Numerical Simulations of Hydraulic Fracture Propagation — A Coupled Eulerian-Lagrangian Approach*

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Abstract

Current numerical simulations of hydraulic fracturing do a poor job of predicting how fracture networks propagate during hydrofrac operations. These simulations can propagate fluid filled cracks in 3D domains using poroelastic governing equations, a realistic, anisotropic, distribution of material properties, initial ambient stress and fluid pressure conditions that include geostatic and tectonic loads, and time dependent fluid pressure loading. They are incapable of modeling branching fluid filled cracks. The relative magnitude of the differential stress controls the direction and morphology of fracture propagation-as the differential stress magnitude diminishes, fracture orientations become random and favor a branching morphology. This study utilizes a Coupled Eulerian-Lagrangian (CEL) approach to simulate hydrofrac propagation using a method that allows branching fractures. A CEL formulation is a Finite Element Method (FEM) technique that has three fundamental components, an Eulerian FEM (EFEM), which models the fluid, a Lagrangian FEM (LFEM), which models the solid and general contact specifications to couple the two FEMs. In the EFEM, fluid, driven by fluid pressure gradients resulting from an injection source, is allowed to move through a fixed mesh. The LFEM has a deformable mesh and can track relatively small deformation and stress in elastic domains. Distributions of material properties can be propagated throughout domain if available. The general contact specifications govern the coupling between the two FEMs, satisfying quasi-static equilibrium over linear piece-wise surfaces normal to the fluid and bound by Lagrangian elements collocated with Eulerian elements having partial saturation. Both FEMs occupy the same space and contain appropriate material properties and initial, boundary, and loading conditions. The LFEM contains a cavity to simulate the injection point, and fluids within the EFEM are initially restricted to the cavity. The system is loaded to achieve the desired geostatic equilibrium and fluid flux is applied to the saturated zone of the EFEM. Fluid pressure increases until it exceeds the strength of some point of the chamber wall, which ruptures, introducing a fracture. The CEL analysis remeshes the LFEM to account for the crack, fluid flows into the fracture, and new coupling interfaces are created. With continuing pressurization, the fracture propagates according to the time dependent stress field and specified rock strength.

Reference Cited

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Motivations

- Current hydraulic fracturing technology is successful at a coarse level
- Predictive modeling is in its infancy
- By developing predictive models that accurately portray the mechanical properties of a reservoir, we can optimize a frac job before actually conducting a frac job

2D Finite element model
(simulates elastic behavior and failure)
Presenter's notes: Geostatic and tectonic loads are a major part of the stress field that controls fracture growth. We are injecting fluids and are quite interested in fluid flow, so poroelastic behavior rather than simple linear elastic behavior is useful. Injection rate varies throughout a frac job, so we have to be able to model this. Fluid filled cracks—we are hydraulically fracturing rock. Branching cracks—we are interested in creating an interconnected fracture network near the wellbore, a model should be able to produce this.
**Existing (and new!) FEM Strategies**

**Within Abaqus**

- Stress trajectories
- Traction separation
- Extended Finite Element Method (XFEM)
- Coupled Eulerian-Lagrangian (CEL)
Stress trajectories

- Simplest model
- Injection pressure is modeled as a pressure load on a cavity wall.
- Fractures are modeled by tracking trajectory of principal stresses.

* Fails to account for mechanical presence of propagating fracture
Traction separation

- Poroelastic
- Injection pressure is modeled as a traction force on the wellbore
- Fractures are modeled by separation along element boundaries

From Abaqus 6.12
Example Problems Manual # 10.1.5

Presenter’s notes: Similar to stress trajectory, but the fracture is modeled and stresses are coupled to the fracture propagation.
Traction separation

Pros and Cons

- Simple = fast
- Satisfies most model requirements…but
- The orientation of the crack must be defined BEFORE the analysis
  - Not a particularly realistic assumption
- The crack must follow the mesh
- The crack cannot branch
- Not particularly realistic

From Abaqus 6.12
Example Problems Manual
# 10.1.5
The Extended Finite Element Method

- Similar to traction separation
- Poroelastic
- Injection pressure modeled as a traction force on the wellbore
- A tip element hosts the actively propagating crack tip, while channel elements model crack expansion
- The crack arbitrarily propagates through an element (and therefore the mesh)


Presenter’s notes: Improvements over traction separation: 1) the fracture picks its own path; 2) not restricted by element boundaries – fracture does not have to follow the mesh.
XFEM

Pros and Cons

- Satisfies most model requirements
- The orientation of the crack is determined by the model
- The crack can travel through the mesh
- The crack may not branch
- More computationally expensive than traction separation
  - 3D simulations can take hours or days to run
Presenter's notes:

- 2D Poroelastic hydraulically driven fracture
- Note how all of the stress is concentrated in the crack tip
- Also note how the crack passes through elements
Presenter's notes:
• XFEM fracture driven by the top and bottom of the block being pulled apart
• Could be driven hydraulically as well, but it’s more computationally intensive.
• Note how fracture changes direction when it hits the oval shaped inclusion on left side of model
CEL

Coupled Eulerian - Lagrangian

- Eulerian – Fixed mesh, material flows through mesh
  - Ideal for modeling liquids
- Lagrangian – Deformable mesh
  - Good for modeling solids
  - If mesh deforms too far, the part will need to be remeshed
- Couple the types together!
- If pressure exerted at the coupling interface is high enough – fail the Lagrangian mesh – create a fracture!
Mesh design is the reason for an unexpected deformation in our proof of concept model.

