Shear Velocity Prediction and its Rock Mechanic Implications*

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Abstract

Shear velocity is important in seismic inversion and petrophysical evaluation, particularly for evaluation of formation geomechanical properties. The shear (S-wave) velocity, compressional (P-wave) velocity and density can be used to estimate the Young's modulus, Poisson Ratio and Lame parameters in a petrophysical evaluation, which are helpful in determination of maximum and minimum horizontal stresses. However, the absence of the dipole shear sonic logs imposes severe limitations to such applications. Fortunately, the S-wave velocity, P-wave velocity and density can be inverted using current advanced pre-stack seismic inversion technique from seismic AVO angle gathers (Russell et al. 2005). Like the petrophysical evaluations, the vertical stress, minimum and maximum horizontal stresses can also be estimated from the seismic inversion if the shear velocity is available. Gray (2010) has proposed method of estimating horizontal stresses for optimizing hydraulic fracturing locations using seismic data. In their stress estimations, however, no consideration was made to corporate petrophysical data.

In this paper, firstly we investigate the possibility to predict the shear velocity from well logs. Several methods have been proposed to evaluate rock mechanics dealing with sandstones with variable shale and hydrocarbon contents. Parameters for characterizing rock mechanical properties, such as Poisson Ratio, pore pressure and minimum/maximum horizontal stress can be estimated for each borehole with proper well logs. Whereas when seismic data is used for the same purpose, pre-stack seismic inversion is performed to estimate the S-wave velocity, P-wave velocity and density, and then calculate the Poisson ratio, pore pressure and minimum/maximum horizontal stress from the derivatives. Because of the uncertainty and resolution in seismic inversion, the direct method to infer the S-wave velocity and P-wave velocity from seismic data usually has limitation of poor resolution. Instead, we propose the geostatistical inversion (Liu, 2009) to make the prediction of the shear velocity and other rock mechanical properties so that both seismic data and rock mechanical properties measured from well logs can be honored. The geostatistical inversion has proven to improve the inversion resolution and better incorporate the well logs.

Shear Velocity Prediction Methods

P-wave velocity (Vp) and S-wave velocity (Vs) show a linear correlation (Eq. 1) in water saturated sandstones (Han, 2004).

$$V_S - 0.79V_P - 0.79 \tag{1}$$

While in a study of the Milk River formation of Western Canada Sedimentary Basin, we found that the S-wave velocity can be estimated using a second-order poly-line equation (Eq. 2).

$$V_S = 0.000158V_P{}^2 - 0.6.2162\ V_P + 2153.32 \tag{2}$$

Eqs. (1) and (2) do not consider the effects of porosity, clay content and differential pressure. Castagna (1985) proposed a method for shear velocity estimation in shaly sandstones from porosity (φ) and clay content V_{cl}.

$$V_S = 4.89 - 7.07\varphi - 2.04\ V_{cl} \tag{3}$$

The φ and V_{cl} can be estimated from well logs.

The relationships in Eqs (1), (2) and (3) are suitable for water saturated sandstone only. Presence of gas drops dramatically the P-wave velocity, thus significantly affect the S-velocity estimation. We propose a method for estimating the shear velocity from P-wave velocity, porosity, shale content and saturation.

The response equation of transit time of sonic logs is:

$$\Delta t_c = \emptyset \Delta t_w + (1 - \emptyset - V_{cl})\Delta t_{ma} + V_{cl}\ \Delta t_{cl} \tag{4}$$

For gas-saturated formation, the (4) has the following form:

$$\Delta t_c = \emptyset(1 - S_g)\Delta t_w + \emptyset S_g \Delta t_g + (1 - \emptyset - V_{cl})\Delta t_{ma} + V_{cl}\ \Delta t_{cl} \tag{5}$$

Where Δt_c is sonic transit time, Δt_w and Δt_g are water and gas transit time respectively, Δt_{ma} and Δt_{cl} are sandstone matrix and shale transit time respectively, and S_g is gas saturation. In the gas-bearing formation, P-wave transit time will increase significantly and the P-wave velocity will decrease accordingly. However, the shear velocity is usually not affected by the gas-saturation. Therefore, for the transit time of S-wave logs, the response equation is:

$$\Delta t_s = \emptyset \Delta t_{sw} + (1 - \emptyset - V_{cl})\Delta t_{sma} + V_{cl}\ \Delta t_{scl} \tag{6}$$

Where Δt_s is S-wave transit time, Δt_{sw}, Δt_{sma} and Δt_{scl} are S-wave water, sandstone matrix and shale transit time respectively. For the clean sandstones, the porosity is often estimated by Wyllie empirical time-average formula:

$$\emptyset = (\Delta t_c - \Delta t_{ma})/(\Delta t_w - \Delta t_{ma}) \tag{7}$$

