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Exploiting a Priori Model Reduction Methods to Accelerate Seismic Simulations*

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Search and Discovery Article #42038 (2017)**
Posted March 27, 2017

*Adapted oral presentation given at AAPG/SPE 2016 International Conference & Exhibition, Barcelona, Spain, April 3-6, 2016

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Abstract

Seismic modeling and applications (imaging, migration and inversion techniques, e.g., Baysal et al., 1983; Pratt, 1999; Virieux and Operto, 2009) usually require a huge number of very expensive simulations to provide accurate results. Solutions are obtained from different values of some design parameters, such that frequency and position of source/receivers, while both geometry and boundary conditions remain fixed between simulations. Actual practice adopts an expensive and time-consuming brute force approach that generates a direct solution for each required set of parameter values. This imposes workable limits to the number of simulations that are feasible to compute in practice. In this work, an a priori reduced order method based on proper generalized decompositions (PGD) is exploited as an attractive alternative strategy to the usual practice. More precisely, the wave field is generalized to provide any particular solution of the seismic problem at negligible computational cost. The PGD technique is then applied to obtain an approximation of this generalized wave field, using it as a database for providing any required particular solution in a real-time framework. A simple 2D problem in frequency domain is used to exemplify the potential of this methodology. The strategy will be particularly useful whenever many realizations of modeling are required (i.e. many shots and frequencies are involved) such as in RTM and FWI applications.

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EXPLOITING A PRIORI MODEL REDUCTION METHODS TO ACCELERATE SEISMIC SIMULATIONS

David MODESTO and Josep DE LA PUENTE

ICE 2016, 3-6 April, Barcelona, Spain

Reduced order modeling… why?

- Parametric / high-dimensional solutions

Variables: space, time, model parameters, boundary conditions,...

$$p(\boldsymbol{x}) = p(x_1, \ldots, x_D)$$

GENERALIZED model solution

Physical field: displacements, velocities, pressure, chemical concentration, wave amplitude,...

- Variables in seismic exploration?
  - space ($\approx 10^8$)
  - time / frequency ($\approx 10^2$)
  - source / receiver position ($\approx 10^5$)
  - model uncertainties (velocity,...)

- Goal: build a surrogate model

$$p(\boldsymbol{x}) \approx p^n(\boldsymbol{x}) = \sum_{m=1}^{n} \alpha_m \Phi_m(\boldsymbol{x})$$

Reduced order modeling... why?

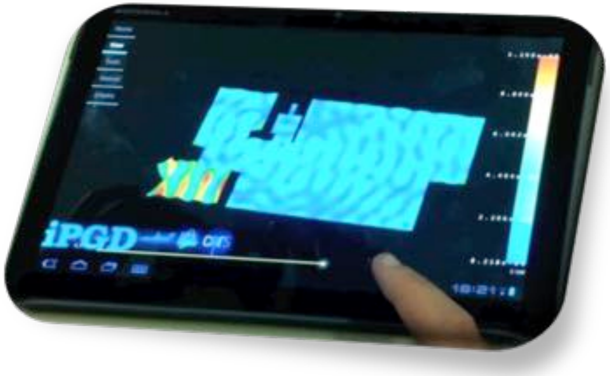
$$p(\boldsymbol{x}) \approx p^n(\boldsymbol{x}) = \sum_{m=1}^{n} \alpha_m \Phi_m(\boldsymbol{x})$$

Reduced basis

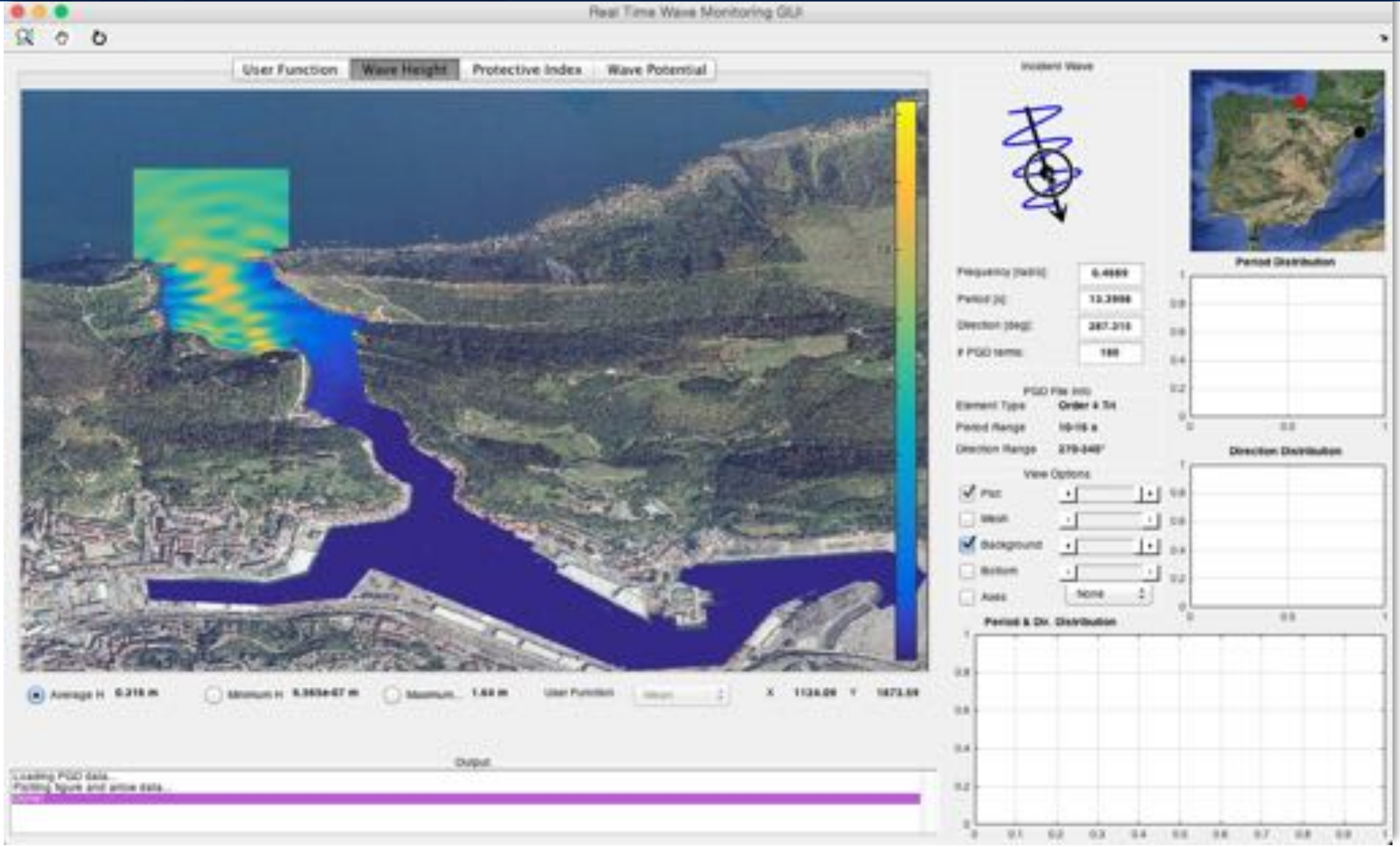
1 **OFFLINE** stage: compute the reduced basis only once in a lifetime → expensive!

