

## **On the Extraction of Seismic AVF Information and the Inversion of Anelastic Reflectivity**

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### **Summary**

Strongly dispersive reflection coefficients associated with highly absorptive, hydrocarbon charged targets, have been observed in seismic data. A seismic AVF (Amplitude variation with frequency) inversion method for determining  $Q$  of a highly absorptive target from measurements of the dispersive reflection coefficient is reviewed in this paper. In order to implement the AVF technique to invert for  $Q$ , it is necessary that we have a method of estimating the local spectrum of the reflection coefficient. We develop a method of implementing AVF inversion by using a calibrated, fast, non-redundant S-transform (FST) algorithm to estimate the local spectrum of dispersive reflection coefficients associated with a highly absorptive target. Using forward modeling, we test the effectiveness of the FST for estimating the spectrum of the frequency dependent reflection coefficients by comparing with the analytic reflection coefficient.

### **Introduction**

Absorption is the progressive decay of the highest frequencies of a seismic pulse as it travels through Earth material (O'Doherty and Anstey, 1971). The effect is that the peak amplitude decays and the pulse becomes broader (O'Doherty and Anstey, 1971). The quality factor,  $Q$ , is a measure of how absorptive an Earth material is; a low  $Q$  value corresponds to a highly absorptive material. The ability to determine  $Q$  presents an important problem in exploration seismology because absorption is closely related to rock fluid properties such as viscosity, porosity and fluid saturation (Odebeatu et al., 2006; Quan and Harris, 1997; Vasheghani and Lines, 2009). Absorption is not easy to measure in the field as it is difficult to isolate the effect of absorption on the seismic pulse from other attenuation mechanisms (Sheriff and Geldart, 1995) such as geometrical spreading and scattering. However, methods of determining  $Q$  exist such as the centroid frequency method which uses the shift in average frequency between two points to estimate  $Q$  (Quan and Harris, 1997).

It has been observed that strong absorptive reflection coefficients caused by reservoirs with very low  $Q$ , cause frequency dependent seismic anomalies (Odebeatu et al., 2006). In this paper a frequency-by-frequency (AVF) method of inverting for  $Q$  from absorptive reflections is reviewed and a method of implementing this technique using a fast S-transform algorithm (FST) is proposed. The method is tested using synthetic traces modeled with a single absorptive reflection.

### **Theory and Method**

If a plane wave is incident upon a planar boundary, oriented perpendicularly to  $z$ , and separating two media then the reflection coefficient will be given by

$$R = \frac{k_{z1} - k_{z2}}{k_{z1} + k_{z2}} \quad (1)$$

Where  $R$  is the reflection coefficient,  $k_{z1}$  and  $k_{z2}$  are the vertical wavenumbers for the media above and below the interface respectively. In order to model absorptive reflection coefficients, we use an expression for wavenumber which includes a model for nearly constant  $Q$  described by Aki and Richards (2002) as

$$k = \frac{\omega}{c} \left( 1 + \frac{i}{2Q} - \frac{\log\left(\frac{\omega}{\omega_r}\right)}{\pi Q} \right) \quad (2)$$

Here  $c$  is the seismic velocity,  $Q$  is quality factor,  $\omega$  is the frequency component, and  $\omega_r$  is a reference frequency. For waves at normal incidence the expression for wavenumber given in equation (2) may be used as the vertical wavenumber,  $k_z$ , and implemented in equation (1). Now consider the situation where an elastic overburden, with velocity  $c_1$ , is overlaying a highly attenuative target (with velocity  $c_2$  and quality factor  $Q$ ), then we may write the expression for anelastic reflection coefficient as

$$R(\omega) = \frac{1 - \Omega(\omega)}{\Omega(\omega)}, \text{ where } \Omega(\omega) = \frac{c_1}{c_2} \left[ 1 + \frac{i}{2Q} - \frac{\log\left(\frac{\omega}{\omega_r}\right)}{\pi Q} \right] \quad (3)$$

Now if we make the following substitutions (e.g., Innanen and Lira, 2008)

$$F(\omega) = \frac{i}{2} - \frac{\log\left(\frac{\omega}{\omega_r}\right)}{\pi}, \text{ where } a_Q = \frac{1}{Q} \text{ and } a_C = 1 - \frac{c_1^2}{c_2^2} \quad (4)$$

We can expand equation (3) and linearize (assuming small  $a_Q$  and  $a_C$ ) to obtain the following expression for anelastic reflection coefficients

$$R(\omega) = -\frac{1}{2} a_Q F(\omega) + \frac{1}{4} a_C \quad (5)$$

where  $a_Q$  and  $a_C$  are the perturbation parameters in  $Q$  and acoustic seismic velocity respectively. It is also worth noting that  $F(\omega)$  is a known function. Equation (5) defines the forward problem of calculating the reflection coefficient,  $R(\omega)$ , given  $a_Q$  and  $a_C$ . The AVF inverse problem is to determine  $a_Q$  or  $a_C$  from measurements of  $R(\omega)$ . In this paper, we wish to obtain reliable measurements of  $R(\omega)$  in order to determine  $a_Q$ . If we can determine  $R(\omega)$  for two different frequencies,  $\omega_1$  and  $\omega_2$ , then we can take the difference between  $R(\omega_1)$  and  $R(\omega_2)$  and obtain an expression for  $a_Q$  given by

$$a_Q = -2 \left( \frac{R(\omega_2) - R(\omega_1)}{F(\omega_2) - F(\omega_1)} \right) \quad (6)$$

Equation (6) provides an expression which solves for  $a_Q$  provided we have exact knowledge of  $R(\omega)$  for at least two different frequencies. The problem is how to determine  $R(\omega)$  from recorded seismic data. It is necessary that we be able to estimate the local spectra of seismic reflection events. We have a calibrated

fast S-transform algorithm (FST) which our testing indicates offers high fidelity estimates of local spectra (see Bird et al., 2010).

