Ground Roll Polarization Filtering with Spatial Smoothness Constraints

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Summary

This paper describes the theoretical basis for polarization filters with strong spatial constraints. In essence this means that both the amplitude and polarization properties of the ground roll at any receiver location are estimated using data from that station as well as some of its neighboring stations. This greatly improves the separation of signal and noise, resulting in better preservation of reflected P-wave and S-wave energy in the filtered data.

Introduction

For some years now polarization filters have been recognized as a useful tool for removing ground roll energy from multi-component data. In principle, any filter that estimates both the amplitude and polarization properties of some signal in order to remove it from three-component (3C) data can be called a polarization filter. There are however variations in how polarization filters model these properties. In the case of ground roll this greatly affects the preservation of reflected P-wave and S-wave energy. Using a complex-numbered vector representation for multi-component data this paper discusses three alternative polarization filter implementations. For the simple case of a single station 3C filter we show that some desired energy will be removed from the data. This definition is then extended to introduce weak spatial constraints and then strong spatial constraints. Both of which significantly reduce leakage of reflected energy into the noise estimate.

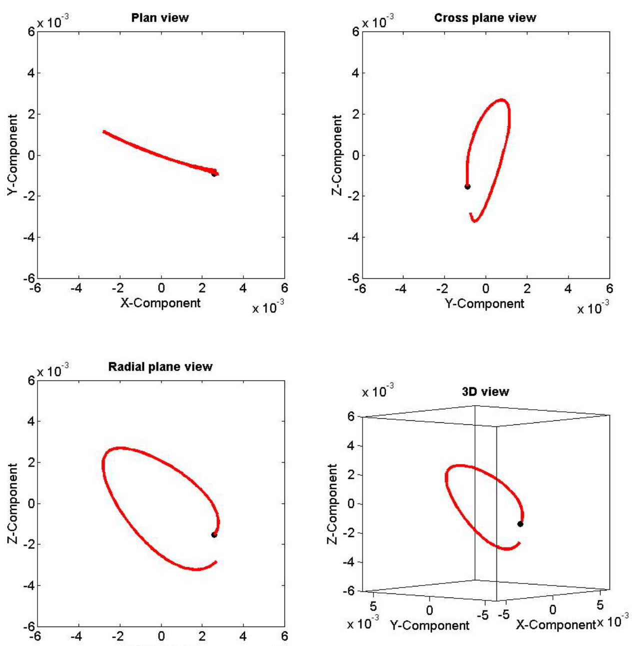
Basic Polarization Filtering Theory

Any 3C trace can be written as a time-dependent vector $\mathbf{d}(t) = \left[x(t) \quad y(t) \quad z(t)\right]^T$, where x(t), y(t) and z(t) are the recorded seismograms in the radial, transverse and vertical directions. Any vector $\mathbf{d}(t)$ is also defined by its length $a(t) = \left|\mathbf{d}(t)\right|$ direction $\mathbf{v}(t) \,||\, \mathbf{d}(t)$ such that $\mathbf{d}(t) = a(t)\mathbf{v}(t)$ and $\left|\mathbf{v}(t)\right| = 1$. Amplitude can be used as a synonym for length, whereas polarization serves as a synonym for direction. The data $\mathbf{d}(t)$ that is measured on a 3C receiver can also be seen as the sum of P-wave $\mathbf{p}(t)$ and S-wave $\mathbf{s}(t)$ energy as well as surface waves $\mathbf{g}(t)$ and ambient noise $\mathbf{n}(t)$. Each phase has their proper amplitude and polarization behavior so that we get:

$$\begin{aligned} \mathbf{d}(t) &= \mathbf{p}(t) + \mathbf{s}(t) + \mathbf{g}(t) + \mathbf{n}(t) + \ldots \\ \mathbf{d}(t) &= a_p(t)\mathbf{v}_p(t) + a_s(t)\mathbf{v}_s(t) + a_g(t)\mathbf{v}_g(t) + a_n(t)\mathbf{v}_n(t) + \ldots \end{aligned} \tag{1}$$