2 **ONLINE** stage: evaluate the surrogate model as many times as required → cheap!

- ✓ Database of any solution available to the user.
- ✓ Immediate, real-time access via linear combination of modes.
- ✓ Acceleration of seismic FM, RTM and IM (derivatives of $p(\boldsymbol{x})$ also readily available!).
- ✓ Potential applications in control tasks, tools development, parameter identification, shape optimization,...



Reduced order modeling... why?



Real-time agitation in harbors [Modesto et al. CMAME, 2015]

Reduced order models
for seismic geophysics

BSIT

- In frequency domain: $$p^n(\boldsymbol{x}, \omega, \mathbf{s}) = \sum_{m=1}^{n} \alpha_m \Phi_m(\boldsymbol{x}, \omega, \mathbf{s})$$

signal frequency

space

source position

(1) **OFFLINE** stage: compute the reduced basis only once in a lifetime → expensive!

A posteriori **POD-based** methods

[Fernández-Martínez, Zaslavsky et al. 2015, Pereyra 2016]

- x r precomputed solutions p_r required
- x SVD with computational cost $\mathcal{O}(r^3)$
- x Projection required to compute online the coefficients α_m
- ✓ Non-intrusive in codes
- ✓ Optimal number of modes (n) in a least square sense $\|p^n - p_r\|_{\mathcal{L}_2} = 0$

A priori **PGD-based** methods

[Ammar et al. 2006, 2007, Modesto et al. 2015]

- ✓ High-dimensional solvers taking advantage of HPC tools
- ✓ Neither SVD nor precomputed solutions are required
- ✓ Suitable for real-time constraints
- ✓ Small impact in codes
- x Optimality highly dependent on the model

A priori PGD solver: 3 main ideas

for seismic geophysics

- Separated representation: (1) $p^n(\boldsymbol{x}, \omega, \mathbf{s}) = \sum_{m=1}^{n} F_m^1(\boldsymbol{x}) F_m^2(\omega) F_m^3(\mathbf{s})$

  space — signal frequency — source position

- Acoustic problems (+ boundary conditions):

$$\Delta p^n + k^2(\omega, \mathbf{x}) p^n = f(\omega)\delta(\mathbf{x} - \mathbf{s})$$

Linear equation

(2) $$F_n^2 F_n^3 \Delta F_n^1 + k^2 F_n^1 F_n^2 F_n^3 = f\delta - (\Delta p^{n-1} + k^2 p^{n-1})$$

Nonlinear equation (greedy) !!

- In variational (weak) form:

$$\boxed{A(F_n^1 F_n^2 F_n^3, v) = L(v) - A(p^{n-1}, v)}$$

high-dimensional test function

$$v = v^1(\mathbf{x}) F_n^2 F_n^3 + F_n^1 v^2(\omega) F_n^3 + F_n^1 F_n^2 v^3(\mathbf{s})$$

(3) Need for linearization techniques:
alternate direction strategy

A priori PGD solver: offline cost
for seismic geophysics

$$A(F_n^1 F_n^2 F_n^3, v) = L(v) - A(p^{n-1}, v)$$

$$v = v^1(\mathbf{x}) F_n^2 F_n^3 + F_n^1 v^2(\omega) F_n^3 + F_n^1 F_n^2 v^3(\mathbf{s})$$

Algebraic equation !!