The shear transit time in (6) can be simplified as:

$$\Delta t_s = \left(\frac{\Delta t_c - \Delta t_{ma}}{\Delta t_w - \Delta t_{ma}}\right)\Delta t_{sw} + \left(1 - \frac{\Delta t_c - \Delta t_{ma}}{\Delta t_w - \Delta t_{ma}}\right)\Delta t_{sma} \tag{8}$$

and shear velocity is:

$$V_s = 1e6/\Delta t_s \tag{9}$$

Because Eq. (7) does not consider the contributions of gas-saturation or clay content, usually the shear velocity predicted using (8) is less than the measured shear velocity. However, if the measured shear velocity is available, the difference may be an indicator to detect the gas.

Geomechanical Application

The direct way to estimate the rock mechanical properties is using dipole sonic log, from which the P-wave velocity and S-wave velocity can be extracted. If not available, we can use sonic log coupled with one of the methods mentioned in the previous section to predict the shear velocity. According to the acoustic wave propagation theory, the P-Velocity (Vp) and S-velocity (Vs) can be expressed as:

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} \qquad V_s = \sqrt{\frac{\mu}{\rho}} \tag{10}$$

If the Vp, Vs and density are available, other elastic coefficients can be calculated in terms of the acoustic velocity.

$$M = \rho V_s^2 \qquad E = \frac{\rho V_s^2 (3V_p^2 - 4V_s^2)}{V_p^2 - V_s^2}$$
$$\lambda = \rho V_p^2 - 2\rho V_s^2$$
$$k = \rho V_p^2 - \frac{4}{3}\rho V_s^2 \qquad \nu = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)} \tag{11}$$

where **E** is young's modulus, ν is Poisson ratio, **k** is bulk modulus, **M** is shear modulus, and λ is lame parameter.

1) Pore Pressure

Based on the generic Karl Terzaghi equation for shale, the pore pressure is given:

$$\lambda_p = \lambda_v - \lambda_e \tag{12}$$

Where λ_v is the overburden, λ_p is the pore pressure and λ_e is the effective vertical stress. The overburden is derived from the density data:

$$\lambda_v = \int_0^H \rho(h)gdh \tag{13}$$

where g is the gravitational constant, $\rho(h)$ is the bulk density. The effective vertical stress can be estimated from the interval velocity, which can be inverted from seismic inversion or sonic logs:

$$\lambda_e = \lambda_0 \left[\frac{V - V_{min}}{V_{max} - V_{min}} \right]^\beta \tag{14}$$

where λ_0 and β are coefficients, V_{max} and V_{min} are maximum and minimum velocities respectively.

2) GeoStress

The state of stress at a point in the formation is characterized by the magnitudes and orientations of the three principal stresses and the pore pressure. Initially it is assumed the vertical stress is a principal stress, another two principal stresses are the maximum and minimum horizontal stresses. The magnitudes can vary with the changes in lithology and pore pressure, but the orientations are much stable over wide areas, except near active faults. The direction of the minimum horizontal stress can be determined by the azimuth of wellbore breakout, which can be detected by the dual-calipers or FMI images.

The vertical stress can be estimated using (13) and another two principal stresses can be determined from empirical relationships based on Hooke's law. The following equation (15) is one of those for calculation of the minimum (σ_h) and maximum (σ_H) stresses (Li et al, 1997):

$$\sigma_h = \alpha\lambda_p + \frac{v}{1-v}(\lambda_v - \alpha\lambda_p) + \frac{E\varepsilon_h}{1-v^2} + \frac{vE\varepsilon_H}{1-v^2} + \frac{E(\alpha_T\Delta T)}{1-v^2} + \Delta\sigma_h$$

$$\sigma_H = \alpha\lambda_p + \frac{v}{1-v}(\lambda_v - \alpha\lambda_p) + \frac{E\varepsilon_H}{1-v^2} + \frac{vE\varepsilon_h}{1-v^2} + \frac{E(\alpha_T\Delta T)}{1-v^2} + \Delta\sigma_H \tag{15}$$

Where α is poro-elastic constant, α_T is the coefficient of linear thermal expansion, ε_h is strain in the minimum horizontal direction, ε_H is strain in the maximum horizontal direction. ΔT is temperature gradient. $\Delta\sigma_h$ is formation minimum erosion contribution for horizontal fracture. $\Delta\sigma_H$ is formation minimum erosion contribution for horizontal fracture.

From the orientations and magnitudes of the estimated three principal stresses, one can infer some useful features of faults and factures. For example, the normal fault: $\lambda_v > \sigma_H > \sigma_h$; reverse fault $\lambda_v < \sigma_H < \sigma_h$; the vertical fracture: $\lambda_{v >} \sigma_h$ and horizontal fracture: $\lambda_v < \sigma_h < \sigma_H$.

