Geostatistical Prediction of Reservoir Petrophysical Properties by Copula Based Dependence Models between Seismic Attributes and Petrophysical Properties*

Martín A. Díaz Viera[1], Arturo Erdely[2], Raúl del Valle García[1], Tatiana Kerdan[1], and Francisco Mendoza-Torres[3]

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Abstract

In the context of geological and petrophysical reservoir modeling proper prediction of petrophysical property spatial distributions is a crucial task because the correct estimation of reserves and the development and optimal exploitation of the reservoir heavily depend on it. Only in recent years, it has imposed on the oil industry a comprehensive and multidisciplinary approach that combines all available information sources such as core data, geological models, seismic surveys and well logs, by the application of geostatistical models in a systematic way. One of the most common ways to combine seismic data with well logs is to establish correlations between seismic attributes and petrophysical properties. These models are quite restrictive because in most cases they assume that variables follow a Gaussian distribution and a strong linear dependence exists between them. Moreover, also the classical multivariate geostatistical models as Cokriging and Sequential Gaussian simulation method (Parra and Emery, 2013) also consider these assumptions. A non-parametric (distribution-free) method is proposed, which does not assume linear dependence, but rather seeks to represent, reproduce and exploit the underlying dependency between attributes and petrophysical properties: a Bernstein copula dependence model that was successfully applied for petrophysical simulation at well log scale. The methodology consists of two steps: firstly, a dependence model between seismic attributes and petrophysical properties at well log scale is established and then this model is used to estimate (median regression approach) or to simulate (stochastic approach using simulated annealing) petrophysical properties to seismic scale. The application of the methodology is illustrated in a case study where the results are compared with sequential Gaussian method.

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Outline

- **Introduction**
- **Methodology**
- **Case study**
- **Final remarks and future work**
- **Bibliography**

Introduction

- Modeling the ***spatial distribution*** of petrophysical properties in reservoir characterization is a crucial and difficult task due to the lack of enough data and hence the degree of uncertainty associated with it.
- For this reason, a ***stochastic simulation*** approach for the spatial distribution of petrophysical properties has been adopted.
- Seismic attributes have been extensively used as secondary variables in static reservoir modeling for petrophysical property prediction but usually assuming ***linear dependence*** and Gaussian distribution (Parra & Emery, 2013).

Introduction

- Quite recently, ***copulas*** have become popular for being a flexible means of representing dependency relationships in the financial sector and applications are already emerging in the field of geostatistics (Bardossy & Li, 2008) (Kazianka & Pilz, 2010).
- A geostatistical simulation method, based on ***Bernstein copula*** approach as a tool to represent the underlying dependence structure between petrophysical properties and seismic attributes, is proposed.

Introduction

- The procedure basically consists of applying the ***simulated annealing*** method with a joint probability distribution model estimated by a ***Bernstein copula*** in a completely non-parametric fashion (Hernández-Maldonado, Díaz-Viera, & Erdely, 2012).
- The method has the advantages of not requiring linear dependence or a specific type of distribution.
- The application of the methodology is illustrated in a case study where the results are compared with sequential Gaussian co-simulation method.

Methodology

- The main goal of this work
- show the application of a ***Bernstein copula-based spatial stochastic co-simulation (BCSCS)*** method for petrophysical property predictions using seismic attributes as secondary variables and its
- compare with the classical ***Sequential Gaussian co-simulation (SGCS)*** method

Sequential Gaussian co-simulation (SGCS)

- Usually this method is applied with a ***linear model of coregionalization*** (Chiles & Delfiner, 1999) which is mostly unnatural, forced, very complicated and difficult to establish.
- The method assumes the existence of very strong ***linear dependence*** between primary and secondary variables, which is its main assumption and at the same time its main drawback.
- Here we choose to use an alternative variant, the ***Markov Model***, given in (Chiles & Delfiner, 1999, p. 305) and implemented in SGeMS (Remy, Boucher, & Wu, 2009).

Bernstein copula-based spatial stochastic co-simulation (BCSCS)

- The procedure consists of two stages:

1. A ***dependence model***, using a ***Bernstein copula***, is established from which a number of sample values are generated.

2. A ***stochastic spatial simulation*** is performed using a ***simulated annealing*** method with a variogram model and a bivariate distribution functions as objective functions (Deutsch & Cockerham, 1994), (Deutsch & Journel, 1998).

Copula-based dependence modeling

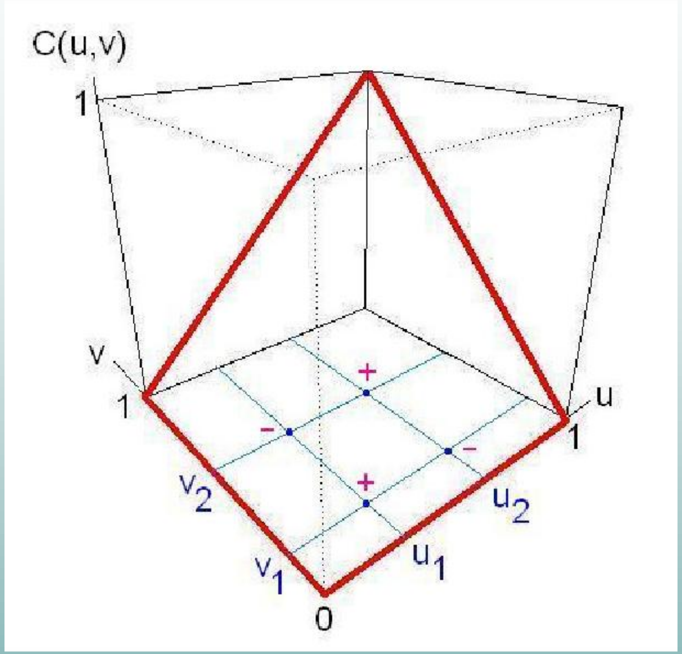
- ***Sklar*** (1959) proved that there exists a function $C_{XY}: [0,1]^2 \rightarrow [0,1]$ such that

$$H_{XY}(x,y) = C_{XY}(F_X(x), G_Y(y))$$

- C_{XY} is called ***copula function*** associated to (X,Y) and contains all the information about the dependence relationship between X and Y, independently from their marginal probabilistic behavior

Copula-based dependence modeling

- Some copula function properties:



- $C(u, 0) = 0 = C(0, v)$
- $C(u, 1) = u, \quad C(1, v) = v$
- $C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0$ if $u_1 \leq u_2, v_1 \leq v_2$
- C is uniformly continuous on its domain $[0,1]^2$.