In addition to this time-domain definition of the 3C data-vector one can also define it in the frequency domain using the spectra, or define it using the analytic signal of the recorded traces. In either case we obtain a complex-numbered 3-element data-vector that varies with frequency (spectral definition) or time (analytic signal definition). The main advantage of the complex numbered analytic signal definition is that the 3C data can be described in terms of amplitude and phase variations as a function of time and component. Using a real numbered approach we would only be able to describe the 3C data in terms of amplitude variations with time and component. Handling both amplitude and phase information is particularly useful when dealing with ground roll. In the simple case, ground roll energy appears as an elliptical particle motion in the vertical radial plane. Surface anisotropy, layering and scattering may cause a tilting or perturbation of the ellipse, but in general, the elliptical ground roll model is a pretty good assumption. The main consequence of this elliptical motion is that the ground roll energy recorded on either component is simply a scaled and phase shifted copy of the other components. The use of a complex numbered parameterization makes sense in this case. Another distinction between real and complex numbered polarization vectors is that real vectors define directions, or linear polarizations whereas complex vectors can define elliptical as well as linear polarizations. Linear polarizations can be treated as ellipses with zero eccentricity and are therefore a subset of elliptical polarizations.

Figure 1 compares a real 3C example of ground roll with a synthetic 2D example of ground roll. The synthetic example was generated using the complex vector model just described for analytic signals. Vidale (1986) shows that a complex polarization vector $\mathbf{v}_g(t)$ can be represented by a real ellipse (green) and a complex scalar $a_g(t)$. This complex scalare can be decomposed into an instantaneous phase angle (blue) and amplitude (cyan). The particle position is found by multiplying the ellipse axis at a phase angle away from the semi-mayor axis with the instantaneous amplitude.



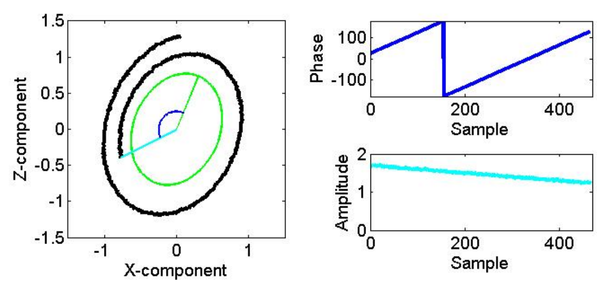


Figure1: *(left)* Example of ground roll particle motion taken from a real 3C dataset. Motion is approximately elliptical and confined to the radial plane. In this example some motion appears in the cross/transverse plane, probably as a result of near surface effects. *(top)* Modeled 2C ground roll particle motion. 3C ground roll motion can be modeled as a straightforward extension of the 2C case. The particle motion is explained using the complex vector model $\mathbf{g}(t) = a_g(t)\mathbf{v}_g(t)$. The complex polarization vector $\mathbf{v}_g(t)$ can be represented by a real ellipse (green) and the complex scalar $a_g(t)$ can be decomposed into an instantaneous phase angle (blue) and amplitude (cyan).

From Equation 1 we learn that any ground roll polarization filter has to estimate both the ground roll amplitudes $a_g(t)$ and polarization properties $\mathbf{v}_g(t)$ in order to remove it from the 3C data. The synthetic example in Figure 1 indicates that it is possible to model ground roll in terms of a stationary elliptical polarization and a rapidly changing complex numbered amplitude. The stationarity assumption for the polarization ellipse is quite flexible. On real data the polarization properties have to be stationary only over short time-windows (few noise periods), but are allowed to vary slowly with time. If these assumptions are met and if the ground roll energy represents more than 50% of the recorded data then it is possible to estimate the polarization ellipse from the data. If $\mathbf{D}_t$ is an N by 3 matrix that contains the data $\mathbf{d}(t)$ over a sliding time-window $t \in \left[t_{-N/2} \quad t_{N/2}\right]$ then the ground roll polarization vector estimate $\tilde{\mathbf{v}}_g(t) \approx \mathbf{v}_g(t)$ can be found as the first eigenvector resulting from a Singular Value Decomposition (SVD) on $\mathbf{D}_t$, or a Principal Component Analysis $\mathbf{D}_t^H \mathbf{D}_t$ with *H* indicating the Hermitian transpose. The ground roll amplitudes can now be found by projecting the data onto the polarization vector so that $\tilde{a}_g(t) = \tilde{\mathbf{v}}_g^H(t)\mathbf{d}(t)$. This projection to estimate the amplitudes of the signal of interest is the weak link in many polarization filters. If we take Equation 1 and combine this with the definition of $\tilde{a}_g(t)$ we find:

$$\tilde{a}_g(t) = \tilde{\mathbf{v}}_g^H(t)\mathbf{d}(t) = a_p(t)\tilde{\mathbf{v}}_g^H(t)\mathbf{v}_p(t) + a_s(t)\tilde{\mathbf{v}}_g^H(t)\mathbf{v}_s(t) + a_g(t)\tilde{\mathbf{v}}_g^H(t)\mathbf{v}_g(t) + a_n(t)\tilde{\mathbf{v}}_g^H(t)\mathbf{v}_n(t) \quad (2)$$

From this we find that even if the ground roll polarization estimate is exact ($\tilde{\mathbf{v}}_g(t) = \mathbf{v}_g(t)$ and $\tilde{\mathbf{v}}_g^H(t)\mathbf{v}_g(t) = 1$) the ground roll amplitude estimate $\tilde{a}_g(t)$ will be contaminated with P-wave energy and S-wave energy unless $\tilde{\mathbf{v}}_g^H(t)\mathbf{v}_p(t) = 0$ and $\tilde{\mathbf{v}}_g^H(t)\mathbf{v}_s(t) = 0$. In a typical example where the P-waves and S-waves are polarized vertically and radially, and where the ground roll is polarized elliptically in the vertical radial plane this condition is *never* met. In other words: any polarization filter where the noise amplitudes are estimated by projecting the 3C data from a single receiver onto the noise polarization vector removes part of the desired signal.

Spatially Constrained Polarization Filtering Theory

Spatially constraining the polarization filter may offer a solution to this problem. De Meersman and Ansorger (2007) used an extension where the sliding data-window $\mathbf{D}_t$ becomes an N by 3M matrix, with M the number of receiver stations. If the station of interest is positioned at the center of the data-window and if we thake the case of M=3 then we find:

$$\mathbf{d}(t) = \begin{bmatrix} \mathbf{d}_1(t+\tau_1) \\ \mathbf{d}_2(t+\tau_2) \\ \mathbf{d}_3(t+\tau_3) \end{bmatrix} = a(t)\mathbf{v}(t) \qquad \text{and} \qquad \tilde{a}_g(t) = \tilde{\mathbf{v}}_g{}^T\mathbf{d}(t). \quad (3)$$

The values τ_i are time-shifts so that the data from each receiver station $\mathbf{d}_i(t+\tau_i)$ is approximately corrected for ground roll moveout. Typically no moveout correction is applied to data for the station at the window centre so that τ_2=0. The 3M-element polarization vector $\tilde{\mathbf{v}}_g(t)$ can be found in just the same way as for the single-station case by performing an SVD on $\mathbf{D}_t$. Each 3-element subvector in $\tilde{\mathbf{v}}_g(t)$ defines the polarization ellipse at its respective receiver station. When we look at the definition of the estimated noise complex amplitudes $\tilde{a}_g(t) = \tilde{\mathbf{v}}_g{}^T\mathbf{d}(t)$ we find that this is in fact a weighted 3C stack of the moveout corrected data from stations in the sliding window. The data has been moveout corrected for ground roll so that ground roll energy adds up constructively and P-wave and S-wave energy interfere destructively. The advantage of this is clear: we obtain an estimate of the ground roll with an improved separation between wanted and unwanted signal compared to a single-station polarization filter aproach. This method is referred to as polarization filtering with weak spatial constraints.