Geostatistical Inversion

The geostatisitcal inversion used in this paper is based on Bayesian Inference and uses a Support Vector Machine (SVM) (Liu, 2009] method to honor seismic attributes, such as AVO attributes, Vp, Vs and density from seismic inversion, well-log data and geological knowledge. Given a training set of seismic attributes with the corresponding well rock mechanical property target from available boreholes, the SVM establishes the mapping functions between seismic attributes $\{\tilde{x}_n\}$ and well rock mechanical property $\{t\}$, such as Poisson Ratio and geostresses. The mapping function may have the following formula:

$$t(x) = \sum_{n=1}^{N} \omega_n K(\tilde{x}, \tilde{x}_n) + \omega_0 + \text{space constraint} + \text{geological constraints} \qquad (16)$$

Where $\{\omega_n\}$ are the model weights, which are determined during the training phase. $\mathbf{K}(\tilde{x}, \tilde{x}_n)$ denotes the kernel function, such as Gaussian function. The space constraint means the Cartesian coordinate system of both seismic and well. If well rock mechanical property is preferred orientation, the anisotropic settings, such as a ratio and angle, could be applied.

After the training phase, the SVM can propagate the borehole rock mechanical properties onto 2D/3D seismic space using the seismic attributes and geological constraints, and then to generate the 3D rock mechanical model.

Examples

Figure 1 is one of the well evaluation results. Track 1 to 7 are original wells logs, track 9 is hydrocarbon volume indicator, track 10 is the volume of lithology, track 11 is Vp, Vs and the ratio of Vp and Vs, track 13 to 15 are the rock mechanical properties, such as Young's modules, track 16 is pore pressure and overburden vertical stress, and track 17 is the three principal stresses and its stress ratio. Because the rock properties from well evaluation are dynamic, the laboratory data will be helpful to convert the dynamic to the static properties. Figure 2 shows pore pressure distribution referred from our geostatisitcal inversion. Horizontal axis is CDP direction and vertical axis is true vertical depth. The top two high abnormal pore pressure areas could be affected by the gas hydrate. In addition, the middle abnormal pore pressure could be indication of free gas.

Conclusions

We demonstrate that the S-wave velocity can be estimated from P-wave velocity, porosity and clay contents if the dipole sonic log is not available, and other rock mechanical properties can be estimated from Vp, Vs and density. The pore pressure and three principal stresses can be inferred from borehole logging, which will be helpful to detect the hydraulic fracture and to study the wellbore stability. The P and S-velocity simultaneous inversion and integrated geostatistical techniques from pre-stack seismic gathers will extend the rock mechanical application from borehole to 3D rock mechanical properties space, which will be useful to optimize the hydraulic facture locations and to analyze the well trajectories for optimal well placement.