- PGD cost: acoustic solver × number of terms (n) × number of linearized iterations
- Typical numbers: ≈10^3 ≈3 **<< 10^7 (brute force) !!**
- **Explicit evaluation** for all the source position terms $F_m^3(\mathbf{s})$, $m = 1, \ldots, n$

$$F_m^3 = g(F_m^1, F_m^2) + \sum_{i=1}^{m-1} h(F_i^1, F_i^2, F_i^3)$$

A priori PGD solver: offline cost
for seismic geophysics

$$A(F_n^1 F_n^2 F_n^3, v) = L(v) - A(p^{n-1}, v)$$

$$v = v^1(\mathbf{x}) F_n^2 F_n^3 + F_n^1 v^2(\omega) F_n^3 + F_n^1 F_n^2 v^3(\mathbf{s})$$

Algebraic equation !!

- PGD cost: acoustic solver × number of terms (n) × number of linearized iterations
- Typical numbers: ≈10[3] ≈3 << 10[7] (brute force) !!
- **Explicit evaluation** for all the frequency terms $F_m^2(\omega)$, $m = 1, \ldots, n$

$$F_m^2 = g(F_m^1, F_m^3) + \sum_{i=1}^{m-1} h(F_i^1, F_i^2, F_i^3)$$

A priori PGD solver: offline cost
for seismic geophysics

$$A(F^1_n F^2_n F^3_n, v) = L(v) - A(p^{n-1}, v)$$

$$v = v^1(\mathbf{x}) F^2_n F^3_n + F^1_n v^2(\omega) F^3_n + F^1_n F^2_n v^3(\mathbf{s})$$

Acoustic solver

- PGD cost: acoustic solver × number of terms (n) × number of linearized iterations
- Typical numbers: ≈10^3 ≈3 **<< 10^7 (brute force) !!**
- **Acoustic propagation** required to evaluate all the spatial terms $F^1_m(\boldsymbol{x}), m = 1, \ldots, n$

$$\Delta F^1_m + \alpha(F^2_m) F^1_m = g(F^2_m, F^3_m) + \sum_{i=1}^{m-1} h(F^1_i, F^2_i, F^3_i)$$

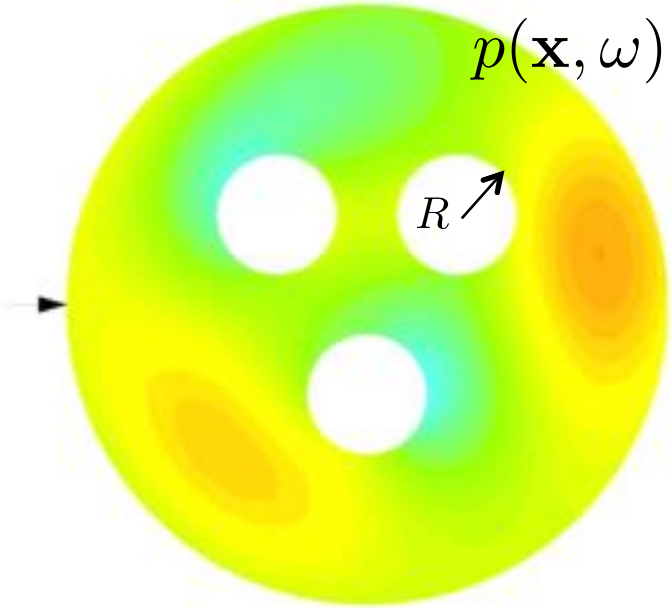
"new" velocity model "new" source

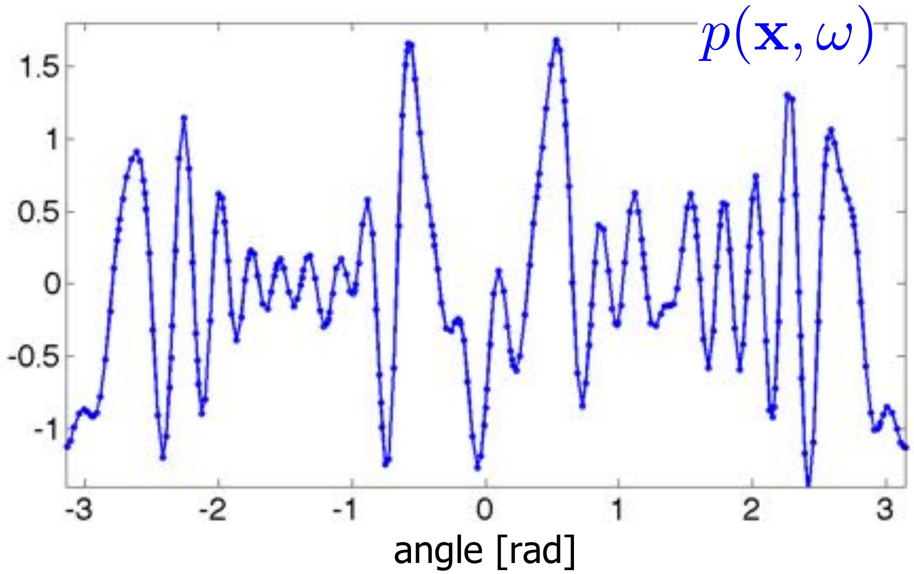
$$\Delta p + k^2 p = f\delta$$

- Non-intrusive → HPC-based software can be used !!