In contrast to this (and to the original filter without any spatial constraints), a more robust extension of the method is presented in this paper and is referred to as a polarization filter with strong spatial constraints. The weak spatial constraints allow a different solution for the polarization ellipse for each receiver station in the window. The strong spatial constraint imposes a solution where the polarization ellipse is the same at each receiver station. This constraint makes sense if the distance between the receiver stations is less than 1-2 noise wavelengths since we don't expect the polarization properties to change rapidly between receiver stations over distances this small. Strong spatial constraints only allow for (1) a constant phase angle difference and (2) a constant scaling difference between stations in the data window. Small constant phase angle differences can accommodate small time-shifts between the stations in the data-window in cases where there is residual ground roll moveout after correction. Constant scaling differences between the polarization ellipses at different locations are true amplitude processing compliant. This allows the modelling of receiver and offset dependent amplitude variations. Figure 2 contains a graphical comparison of spatially constrained polarization filters for a 2C 3-station case.

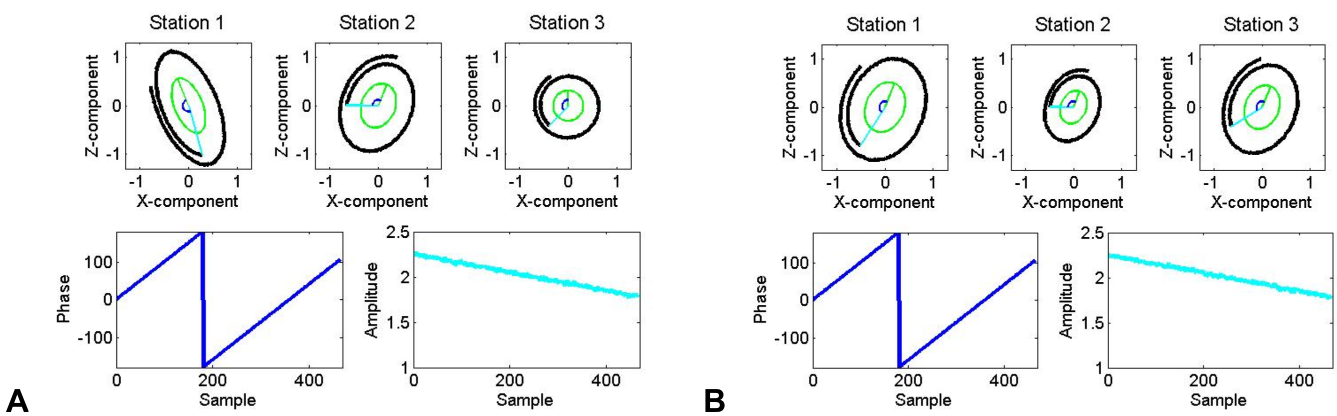


Figure 2: *(A)* 2C 3-station example of a polarization filter with weak spatial constraints. The data particle motion at the station of interest (station2) is estimated from the 'raw' data using a model where the polarization ellipse varies between stations and where instantaneous amplitudes and phases are shared between stations.

(B) 2C 3-station example of a polarization filter with strong spatial constraints. Here the particle motion at the station of interest is estimated from the 'raw' data using a model where the polarization ellipses and instantaneous amplitudes and phase angles are the same for all three stations. This model only allows a constant amplitude and scaling difference between the stations.

Conclusions

In this work we explain how 3C polarization filters for ground roll work by estimating both the amplitude and polarization properties. If the amplitude properties of the noise are obtained by projecting the 3C data from a single receiver onto the estimated polarization vector then the noise estimate is contaminated by leaked P-wave and S-wave energy. Hence, a single receiver polarization filter typically removes some desired signal energy. Signal leakage can be reduced significantly by using a spatially constrained polarization filter. Two alternative models to achieve this have been demonstrated. In both cases the amplitude behavior at the station of interest is estimated using the data from that station as well as some neighboring stations. In the case of strong spatial constraints the polarization properties at the station of interest are also estimated using information from neighboring receivers.

References

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