Copula-based dependence modeling

- The word ***copula*** is a Latin noun that means “A link, tie, bond”
- Copula function is a flexible tool for building ***joint probability distributions***
- Univariate models for the random variables of interest and the copula function may be chosen separately
- There are parametric, semi-parametric and non-parametric approaches.
- Particularly appropriate for non-linear dependencies

Copula-based dependence modeling

- When F_X and G_Y are known and H_{XY} is unknown, if $\{(x_1, y_1), \dots, (x_n, y_n)\}$ is an observed random sample from (X, Y), the set $\{(u_k, v_k) = \big(F_X(x_k), G_Y(y_k)\big) : k = 1, \dots, n\}$ would be an observed random sample from (U, V) with the same underlying copula C as (X, Y), and since $C = F_{UV}$ we may use the (u_k, v_k) values (called copula observations) to estimate C as a joint empirical distribution:

$$\hat{C}(u, v) = \frac{1}{n} \sum_{k=1}^{n} 1_{\{u_k \leq u \,,\, v_k \leq v\}}$$

- Strictly speaking, the estimation $\hat{C}$ is not a copula since it is discontinuous and copulas are always continuous.

Copula-based dependence modeling

- F_X and G_Y are estimated by univariate empirical distribution functions:

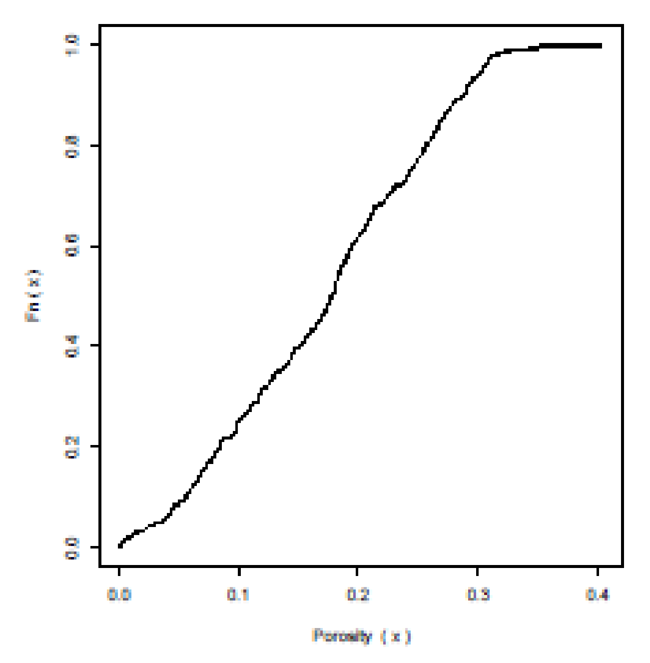
$$\widehat{F_X}(x) = \frac{1}{n}\sum_{k=1}^{n} 1_{\{x_k \le x\}} \qquad \widehat{G_Y}(y) = \frac{1}{n}\sum_{k=1}^{n} 1_{\{y_k \le y\}}$$

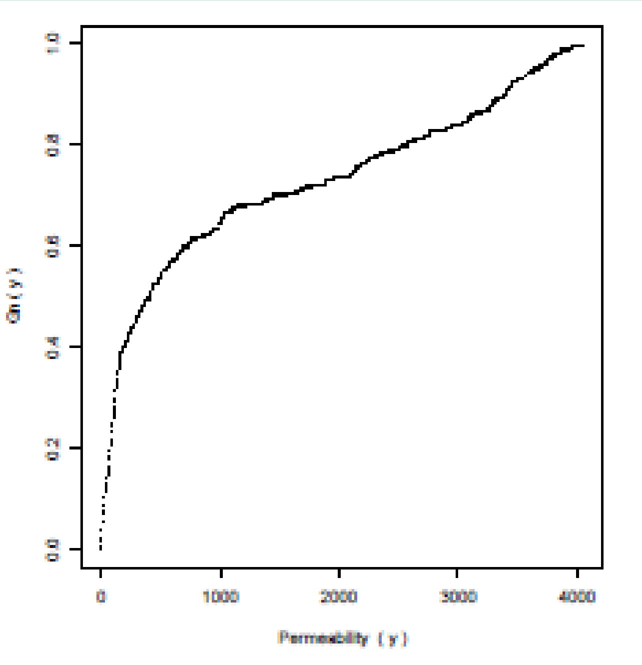
- Now the set of pairs $\{(u_k, v_k) = \left(\widehat{F_X}(x_k), \widehat{G_Y}(y_k)\right) : \ k = 1, \dots, n\}$ is referred to as ***copula pseudo-observations***.

Copula-based dependence modeling

- F_X and G_Y are estimated by univariate empirical distribution functions:

$$\widehat{F_X}(x) = \frac{1}{n}\sum_{k=1}^{n} 1_{\{x_k \leq x\}} \qquad \widehat{G_Y}(y) = \frac{1}{n}\sum_{k=1}^{n} 1_{\{y_k \leq y\}}$$

