The Application of The Nearly Optimal Sponge Boundary Conditions for Seismic Wave Propagation in Poroelastic Media

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Summary

For numerical simulation of seismic wave propagation in elastic media, absorbing boundary conditions (ABC) play an important role in eliminating artificial reflections from the edges of the seismic model. In this paper, we present a modified nearly optimal sponge boundary condition. Using a measure of reflected wave energy it is possible to construct a contour map for a range of sponge absorption coefficients and numbers of grid points at tapered zone, and determine the best set of parameters to minimize this energy metric. We apply this optimal scheme to the numerical simulation of seismic wave propagation in 2-D transversely isotropic poroelastic media using staggered-grid finite-difference operator. Modeling studies indicate remarkably good results, the nearly optimal sponge boundary conditions is simple and effective enough to eliminate the artificial reflections from the boundaries of the poroelastic model.

Introduction

During the last 30 years, numerous techniques have been developed for dealing with the problem of existing artificial reflections mentioned above: absorbing layers and absorbing boundary conditions (ABC). In the context of absorbing layers, Bérenger (1994) introduced a technique called the perfectly matched layer (PML) that has the remarkable property of generating no reflection at the interface between the free medium and the artificial absorbing medium. This method has been proven to be more efficient and has become widely used. However, this method needs larger memory storage.

Absorbing boundary conditions (ABC) are an alternative to absorbing layers. Bording's paper (Bording, 2004) further discussed boundary condition by minimizing wavefield energy and optimizing the parameters of absorption coefficient and the width of tapered zone. Here, we will apply modified Bording's method to the numerical modeling of seismic wave propagation in the transversely isotropic poroelastic media based on Biot/squirt flow theory.

Since the Biot theory (Biot, 1962) does not adequately describe the wave attenuation and dispersion, a consistent theory dealing simultaneously with Biot and squirt-flow (BISQ) mechanisms has been presented by Dvorkin and Nur (1993). Biot/Squirt equations (Yang and Zhang, 2002) are reformulated into a first-

AAPG Search and Discovery Article #90171 CSPG/CSEG/CWLS GeoConvention 2009, Calgary, Alberta, Canada, May 4-8, 2009

order system whose vector of unknowns consists of the solid and fluid particle velocity components, the solid stress components, and the fluid pressure. For the implementation of the staggered-grid finite-difference method, eighth-order accuracy in space and second-order accuracy in time are used, and a harmonic average scheme is applied for effective media parameters. The results of modeling test show us that this method is effective to eliminate the reflections from the edges of model after using the optimal operator length and absorption coefficient.

Biot/Squirt Theory

Yang and Zhang (Yang and Zhang, 2002) extend the Biot/Squirt theory to include the solid/fluid coupling anisotropy and develop a general 3D poroelastic wave equation in time domain including both mechanisms simultaneously.

$$\sum_{j=1}^{3}\frac{\partial \tau_{ij}}{\partial x_j} = \frac{\partial^2}{\partial t^2}(\rho u_i + \rho_f w_i) \tag{1}$$

$$-\frac{\partial P}{\partial x_i} = \frac{\partial^2}{\partial t^2}(\rho_f u_i + m_{ii} w_i) + \frac{\eta}{\kappa_{ii}}\frac{\partial w_i}{\partial t} \tag{2}$$

where $i, j = 1,2,3$. The parameters describing the physical properties of the medium are defined as follows:

τ_{ij} denotes the total stress component of the bulk material; u_i is the displacement component for solid; $w_i = \phi(U_i - u_i)$ represents the displacement component for the fluid relative to that for the solid; U_i is the displacement component for fluid; ϕ is the porosity; ρ is the overall density of the saturated media determined by $\rho_1 + \rho_2$, $\rho_1 = (1-\phi)\rho_s$ and $\rho_2 = \phi\rho_f$; ρ_s and ρ_f are the density of the fluid and the solid, respectively; P denotes fluid pressure; η represents the viscosity of the pore fluid; m_{ii} denotes the component of coefficient constant; κ_{ii} denotes the permeability of the matrix, these two are 3×3 diagonal matrixes.

