Fast, Fully Probabilistic, Nonlinear Inversion of Seismic Attributes for Petrophysical Parameters*

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Search and Discovery Article #120047 (2012)
Posted December 31, 2012

*Adapted from extended abstract prepared in conjunction with poster presentation at AAPG Hedberg Conference, Fundamental Controls on Flow in Carbonates, July 8-13, 2012, Saint-Cyr Sur Mer, Provence, France, AAPG©2012

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Abstract

We applied a fast, fully probabilistic inversion technique based on neural networks to predict 3D petrophysical properties from inverted pre-stack seismic data. The objective of petrophysical inversion is to estimate the joint probability density function (PDF) of model vectors consisting of porosity, clay content, and water saturation components at each point in the reservoir, from data vectors consisting of compressional- and shear-wave-impedance components, obtained from the inversion of AVO seismic data. The petrophysical inverse problem is significantly non-linear, and due to the large number of data points in a seismic cube we need to apply fast inversion methods. In this study we consider the effect of different sources of uncertainty on the a posteriori PDF of model parameters. These sources include variations in effective pressure, bulk modulus and density of hydrocarbon, random noise in recorded data, and uncertainties in petrophysical forward function. Results show that the standard deviations of all model parameters are reduced after inversion, which shows that the inversion process provides information about all parameters. The reduction of uncertainty in water saturation is smaller than that for porosity and clay content; nevertheless the maximum of the a posteriori PDF of water saturation (MAP) clearly shows the boundary between brine saturated and oil saturated rocks.

Introduction

Seismic data are usually the only source of information available throughout a field that can be used to predict the 3D distribution of petrophysical properties (e.g. porosity, clay content, and water saturation) with appropriate spatial resolution. The main challenge in inferring properties from seismic data is the ambiguous nature of geophysical information. Uncertainty enters into the problem in at least three levels: first, there is non-uniqueness in the inversion of (here, AVO) seismic data for the acoustic impedances of rock, second there is non-uniqueness in the petrophysical inversion of the rock impedances for rock-fluid properties given a petrophysical relationship between

the acoustic properties and rock-fluid properties, and third there is ambiguity in these petrophysical relationships themselves (Doyen, 1988). Therefore, any estimate of rock and fluid property maps derived from seismic data must also represent its associated uncertainty.

The Monte Carlo method has been used to simulate the elastic response of earth models by exploring priori ranges of rock and fluid properties using petrophysical forward relations (Bachrach, 2006, Spikes et al., 2007, Bosch et al., 2007). Statistical rock physics, which is also based on the Monte Carlo sampling, has been used as a general tool to address uncertainties associated with the petrophysical relations (Avseth, 2005). In all of the above studies, it is indicated that while in principle the MC sampling method can map uncertainty of petrophysical parameters, in practice applying it to invert seismic attributes for rock and fluid properties is computationally highly demanding.

As a solution we applied the mixture density network (MDN) (Shahraeeni and Curtis, 2011), to invert seismic attributes for petrophysical properties (i.e. porosity, clay content, water saturation). A Mixture Density Network (MDN) is a particular extension of neural networks that maps a deterministic input vector onto a PDF over uncertain output vectors (Shahraeeni, 2010). In an MDN, any arbitrary PDF is modeled as a mixture (weighted sum) of Gaussian PDF's. A neural network is trained to estimate weights, mean vector, and covariance matrix of each Gaussian from a training data set. We first briefly summarize the MDN method of solving an inverse problem in the petrophysical inversion context and then present the result of inversion of industrial seismic compressional and shear impedances, I_p and I_s, for the joint PDF of effective porosity φ_e, clay content V_{cl}, and water saturation s_w, using calibrated petrophysical relations.

Method

Mixture Density Network (MDN)

A neural network is essentially a flexible function or mapping. By varying the parameters (i.e. weights) within the network, we can change the mapping. Varying the parameters to emulate a specific, desired mapping is called training the network. Networks are usually trained by fitting them to examples of the input and output values of the mapping. The set of examples used is called training data set.