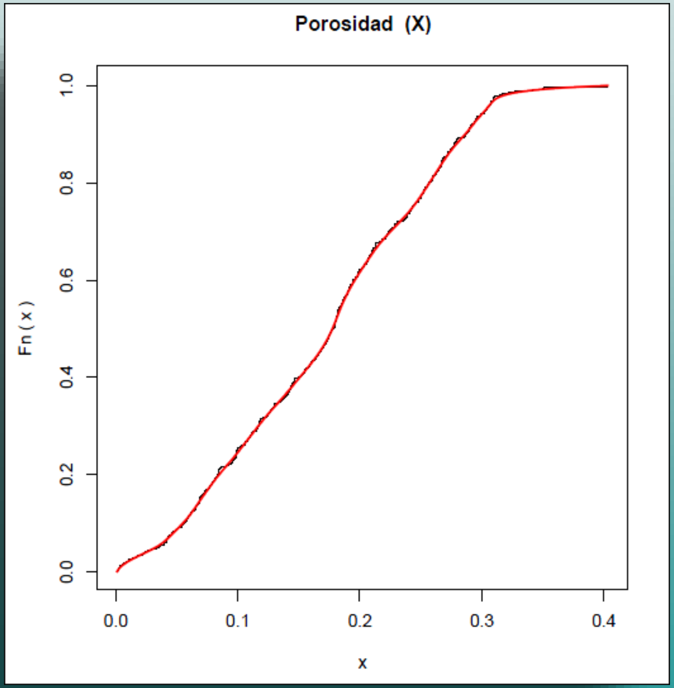


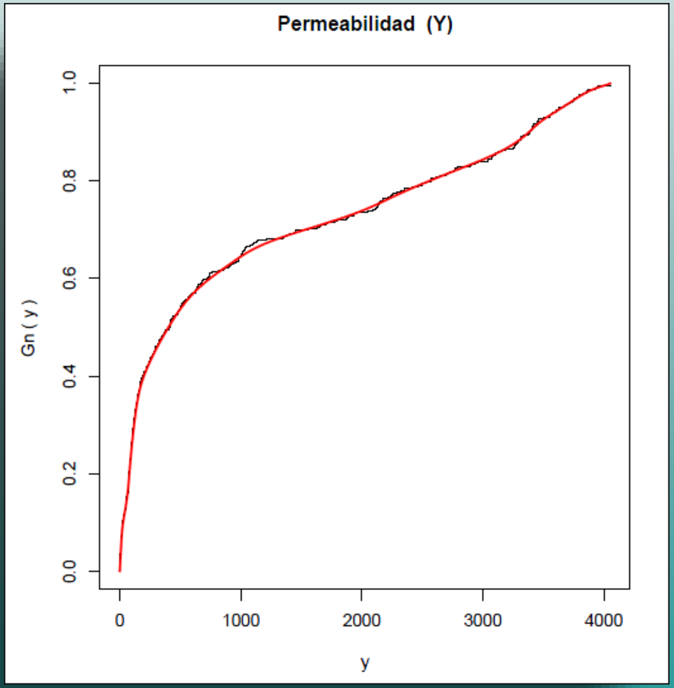


Copula-based dependence modeling

- F_X and G_Y are estimated by univariate empirical distribution functions (Pérez & Fernández-Palacín (1987) :

$$\widetilde{Q}_n(u) = \sum_{k=0}^{n} \frac{1}{2}\left(x_k + x_{k+1}\right)\binom{n}{k} u^k (1-u)^{n-k}$$





Copula-based dependence modeling

- The ***empirical copula*** is defined as the following function $C_n: I_n^2 \to [0,1]$, where $I_n = \{\frac{i}{n} : i = 0, \dots, n\}$, given by:

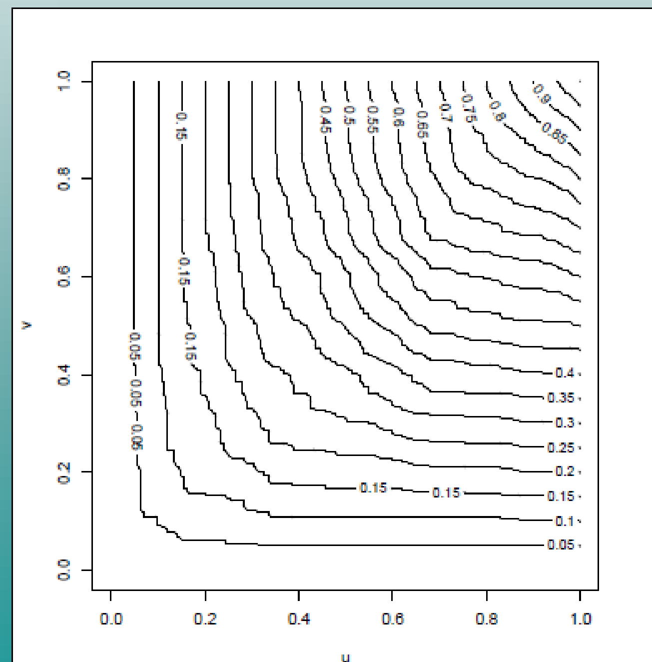
$$C_n\left(\frac{i}{n}, \frac{j}{n}\right) = \frac{1}{n}\sum_{k=1}^{n} 1_{\{rank(x_k) \leq i\,,\, rank(y_k) \leq j\}}$$

- where C_n is not a copula but it is an estimation of the underlying copula C on the grid I_n^2 that may be extended to a copula on $[0,1]^2$ by means of, for example, a polynomial approximation.

Copula-based dependence modeling

- The ***empirical copula*** is defined as $C_n : I_n^2 \to [0,1]$, where $I_n = \{\frac{i}{n} : i = 0, \dots, n\}$, given by:

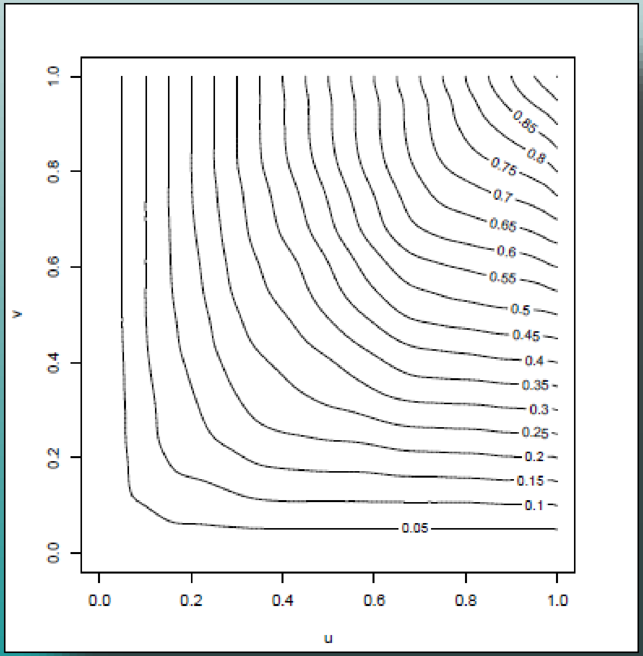
$$C_n\left(\frac{i}{n}, \frac{j}{n}\right) = \frac{1}{n}\sum_{k=1}^{n} 1_{\{rank(x_k) \leq i\,,\, rank(y_k) \leq j\}}$$