A priori PGD solver: offline cost
for seismic geophysics

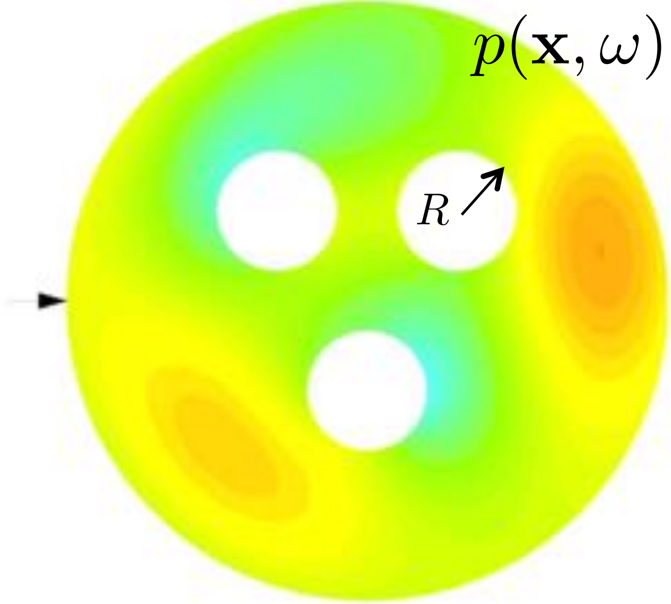
Example: wave propagation in open domain

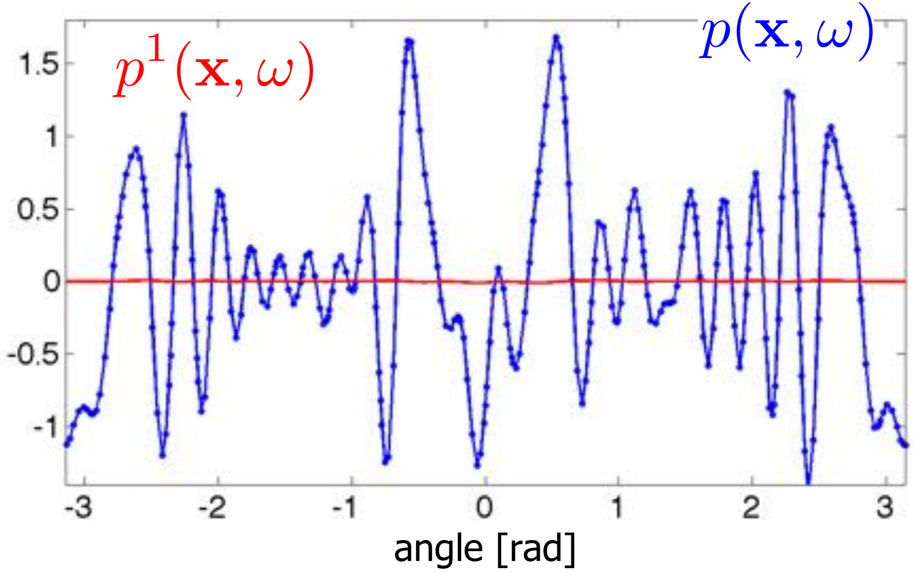




A priori PGD solver: offline cost
for seismic geophysics

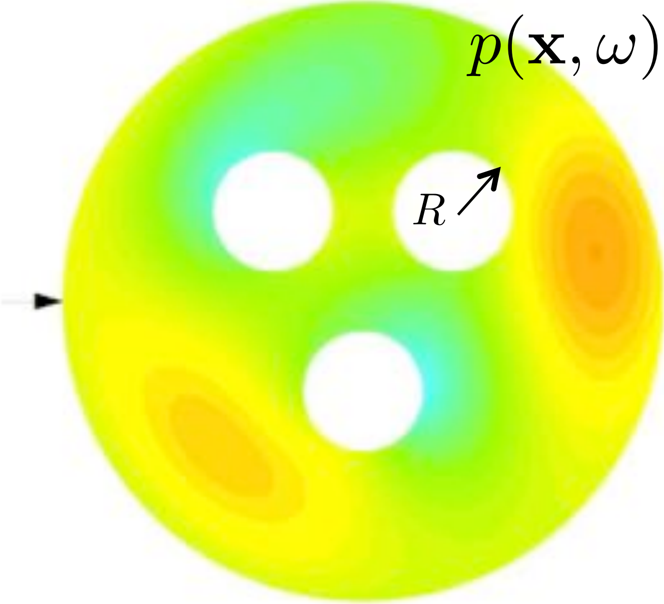
Example: wave propagation in open domain