In this paper, we define  $\tilde{R}(\omega)$  as the estimate of the spectrum of the true reflection coefficient,  $R(\omega)$ , which we obtain using the calibrated FST algorithm. We modify equation (6) to take in  $\tilde{R}(\omega)$  as input (for details see Bird et al., 2010). We use forward modeling to generate synthetic traces with an absorptive reflection and then implement the FST to estimate its spectrum. We compare our estimate of the spectrum using the FST with the analytic spectrum. Finally, we invert for  $a_Q$  using the modified version of equation (6).

## Results

We implement our calibrated FST to invert for Q by generating synthetic seismic traces with a single absorptive reflection, and using the FST to estimate the spectrum of the reflection. The traces are generated for a two-layer, single interface model in which an elastic overburden overlays a highly attenuative target. We use the FST to estimate the spectrum of the reflection and then implement our modified expression of equation (6) to invert for Q. A number of traces were modeled in which the target Q ranged from 200 to 1, the inversion was performed and compared with the actual value for accuracy. This is shown in Figure 1 where the red line is the inverted Q value and the black line is the actual Q value used to model the reflection. From these figures, it appears that the inversion works best for Q values ranging from about 15 to 50 but works satisfactorily until Q drops below 8.

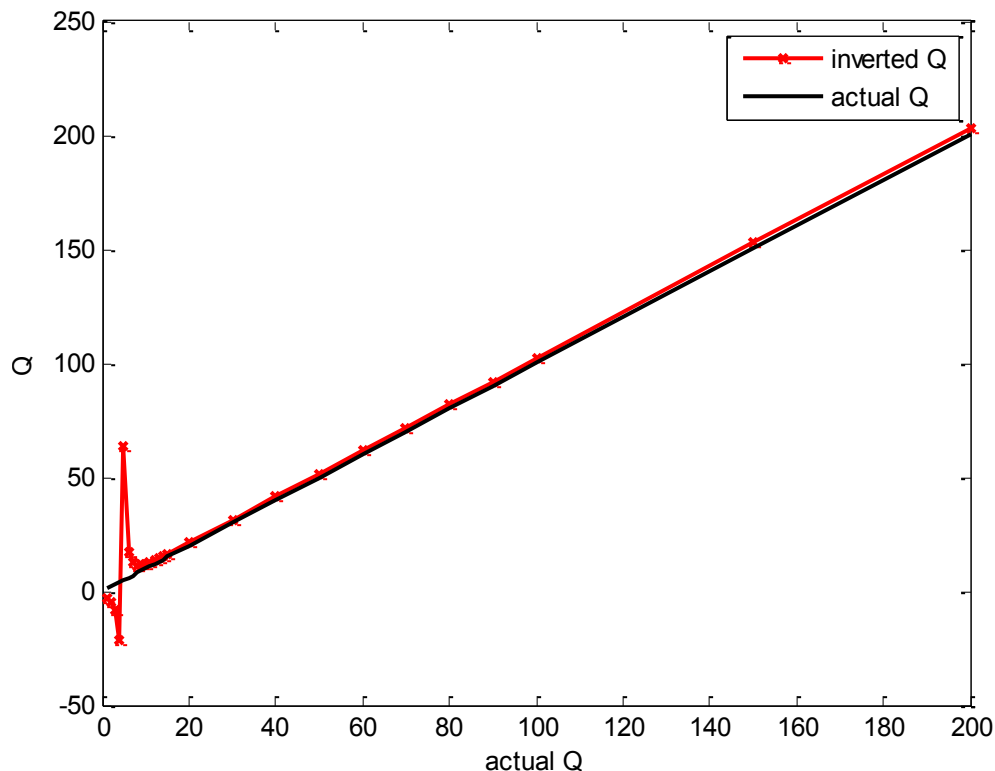


Fig. 1. This figure shows the accuracy of the AVF inversion. The red line indicates the inverted Q value and the black line shows the actual Q value used in the modeling.

## Conclusions

We conclude that the calibrated FST is a promising time-frequency decomposition tool which may be used to extract seismic AVF information. The FST was shown to produce a very good approximation to the average of the spectrum of the analytic reflection coefficients over the frequency bands of the FST algorithm. For a situation where an elastic overburden overlays a highly attenuative target, which has been observed for some gas charged reservoirs (Odebeatu et al., 2006), an expression for inverting for Q using the FST to estimate the spectrum of the reflection was presented. Synthetic traces for a single absorptive reflection were generated and Q was inverted for and it was shown that the inverted values were in close agreement with the true values, except for very low  $Q(<8)$ . Partly this is due to linearization error.

Moving forward we will extend this method of extracting AVF information to more complicated ( and realistic) modeled traces; such as including noise, a source wavelet, and traces with numerous reflection events. We will extend this method into the offset domain investigate aperture limitations. Finally, we will attempt our method on field data. Further, we can apply a non-linear correction to equation (6) to improve the accuracy of the inversion.

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