Copula-based dependence modeling

- As proposed in (Sancetta & Satchell, 2004), using Bernstein polynomials leads to what is known as a ***Bernstein copula*** non-parametric estimation $\tilde{C}$: $[0,1]^2 \rightarrow [0,1]$ given by:

$$\tilde{C}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{n} C_n\left(\frac{i}{n},\frac{j}{n}\right)\binom{n}{i}u^i(1-u)^{n-i}\binom{n}{j}v^j(1-v)^{n-j}$$



Copula-based dependence modeling

- As summarized in (Erdely & Diaz-Viera, 2010) in order to simulate replications from the random vector (X, Y) with the dependence structure inferred from the observed data $\{(x_1, y_1), \dots, (x_n, y_n)\}$ we have the following algorithm:

1. Generate two independent and continuous Uniform (0,1) random variates u and t.
2. Set $v = c_u^{-1}(t)$ where $c_u(v) = \frac{\partial \tilde{C}(u,v)}{\partial u}$.
3. The desired pair is $(x, y) = \left(\widetilde{Q_n}(u), \widetilde{R_n}(v)\right)$ where $\widetilde{Q_n}$ and $\widetilde{R_n}$ are empirical quantile functions for X and Y, respectively.

The general workflow

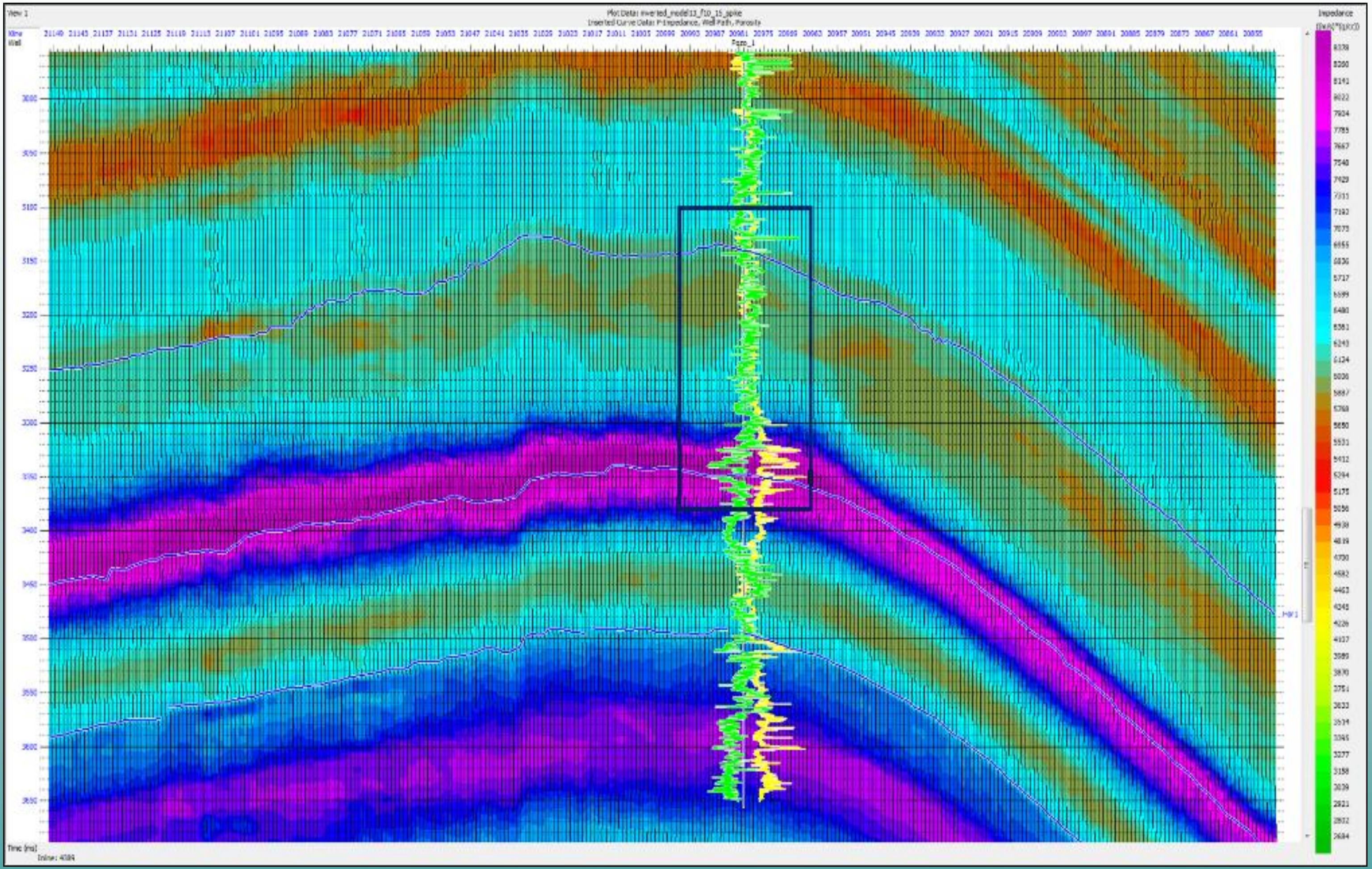
- The general workflow is as follows:

1. Univariate data analysis,
2. Bivariate dependence analysis,
3. Variography analysis,
4. Simulations.

Case study

- Data used in the case study are from a deep water reservoir in the Gulf of Mexico.
- The data consist of a total porosity well-log from a well and seismic attribute (P-impedance) obtained in a vertical (inline) section.
- The well-log has a sample interval of 0.1 m.
- The section has a length of 412.5 m and covers an interval of 336.4 m in depth and was chosen so that the well was located in the middle of it.