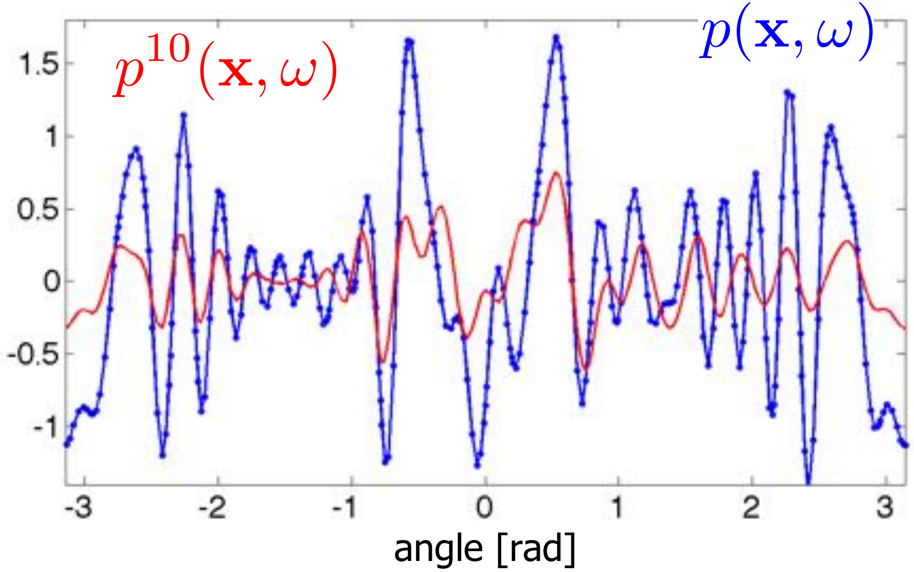




A priori PGD solver: offline cost
for seismic geophysics

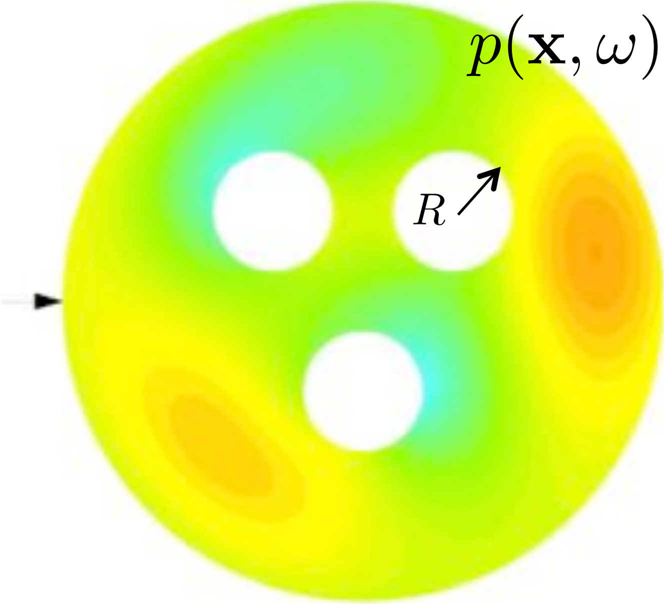
Example: wave propagation in open domain