Presentation notes:

**CEL**

**Procedure and challenges**

1. A fluid flux boundary condition adds fluid to the cavity
2. The stress applied at the coupling interface is calculated
3. Where the stress exceeds Lagrangian material strength, a fracture is modeled using Abaqus' brittle failure and shear functions
4. The fracture is modeled by separating Lagrangian elements
   - In the initial step, only mode I (opening) motion is allowed
   - After the fracture is opened, modes I and II (sliding) are allowed
5. Fluid then flows into the fracture from the cavity, and the process is repeated
   - It can be difficult to design meshes with sufficient refinement to capture detail of the coupling without over-refining, which adds unnecessary computational expense
Pros and cons

- Do not have to specify where fractures are located
- Can propagate multiple branching fractures
- Eulerian meshes can support non-Newtonian fluid flow
- NOT poroelastic like XFEM
  - The rock is elastic and fluids are confined to the cavity
- Fractures propagate along the mesh
- Computationally expensive – CEL is a brute force approach and involves computing 2 simulations simultaneously
  - An issue now, but in 5-10 years…maybe not

Presenter’s notes: Branching fractures are good because what we are interested in is stimulating a fracture network around the wellbore.
CEL

Proof of Concept design

- Eulerian - rectangular prism, 1 element thick
  - Eulerian meshes must be 3D
  - Get as close to 2D as possible
- Lagrangian – rectangular prism, same size as Eulerian, with a cylinder removed from the center
- Z velocity at the boundary is zero for the Eulerian
  - No fluid flow out of the boundary
- Z displacement is zero at the Lagrangian boundaries
- Sides are pinned
- Contact between fluid and solid is frictionless
- Material properties in the Lagrangian are set to allow brittle cracking and shear along cracks
- A fluid flux is introduced to the cavity

\[ U_1 = U_2 = U_3 = 0 \text{ on sides} \]
\[ V_3 = 0 \text{ on square faces} \]
Presenter's notes:

- Lagrangian mesh video of fluid flowing into a wellbore with purely elastic rock surrounding it
- Deformation is exaggerated so that it is visible
- The material is not allowed to fail – this is just a demonstration to show that the two meshes are in fact coupled
- The fluid filled cavity blows up like a balloon – just what we’d expect
- Next slide shows this same model, but with material properties modified to allow brittle failure
Presenter's notes:

- Video of Lagrangian domain
- Strain is highlighted
- Multiple fractures form
- The deformation extending into the "wellbore" is likely due to the mesh being too coarse (?)
Presenter's notes:
- Eulerian mesh video of fluid flowing into hydraulically induced fractures
- Blue – no fluid
- Red – completely saturated
- Note multiple fractures created and filled
Applications and Potential Improvements

- Modify contact specifications to allow fluid leakoff into a poroelastic Lagrangian?
- Use Monte Carlo methods to incorporate uncertainties into analyses

Presenter's notes: On the next page
Presenter’s notes:

We do not know enough, but we have some idea of what we do not know! Along with measured material property values, we also have uncertainties.

Use Monte Carlo methods to utilize these uncertainties:

- VERY expensive computationally – but computers are improving all the time, and this is a problem that can be parallelized
- Also can be done with XFEM and other models
- Run a large number of simulations
- Using known uncertainties, vary the material properties of each element in each model
- We can also randomly add natural fractures to each model run
- Combine all models together, provide estimates of fracture behavior
- These methods are already used in FEM deformation studies of earthquakes and volcanoes and can easily be adapted to fracture propagation problems.
- Example – using a simulated annealing algorithm to locate the magma chamber of Okmok volcano in the Aleutians with FEMs and known InSAR derived surface deformation
  - MH is a homogeneous half space model
  - MT is a heterogeneous model utilizing Young’s Moduli and Poisson’s Ratios derived from tomography
  - MTP is a heterogeneous model utilizing tomography AND uncertainties.
  - All models vary magma chamber position with each model run, MTP also varies material properties with each run
  - Sx, y, and z show the position of the source, delta P is the change in pressure between the two looks
- On left
  - P-Values is a ranking of the individual model run against all model runs
  - Gray areas are 99% confidence intervals for source location and pressure change
- On right
  - The models converging through time to a solution
  - Vertical axis is error in first plot, then X Y and Z estimates of magma chamber location and its change in pressure

So for a frac model, we would be able to give not only an estimate of how the fracture propagates, but how likely it is to propagate in that fashion.