Case study

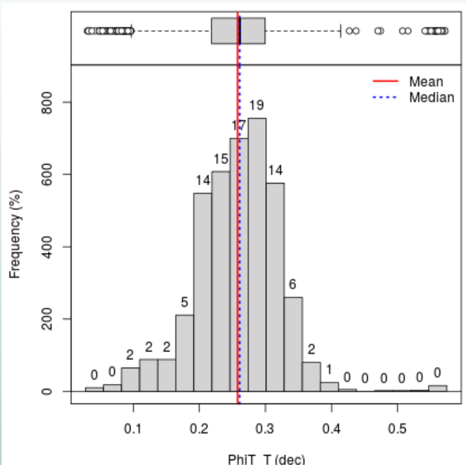


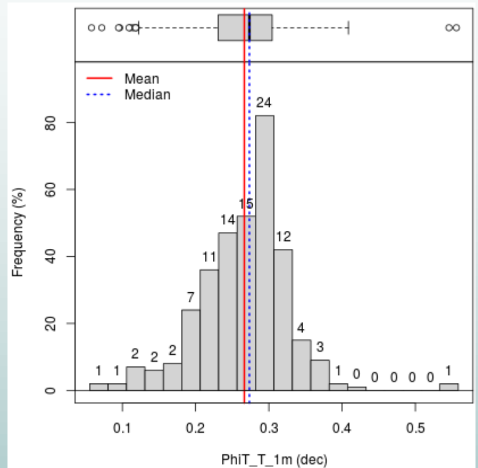
Vertical (inline) section with P-impedance as a result of seismic inversion. The color scale represents impedance values. In the middle of the section two logs are plotted along a well: in yellow P-impedance and in green total porosity.

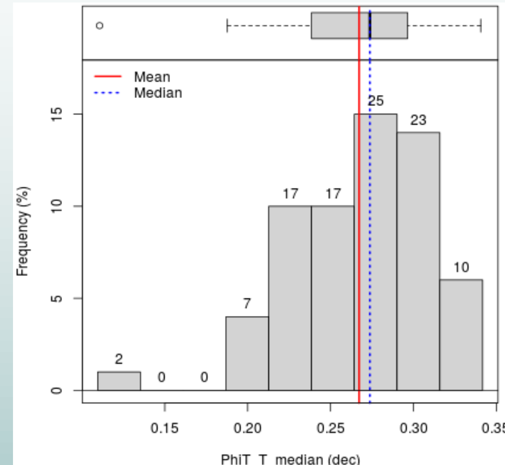
Univariate data analysis

4059 | 337 | 60

P h i T







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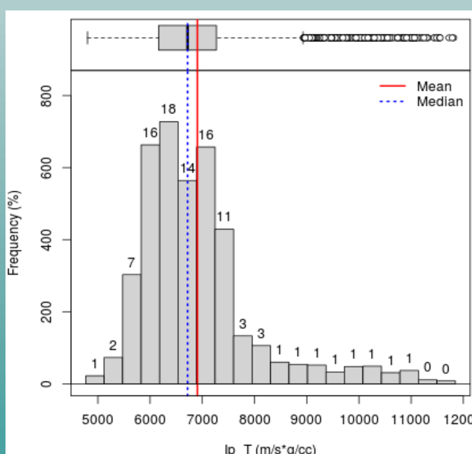
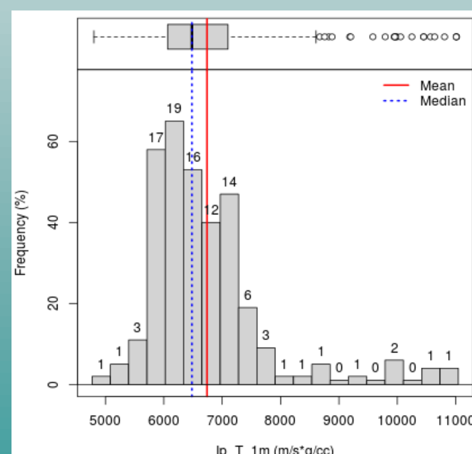
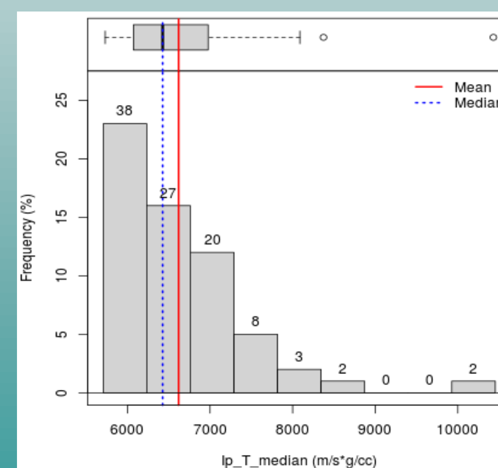