The expression of the stress-strain relation in terms of the effective stress or total stress is defined as

$$\tau = Ae - \alpha P \tag{3}$$

where $\tau = (\tau_{ij})$ indicates the total stress tensor of the saturated porous medium, A denotes the solid-frame stiffness tensor containing 21 independent drained elastic coefficients for general anisotropic media, $\alpha = (\alpha_{ij})$ is the poroelastic coefficient tensor of the effective stress, $e = (e_{ij})$ represents the strain tensor of the porous medium, $e_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$, $i, j = 1,2,3$.

They also provide the expression of fluid pressure P in 3D in their paper(Yang and Zhang, 2002).

The implementation of staggered-grid finite-difference is described in Zeng and Liu (2001). The source time function used in this paper is a band-limited Ricker wavelet, which is distributed over a small region of the grid. The source frequency we select is 25Hz.

Boundary Condition

In Bording's paper (2004), to determine the best parameters for this implementation of the elastic wave equation, the operator length and absorbing coefficient are varied over a range of values, we can construct an energy metric and determine the best set of parameters to minimize this metric. By summing the absolute value of the wave field along x and z direction, respectively, a measure of the total wave-field energy is computed separately. The metric used is similar to the L_2 norm over the domain. In this paper, we draw three lines along x and z direction, respectively.

Examples

We select Taylor sandstone with water-saturated as solid HTI media (Sedimentary rock) based on data from Thomsen (1986). All the physical properties of the Taylor sandstone with water-saturated used in models are given in Table 1.

The size of the model with water-saturated porous media is $N_x \times N_z = 240 \times 240$ nodes. The spatial grid spacings are 10m and 10m whereas the time step is 1ms. The explosive source with 25Hz of dominant frequency is located at the center-grid position of the model. We select three horizontal lines completely across the model along x and z direction, the grid nodes are 80, 120, and 160 at two directions. When seismic wave propagates 360ms after the source is generated, we compute the total wave-field energy along the three lines separately. Figure 1 shows the configuration of the lines where the wave-field energy is measured. In Figure 1, the solid lines represent the measured lines, and the zones between dash lines and model boundary denote the tapered zone. The contour plot of the metric shown in figure2, figure 2a denotes the measurement of wave field energy along z direction, figure 2b denotes x direction. The cross symbol denotes the optimal parameters. Figure 2 clearly indicates an optimal operator length and exponential weight at 25 and 0.0007 along z direction, 22 and 0.0009 along x direction.

Table 1. All physical properties (rock and fluid) of Taylor sandstone

Properties	Solid	Fluid
V_p (m/s)	3368	1500
V_s (m/s)	1829	—
ρ (g/cm3)	2.500	1.000
ρ_{ax}, ρ_{az} (g/cm3)	0.300, 0.33	—
ϕ (%)	19	—
K_s (Pa)	12.402E+9	—
η (Pa • s)	—	1.000E-3
β	0.4	
k_{11}, k_{33} (md)	100, 100	—
R_x, R_z (mm)	—	5, 5

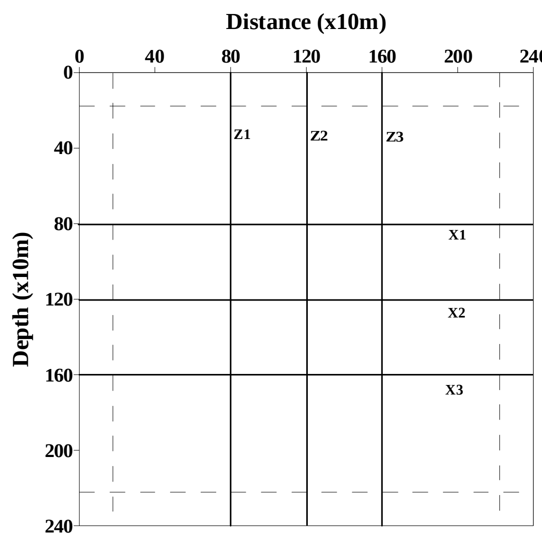

Figure 1. The configuration of the measured lines.