One application of neural networks is therefore to estimate some given mapping from an input vector, **d**, to a target vector, **m**. Any uncertainty associated with the target vector in this mapping can be represented by the probability density of **m** conditioned on (or given) value of **d**, written as $p(\mathbf{m}|\mathbf{d})$. The mixture density network (MDN) is a type of neural network that can be trained to emulate an approximation to $p(\mathbf{m}|\mathbf{d})$. Within the MDN, $p(\mathbf{m}|\mathbf{d})$ is represented by a mixture or sum of known probability densities:

$$p(\mathbf{m}|\mathbf{d}) = \sum_{i=1,\ldots,m} \alpha_i(\mathbf{d})\, \varphi_i(\mathbf{m}|\mathbf{d}) \qquad (1)$$

In the above equation, $\varphi_i(\mathbf{m}|\mathbf{d})$ is a known PDF and is called a kernel, m is the number of kernels, and $\alpha_i(\mathbf{d})$ is called the mixing coefficient that defines the weight of each kernel in the mixture (the sum). This representation of the probability density function is called a mixture model. It can be shown that a mixture of densities with Gaussian kernels can approximate any PDF to any desired accuracy, given a sufficient number of kernels with appropriate parameters (Shahraeeni, 2010).

The solution to any inverse problem is a definition of the extent to which any combination of model parameter values are consistent with the data, given the data uncertainty. In mathematical terms the solution can be expressed as (Tarantola, 2005):

$$\sigma_M(\mathbf{m}) = K\rho_M(\mathbf{m})L(\mathbf{d}|\mathbf{m}) \quad (2)$$

In the above equation **m** is the model vector, **d** is the data vector, L is the likelihood, ρ_M is the a priori PDF of model vector, and K is a normalizing constant.

The MDN is trained to emulate $\sigma_M(\mathbf{m})$ for any measured data **d**. This is achieved using pairs of a priori samples of model and data vector pairs (**m**,**d**). The set of sample pairs is called a training data set and is constructed in the following way: samples $\mathbf{m}_i$, $i = 1,\ldots,N$, are taken according to the a priori model PDF ρ_M, and for each sample the corresponding synthetic data $\mathbf{f}(\mathbf{m}_i)$ is calculated, where **f** is the forward function. Several samples, $\varepsilon_{i,j}$, $j = 1,\ldots,R$, of data measurement uncertainty, as well as of theoretical uncertainty in the forward function **f** (both of which are represented within the likelihood function L), are added to each calculated synthetic data vector $\mathbf{f}(\mathbf{m}_i)$. This results in several samples of possible synthetic data vectors for each sample of the model vector $\{(\mathbf{m}_i,\mathbf{f}(\mathbf{m}_i)+\varepsilon_{i,j}): i=1,\ldots,N; j=1,\ldots,R\}$. Using these sample pairs, which for short we denote $(\mathbf{m}_k, \mathbf{d}_k)$, $k = 1,\ldots,NR$, the network is trained to map any data vector including its uncertainty into an approximation of a posteriori PDF of model vector **m**. This approximation is represented as in equation 1, and during training neural network learns to estimate mixing coefficients $\alpha_i(\mathbf{d})$, mean vector and covariance matrix of each kernel $\varphi_i(\mathbf{m}|\mathbf{d})$ for any given data vector **d** within the range of the training dataset $(\mathbf{m}_k, \mathbf{d}_k)$, $k = 1,\ldots,NR$, (Shahraeeni and Curtis 2011).

Inverted Seismic Data

A 3D simultaneous elastic inversion technique jointly inverted near-, mid- and far-angle sub-stacks to derive estimates of the 3-dimensional distribution of compressional and shear wave impedances, I_p and I_s. The a priori information about elastic impedances at 4 well locations was propagated throughout the reservoir using kriging technique and simulated annealing technique was used to update this a priori information with seismic data. Uncertainty of the estimated value of I_p and I_s is estimated by statistical comparison of upscaled I_p and I_s values derived from well logs with seismic-inversion results (Bachrach, 2006).