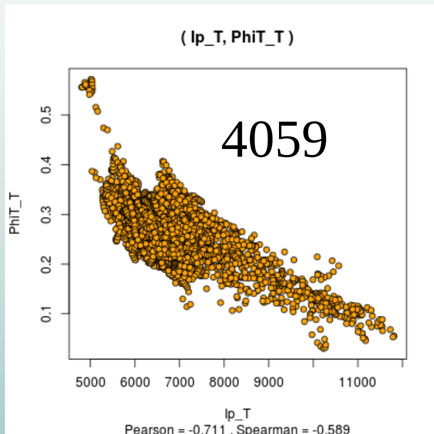




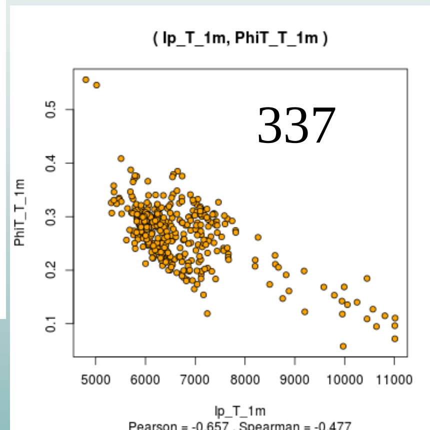
log scale | 1m scale | seismic scale

Bivariate dependence analysis

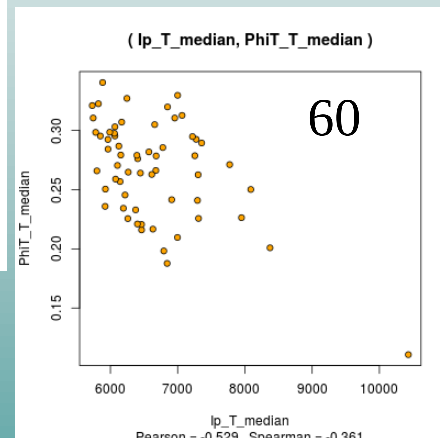
log scale



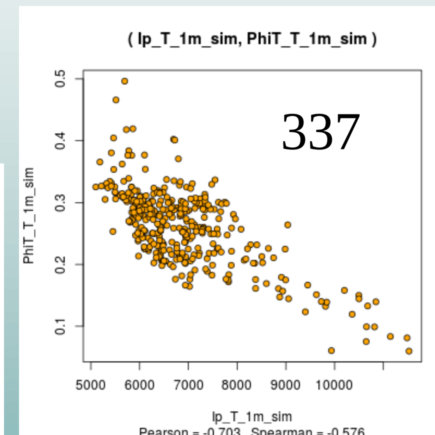
1m scale



seismic scale



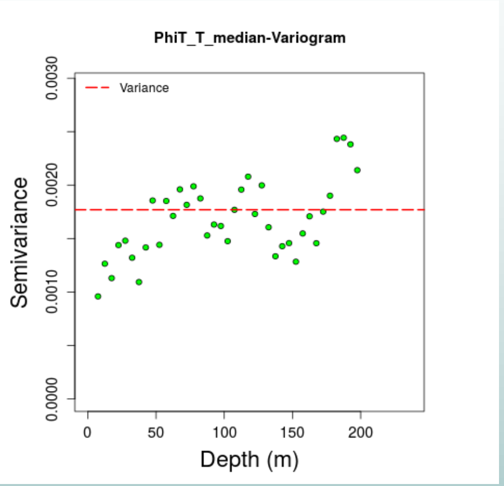
1m scale simulation

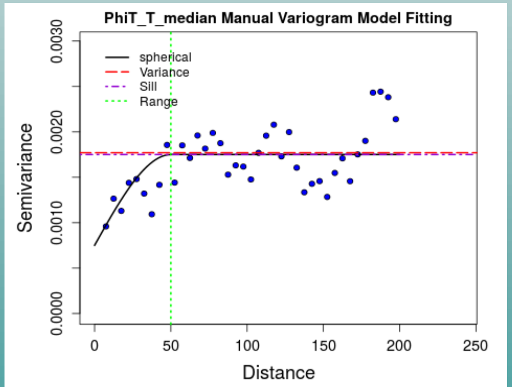


	well-log scale	one-meter scale	seismic scale	1m_BCS
Spearman	-0.589	-0.477	-0.361	-0.576
Pearson	**-0.711**	**-0.657**	**-0.529**	**-0.703**