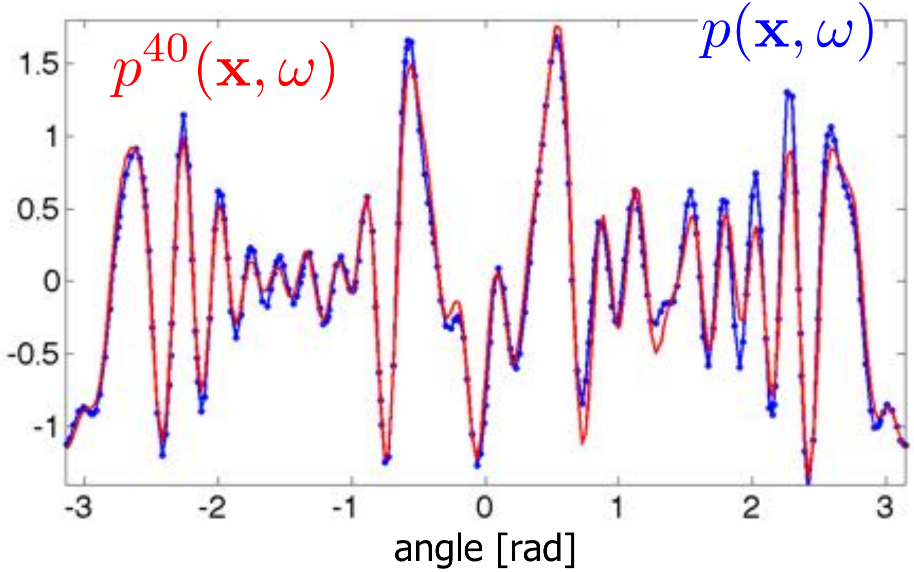




A priori PGD solver: offline cost
for seismic geophysics

Example: wave propagation in open domain

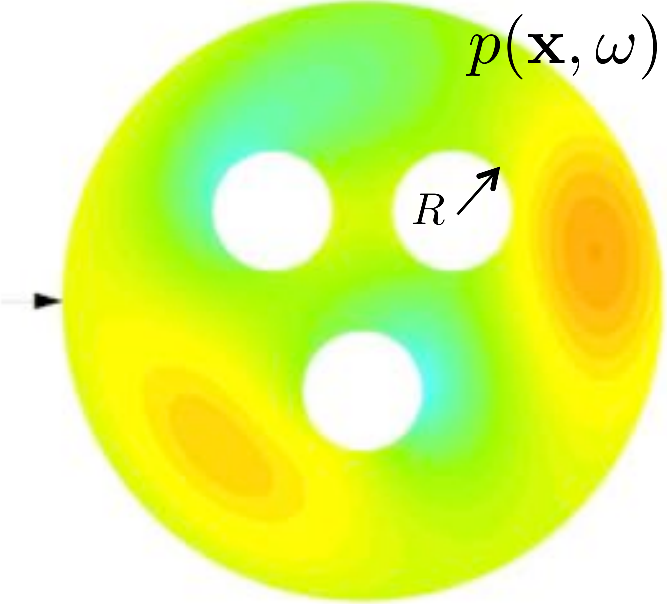


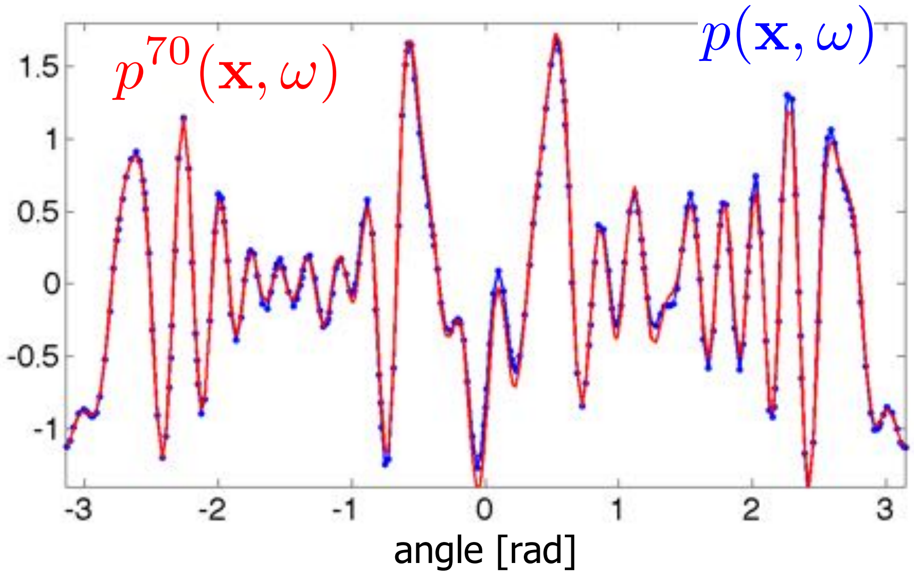


- 120 iterations for a good wave phase approximation **(goal in seismic applications !!)**.

A priori PGD solver: offline cost
for seismic geophysics

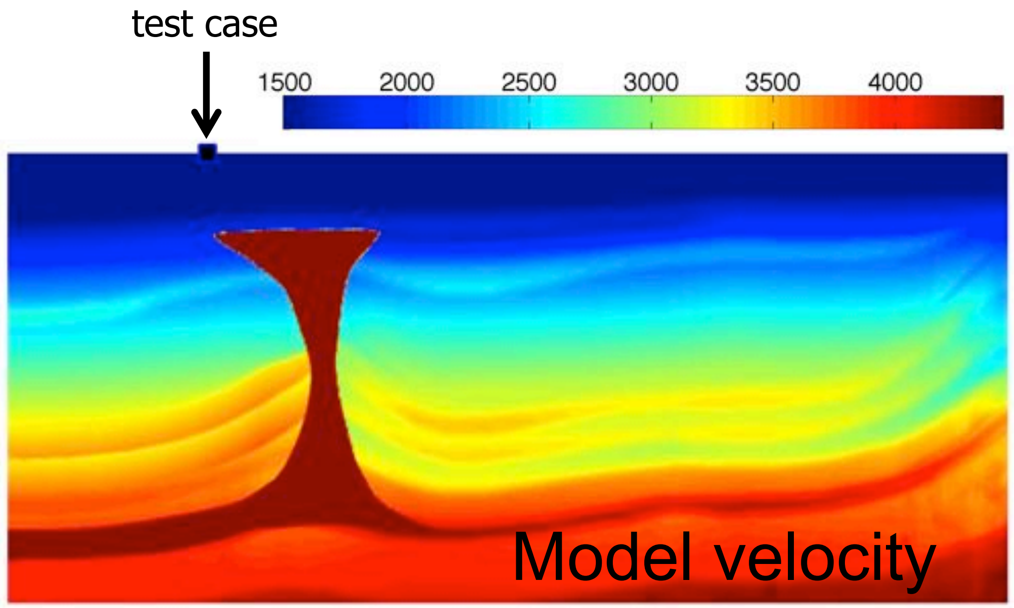
Example: wave propagation in open domain





- 120 iterations for a good wave phase approximation **(goal in seismic applications !!)**
- 210 iterations for an acceptable wave height approximation (less important).