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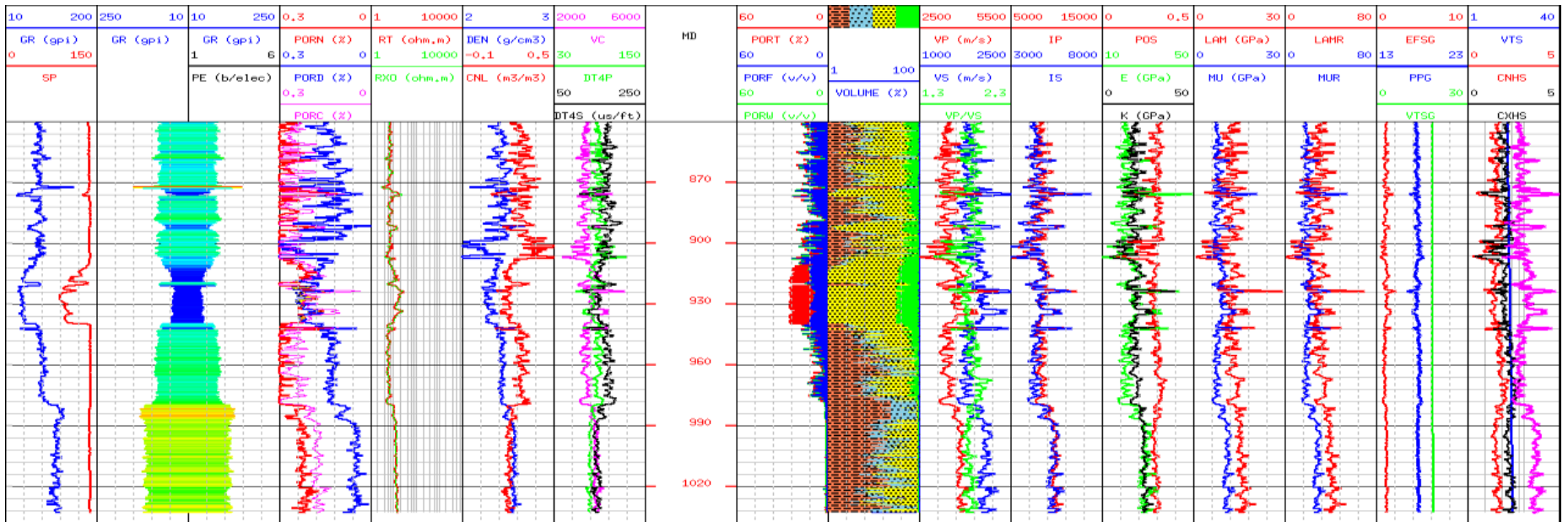
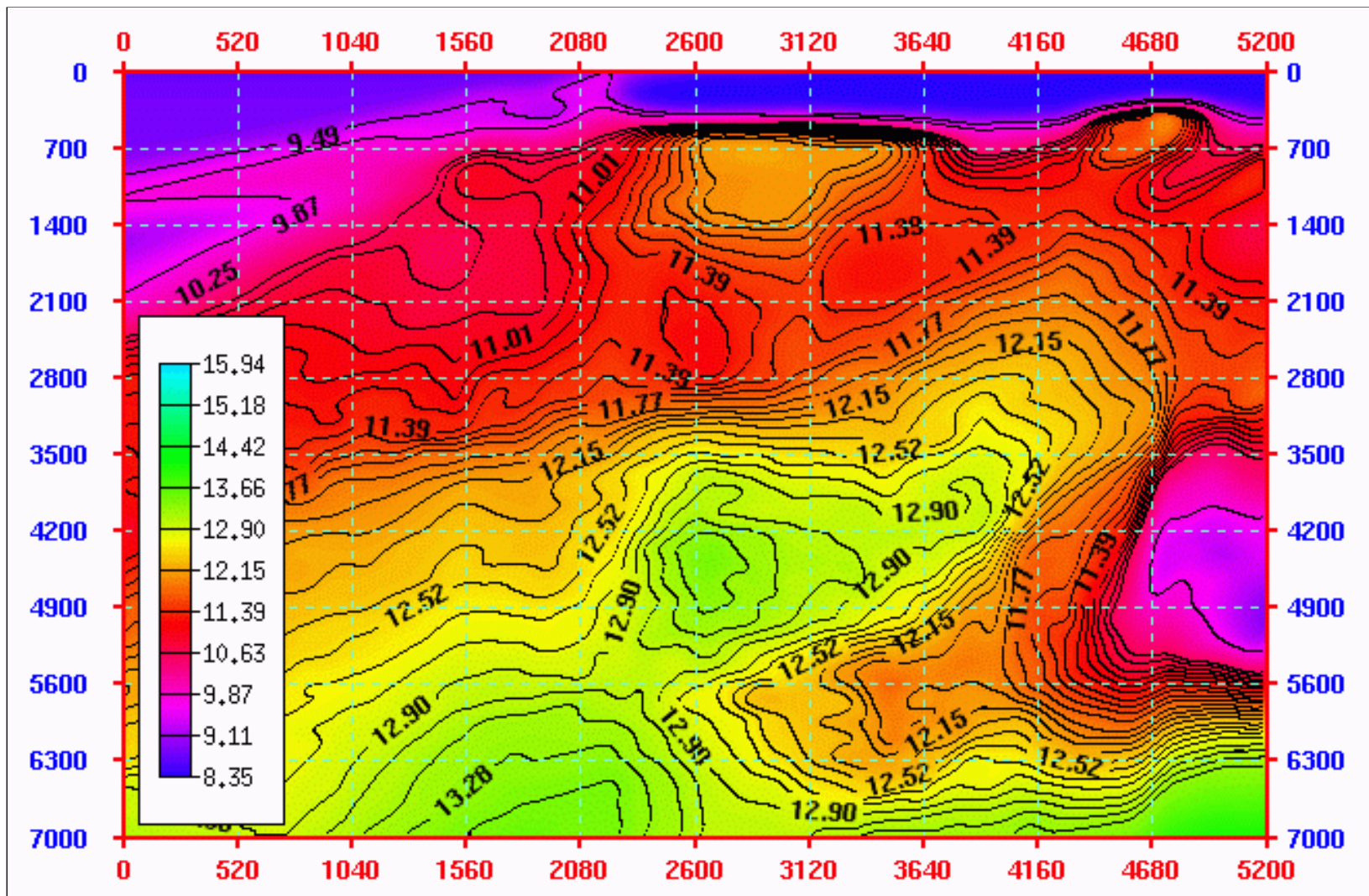Figure 1. Well evaluation to estimate the rock mechanical property and geostress.

Figure 2. Pore Pressure Prediction using seismic inversion.