Variography analysis

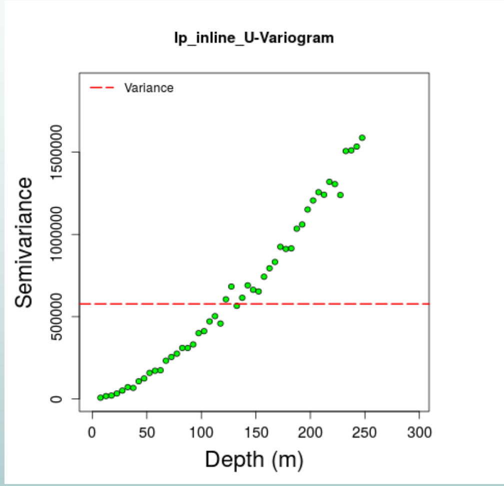
PhiT-upscaled

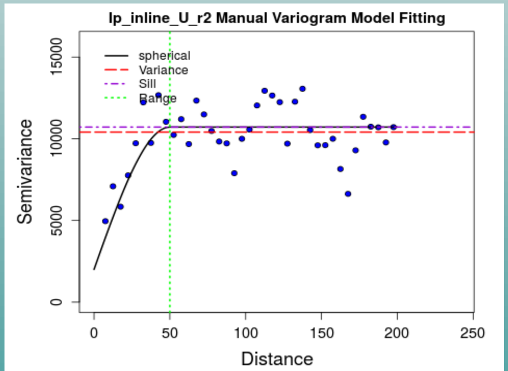




Model	Nugget	Sill	Range	RMSE
spherical	0.0008	0.0018	50.0000	0.0003

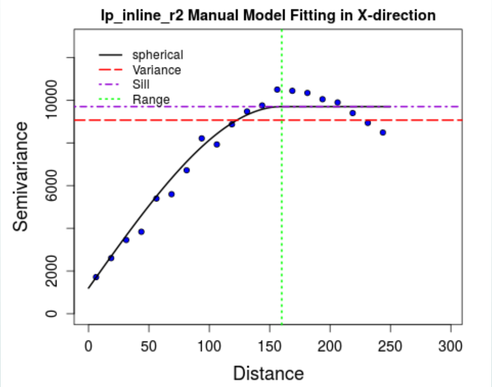
Ip-upscaled



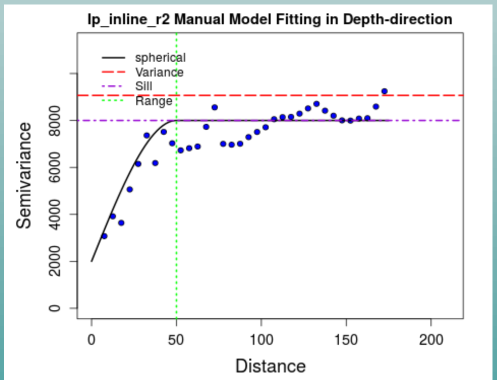


Model	Nugget	Sill	Range	RMSE
spherical	2000.0000	10716.0300	50.0000	1491.1592

Ip-seismic



Model	Nugget	Sill	Range	RMSE
spherical	1200.0000	9700.0000	160.0000	534.8302



Model	Nugget	Sill	Range	RMSE
spherical	2000.0000	8000.0000	50.0000	706.5590

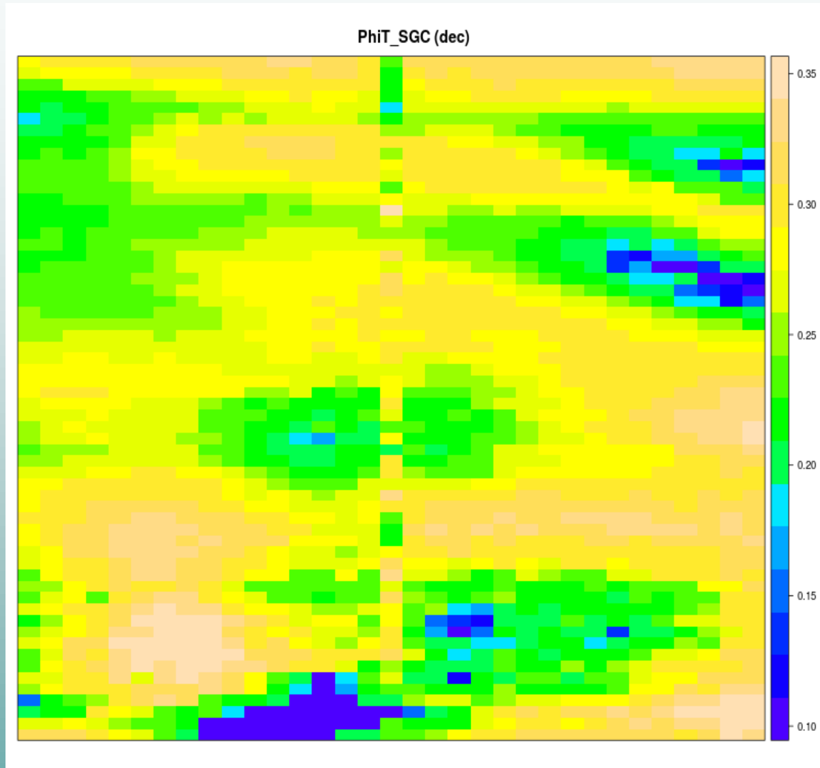
Simulations

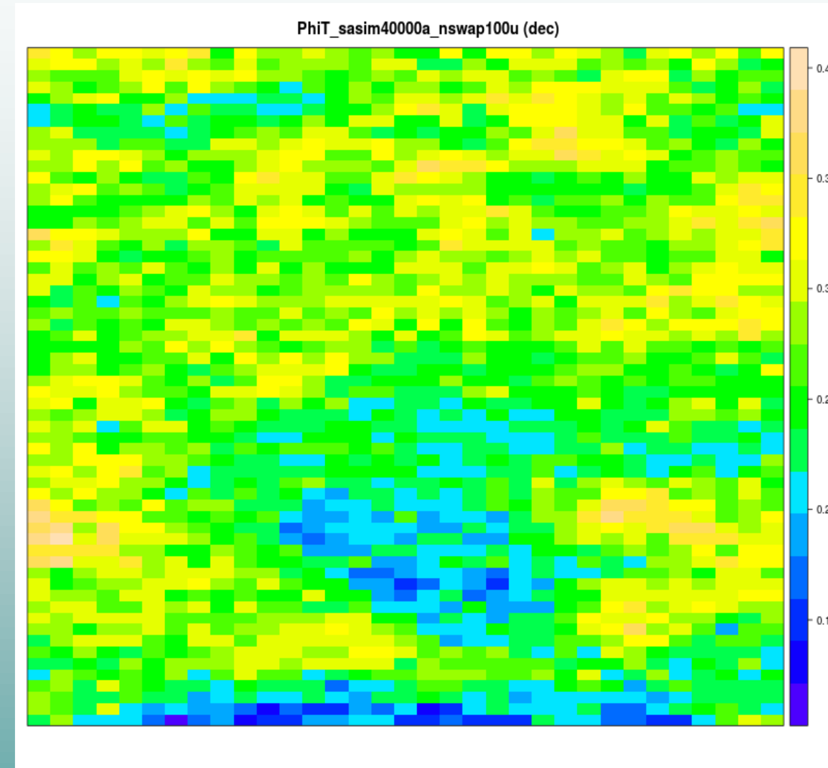
Features	**SGCS**	**BCSCS**
Grid	33x60x1	
Variogram model of primary variable	spherical, nugget= 0.0002, structure contribution=0.0016, ranges: max.=160, med.=50, min.=1, angles: x=90, y=0, z=0	
Dependence Model	Corr. coefficient -0.657	Bernstein copula model
Software	SGEMS	SASIM(GSLIB)

Simulations

X-direction (m)

Depth (m)



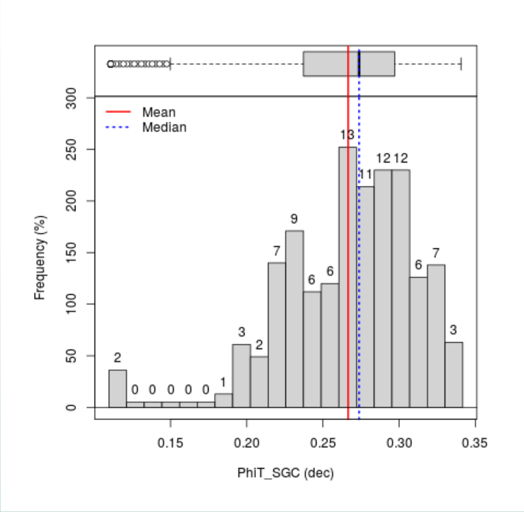


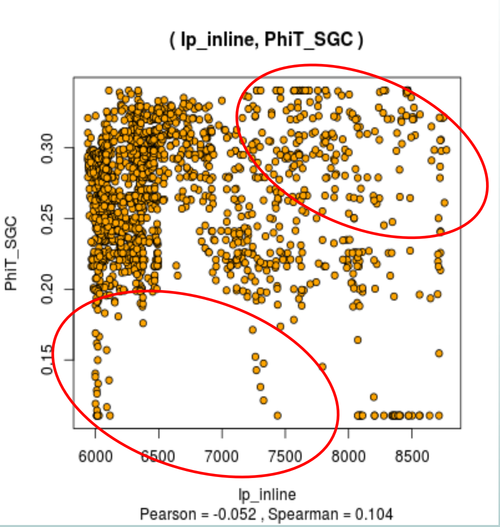
SGCS

BCSCS

Simulation Comparison

S
G
C
S





Stat.	SGCS	BCSCS
n	1980	1980
Min.	0.1106	0.121
1st. Q.	0.2373	0.2375
Med.	0.2737	0.2681
Mean	0.2665	0.2662
3rd. Q.	0.297	0.2969
Max.	0.3406	0.3903
Var.	0.0019	0.0017