Preliminary (test) results

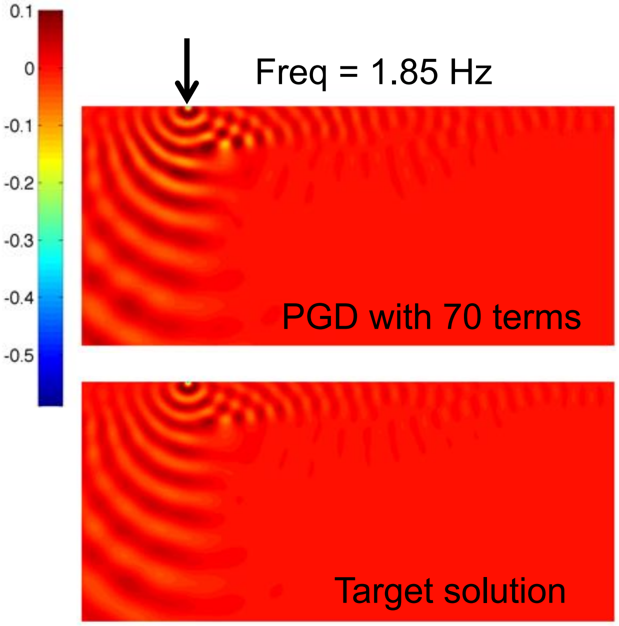


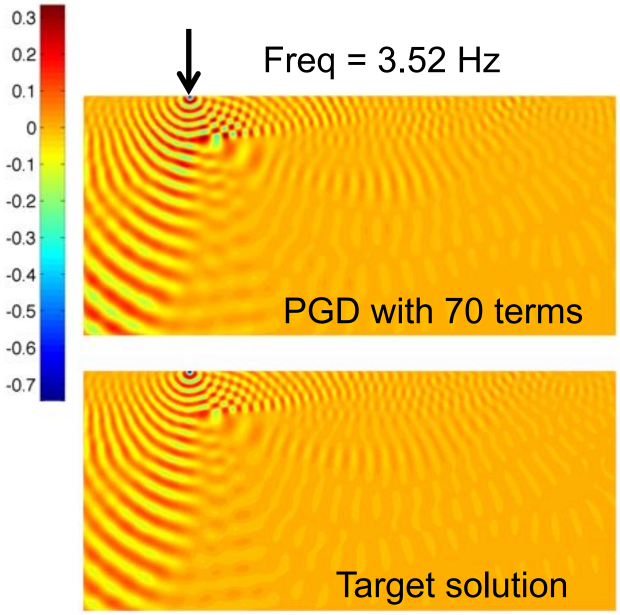
- Isotropic version of the BP 2007 TTI model in frequency domain
- 1~5 Hz variation in frequency
- Source / receiver position varying along all the surface
- Discretization: 100 nodes for both parametric dimensions

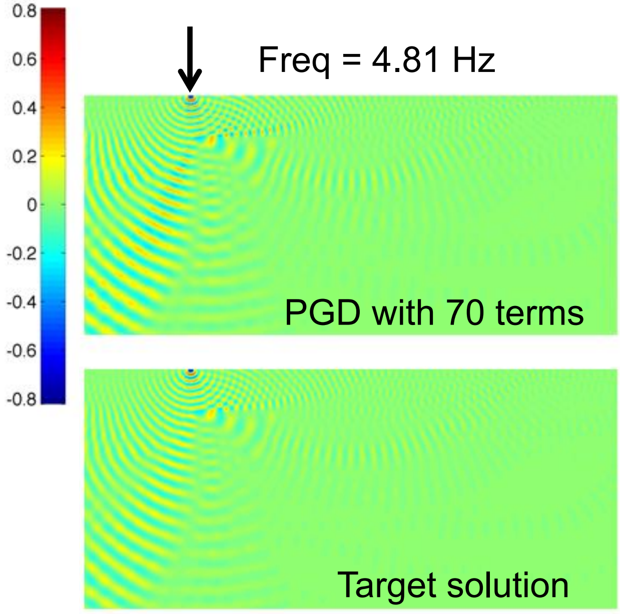
Preliminary (test) results

$$p(\mathbf{x},\omega) \approx \sum_{m=1}^{n} F_m^1(\mathbf{x}) F_m^2(\omega)$$ with fixed source

All PGD approximations with error < 10%

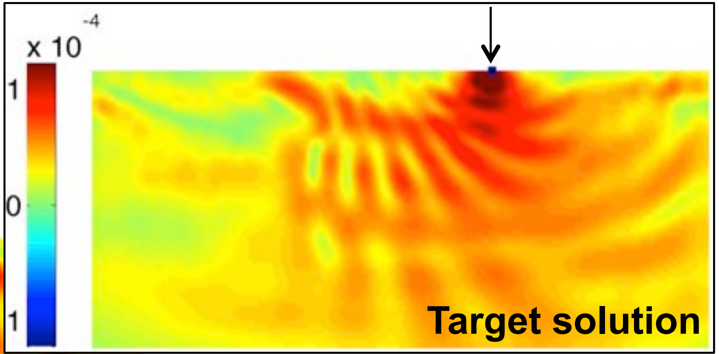




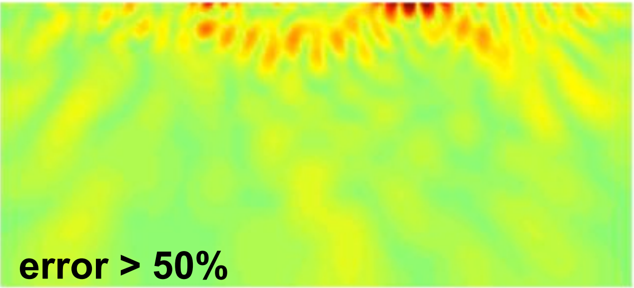


Preliminary (test) results

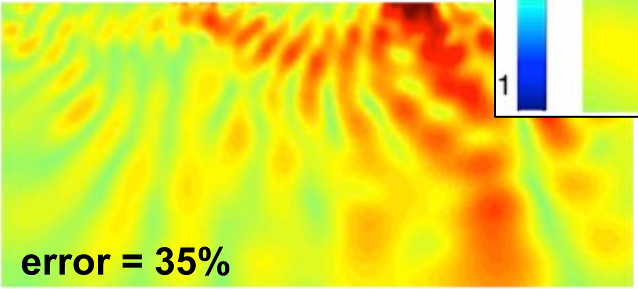
$$p(\mathbf{x},\mathbf{s}) \approx \sum_{m=1}^{n} F_m^1(\mathbf{x}) F_m^2(\mathbf{s})$$ with fixed freq = 1 Hz