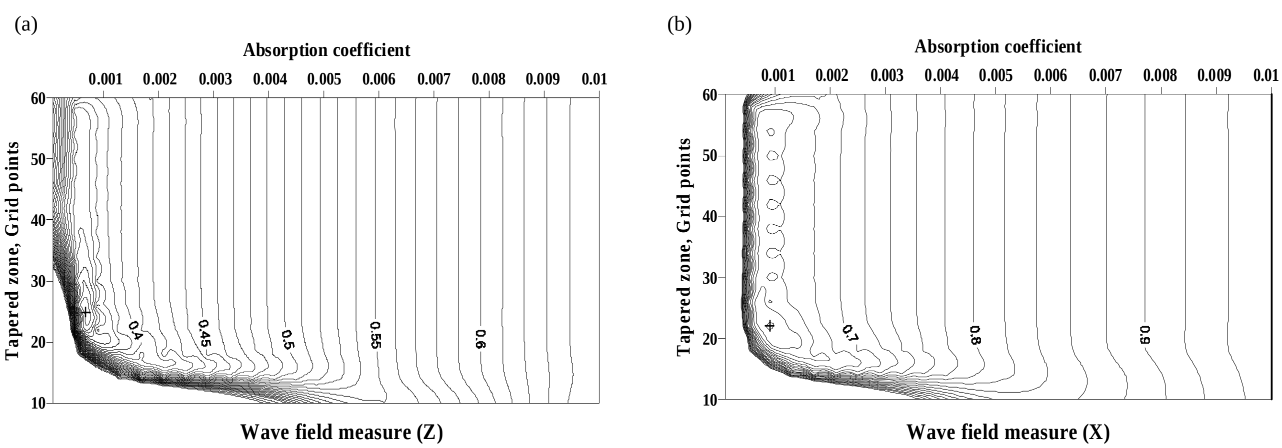

Figure 2. Wavefield energy contour plot of tapered zone grid points versus absorption coefficient.

To compare the effectiveness of the absorbing boundary condition, two cases of the model were run, one without the sponge boundary applied and another with the optimal parameter boundary condition, shown in figure 3 at the snapshots time 360ms, respectively. (a), (b), (c), and (d) are horizontal and vertical components of the solid particle velocity without and with nearly optimal sponge boundary condition, respectively. As illustrated in the fig. 3c and 3d the boundary condition technique is extremely effective.

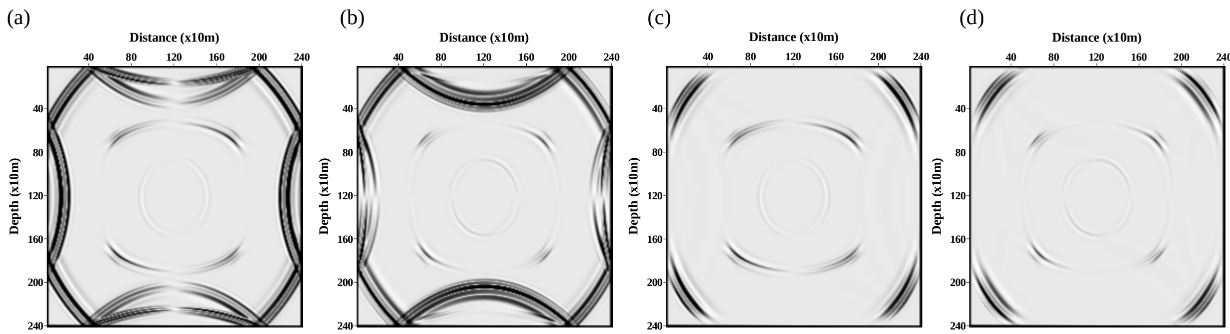

Figure 3. Snapshots of two components of the solid particle velocities without and with optimal boundary condition, respectively.

Conclusions

We present a modified nearly optimal sponge boundary condition. We can find nearly optimal sponge length and absorbing coefficients by this new scheme. Finally, this method is applied to numerical modeling of seismic wave propagation in transversely isotropic poroelastic media, the model test illustrates that this method can effectively eliminate the reflections from the edges of the discrete model.

Acknowledgements

The authors would like to thank Atlantic Canada Opportunities Agency-AIF, IBM, and Husky Energy for their generous support.

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