using PLV algorithm. The gas discovery at well W1 at the targeted reservoir corresponds to a low Hölder exponent value. Other anomalies indiacted by dashd circles: A to K, associated with low Hölder exponent attribute values, may be related to hydrocarbon traps.

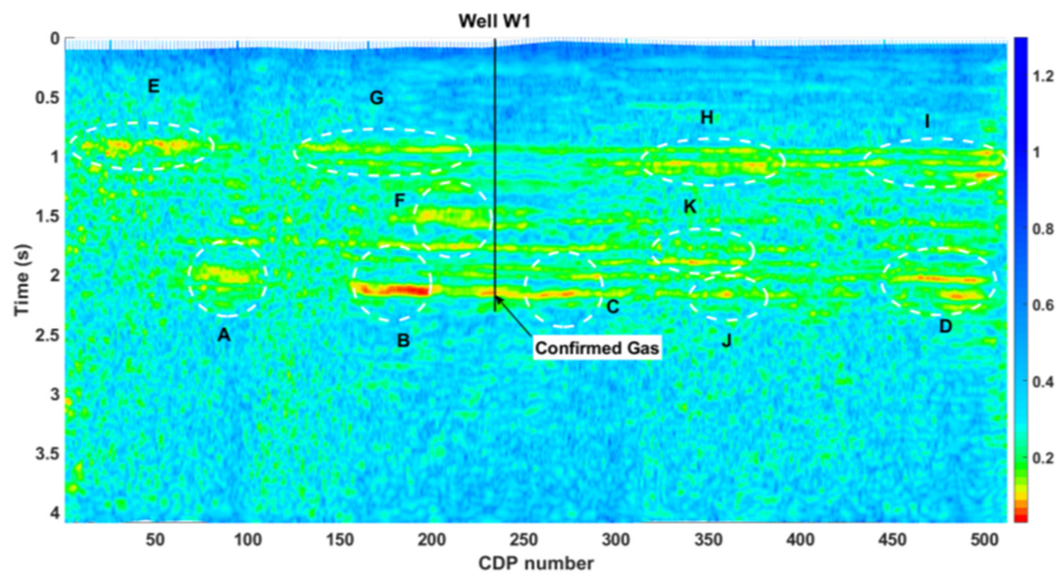

Figure 4. Seismic regularity attribute computed from the seismic line 1 using ALV algorithm. The gas discovery at well W1 at the targeted reservoir corresponds to a low Hölder exponent value. Other anomalies indiacted by dashd circles: A to K, associated with low Hölder exponent attribute values, may be related to hydrocarbon traps.

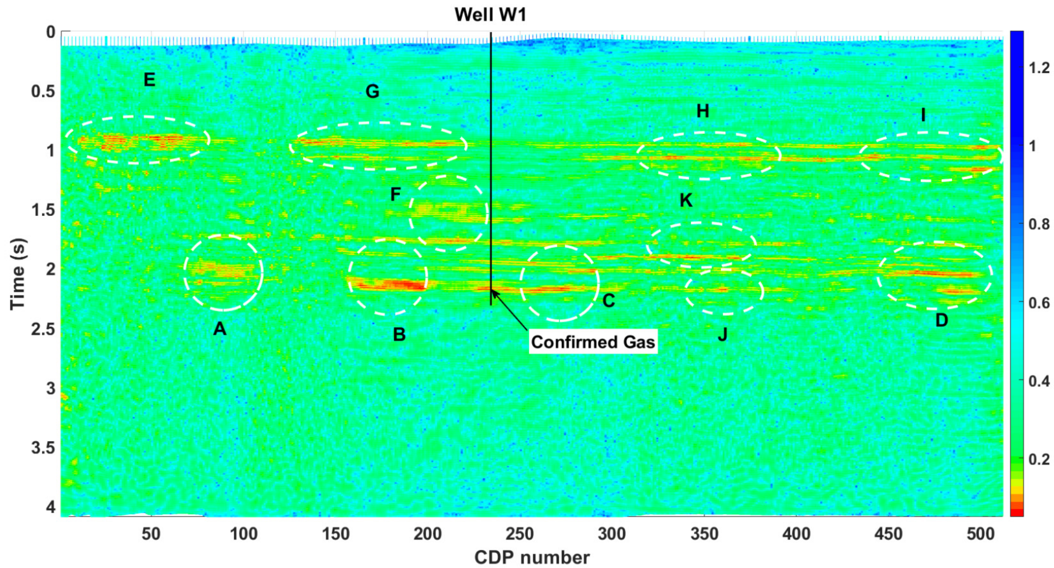

Figure 5. Seismic regularity attribute computed from the seismic line 1 using Bianchi algorithm. The gas discovery at well W1 at the targeted reservoir corresponds to a low Hölder exponent value. Other anomalies indiacted by dashd circles: A to K, associated with low Hölder exponent attribute values, may be related to hydrocarbon traps.