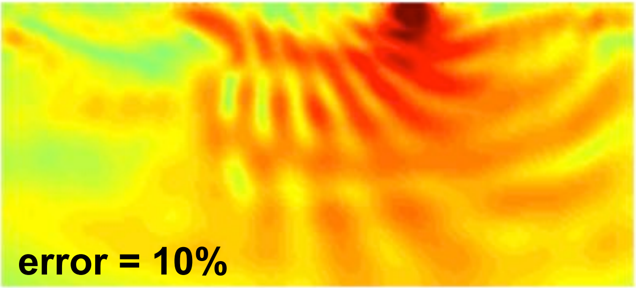
PGD 20 terms



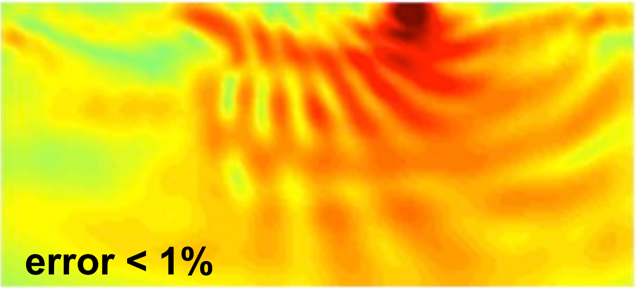
PGD 60 terms



PGD 100 terms



PGD 170 terms

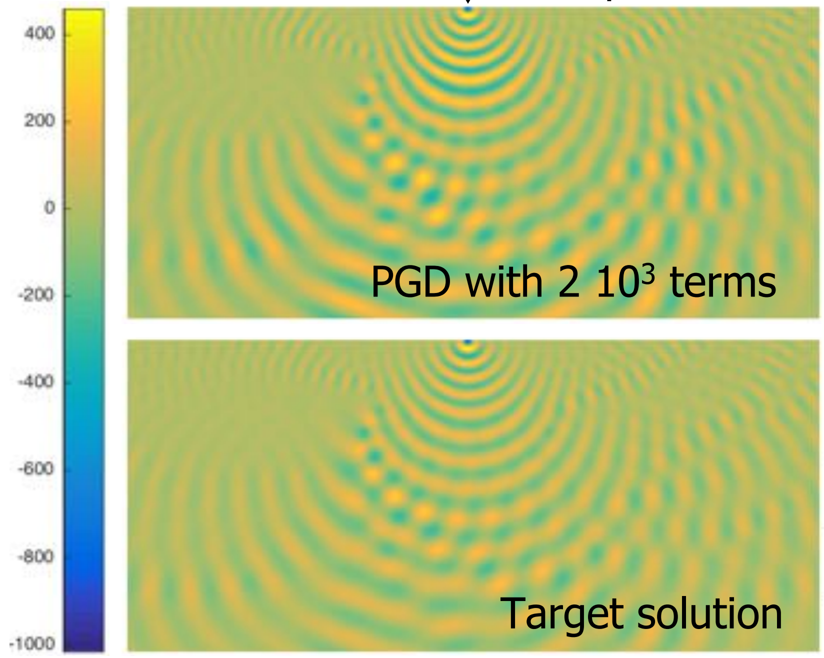


Preliminary (test) results

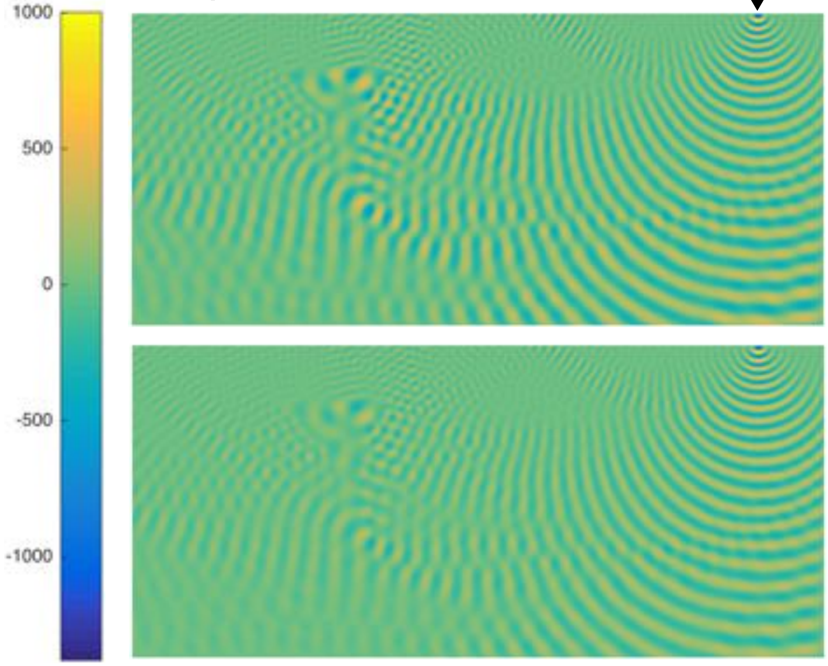
$$p(\mathbf{x}, \omega, \mathbf{s}) \approx \sum_{m=1}^{n} F_m^1(\mathbf{x}) F_m^2(\omega) F_m^3(\mathbf{s})$$

All PGD approximations with error < 10%

Freq = 2.82 Hz



Freq = 4.99 Hz



Final remarks and ongoing work

- A priori reduced order approximations can be exploited for seismic applications:
  - ✓ Avoids SVD and precomputed solutions
  - ✓ Provides immediate access to seismic propagations and derivatives
  - ✓ Take benefit from HPC software and non-intrusive implementations
  - ✓ More suitable for wave phase errors than for wave height errors
- First tests provide promising results with sufficient accuracy and moderate number of terms
- Currently working in:
  - ✓ Implementation of PGD solutions to accelerate imaging tools (RTM)
  - ✓ Reducing the offline cost:
    - ✓ Exploring spatial solvers that take profit from the PGD structure (iterative refinement techniques, preconditioners,...)
    - ✓ Adapted meshes

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