

The Theory Behind the Carman-Kozeny Equation in the Quest for Permeability of the Rocks

Carlos Haro¹

¹*Occidental Oil & Gas, Houston, USA*

Abstract

Permeability is one of the most fundamental petrophysical parameters in petroleum engineering calculations. The Carman-Kozeny equation has been used to obtain permeability (k). This equation results from the mixture of Darcy's and Poiseuille's laws. While Darcy's law macroscopically quantifies fluid flow, Poiseuille's law describes the parabolic displacement of a viscous fluid in a straight-circular tube. This denotes a common scheme in hydraulics (like the boundary layer theory) that, for the sake of simplification, solves separately for inviscid (zero viscosity) and viscous effects. Yet, in some cases this is only a good approximation (White 2003).

Darcy's law, intended for hydraulic systems, has been catalogued as equivalent to Ohm's law, best suited for electric problems. Ohm's law has been proven insufficient in porous media, because it is correct only for the straight paths of the electric current, not for the meandering routes encountered in the rocks (Haro 2010). Thus, by similarity, Darcy's law also requires modification to compensate for the tortuous paths. It should be derived by use of the streamlines that describe the hydraulic flow distribution within the porous network. This law should invoke a corner-angle solution of the Laplace differential equation, to include an exponent (A), associated with the corresponding cross section, that considers the velocity variations in direction caused by the hydraulic tortuosity of the rock (White 2003).

The mathematical similarity between Darcy's and Ohm's laws permits us to connect Carman-Kozeny's and Archie-Haro's (modified Archie's equation, Haro 2010) equations, which are also mathematically analogous (Haro 2004, 2006). This means that resistivity log measurements can help us find the unknowns in Carman-Kozeny's equation. However, the electric tortuosity exponent (m) differs from the hydraulic tortuosity exponent (A) because of fluid viscosity and the inertial forces created by the intricacy of the rock. Although a rough estimate might be $(m + 2)$, hydraulic tortuosity is mainly uncertain because flow is not always laminar Darcy flow everywhere within the pores. In fact, hydraulic tortuosity is the main unknown to find k .

This theory in conjunction with a database approach (lookup table) facilitates enhanced permeability modeling (Haro 2004, 2006). This method enables the creation of static and dynamic synthetic production logs, which can assist effectively in fluid allocation, production forecasting, history matching, and well completion (Haro 2012). These are critical variables to succeed in reservoir management.