B
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S
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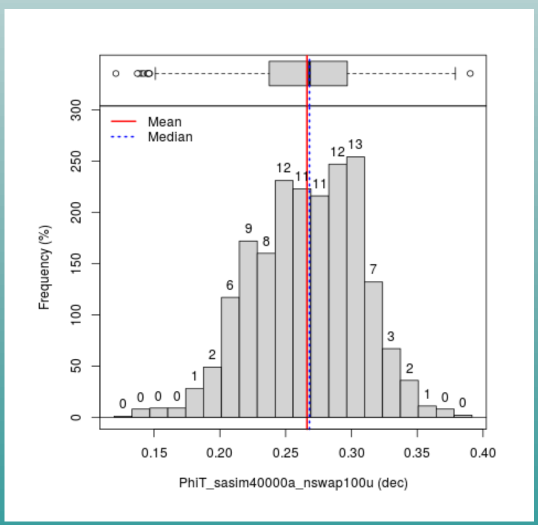
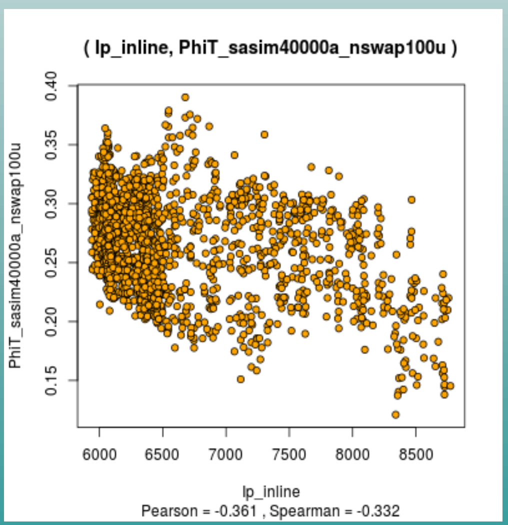
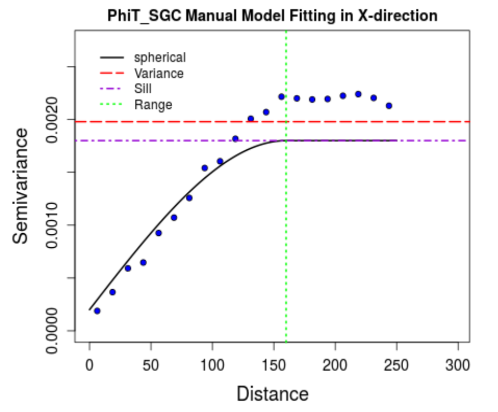
S



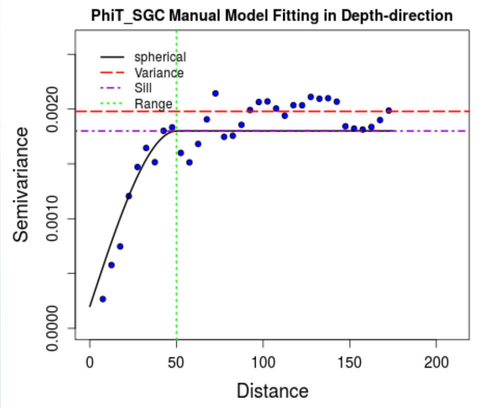


Simulation Comparison

SGCS

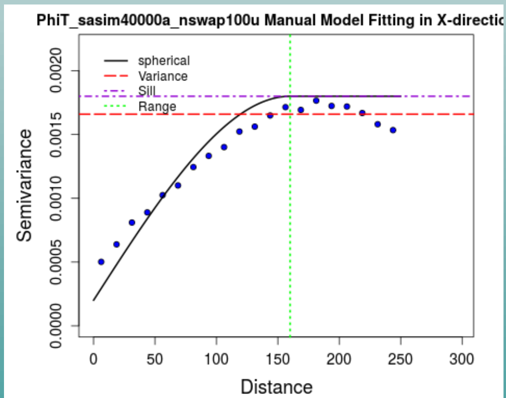


Model	Nugget	Sill	Range	RMSE
spherical	0.0002	0.0018	160.0000	0.0003

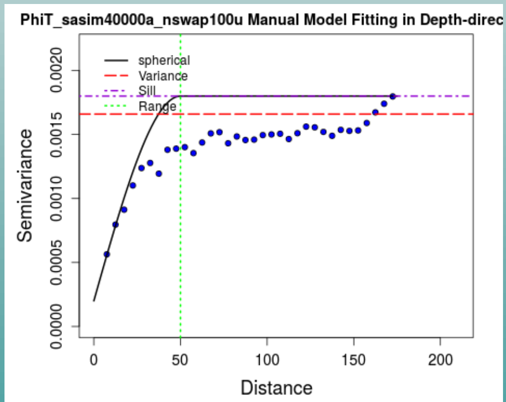


Model	Nugget	Sill	Range	RMSE
spherical	0.0002	0.0018	50.0000	0.0002

BCSCS



Model	Nugget	Sill	Range	RMSE
spherical	0.0002	0.0018	160.0000	0.0001



Model	Nugget	Sill	Range	RMSE
spherical	0.0002	0.0018	50.0000	0.0003

X-direction

Depth

Comparison

Features	SGCS	BCSCS
Linear	Yes	No
Parametric	Yes	No
Transformation	Yes	No
Computational cost	Lower	Higher

Final remarks

- A Bernstein copula-based spatial stochastic co-simulation (BCSCS) method presented in this paper possess several advantages over the classical sequential Gaussian co-simulation (SGCS), among others:
- Does not require of a strong linear dependence
- Captures and reproduces the existing dependence
- Is non-parametric (does not need a specific distribution)
- Reproduces the variability and the extreme values.
- Does not need to make back transformations

Future work

- Used a linear combination of attributes (principal component and factorial analysis).
- A multivariate copula with three or more variables.
- 3D extension, but it depends on the computing power available.
- A simpler and efficient alternative, the median regression approach

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!!!Thank you
for your attention¡¡¡