Petrophysical Forward Function and Model Vector

The petrophysical forward relations are a combination of Gassmann's law to account for fluid substitution, a mixing law to account for the mixed lithology (sand and shale), and empirical depth trend curves to describe pressure effects on the bulk and shear moduli of reservoir rocks. The components of the output vector of this petrophysical relation, **d**, are estimated compressional- and shear-wave impedances, $\mathbf{d} = (I_p, I_s)$. The input vector is $\mathbf{m}' = (\varphi_e, V_{cl}, s_w, p_e, K_{hc}, K_w, \rho_{hc}, \rho_w)$ where p_e is effective pressure, K_{hc} and K_w are bulk modulus of hydrocarbon and brine, and ρ_{hc} and ρ_w are density of hydrocarbon and brine, respectively. Error for predicted values of I_p and I_s using the petrophysical forward function are equal to 4% and 6%, respectively (Chao, 2009).

The a priori PDF of the input vector of the model parameters are obtained from well log samples. In our inverse problem the data vector $\mathbf{d} = (I_p, I_s)$ is inverted for the marginal joint PDF of porosity, clay content, and water saturation, $\mathbf{m} = (\varphi_e, V_{cl}, s_w)$. Other input parameters of the petrophysical forward relations are treated as confounding parameters, $\mathbf{m}_{\text{conf}} = (p_e, K_{hc}, K_w, \rho_{hc}, \rho_w)$ – parameters that can increase uncertainty of the model vector $\mathbf{m}$ in the inversion process. In mathematical terms, the confounding petrophysical parameters are integrated out in the marginal a posteriori PDF of the model vector $\mathbf{m}$:

$$\sigma_M(\mathbf{m}|\mathbf{d}) = \int_{\mathbf{m}\_\text{conf}} \sigma_M(\mathbf{m}'|\mathbf{d})\, d\mathbf{m}_{\text{conf}} \qquad (3)$$

Where, $\mathbf{m}_{\text{conf}}$ and $\mathbf{m}'$ are defined above. The above equation shows that the effect of confounding model parameters is to increase uncertainty in the a posteriori model vector $\mathbf{m} = (\varphi_e, V_{cl}, s_w)$.

Training Data Set and MDN Specifications

Model vectors in the training data set are constructed using uniform sampling from their a priori PDF. Effective porosity and clay content are dependent variables and 846 samples from their joint a priori PDF are selected. Water saturation is uniformly distributed between 0 and 1 and 40 samples were selected from this interval. Effective pressure changes between 173 and 332 bar and 6 samples were selected from this interval. Bulk modulus and density of hydrocarbon were given as functions of effective pressure. Bulk modulus and density of brine was constant. The total number of model samples constructed from the a priori PDF was 642,960.

The above number of samples of the desired model parameters, i.e. (φ_e, V_{cl}, s_w), will reduce the interpolation error of the MDN inversion result. We selected smaller number of samples from the confounding model parameters, i.e. $(p_e, K_{hc}, K_w, \rho_{hc}, \rho_w)$, because the effect of these parameters are integrated out by the MDN.

A denser sample selection of the above parameters will improve the accuracy of the MDN, however, it will increase the training time significantly. The number of kernels in the MDN was equal to 10 and the number of hidden units was equal to 27. The procedure of selecting the parameters of the MDN is explained in Shahraeeni and Curtis, 2011.

Example

Figure 1 shows the marginal PDF's of porosity, clay content, and water saturation, which are derived from the inversion of seismically derived I_p and I_s in one well. This figure shows that the marginal a posteriori PDF of the model parameters is a reasonable estimate of log values.

Figure 3 shows the maximum a posteriori (MAP) and the standard deviation of the marginal PDF of porosity, clay content, and water saturation obtained from inverting one cross-section of inverted I_p and I_s seismic cube. The highest value of the color bar (dark red) in the standard deviation maps represents the a priori standard deviation of the parameters. The estimated standard deviations of the parameters in the cross section are always smaller than this value (darkest color in the standard deviation cross-sections is orange). This means that after

inversion uncertainty about all model parameters is decreased. However, it is clear from the figure that uncertainty of water saturation is much higher than porosity and clay content.

Discussion and Conclusions

The difference between the estimate of the model parameters and measured values of these parameters at the well location (Figure 1) can be related to three factors: (1) accuracy of inverted I_p and I_s, (2) accuracy of the forward petrophysical relationship, and (3) differences between bandwidth of seismic data and well log data.

The petrophysical forward function is calibrated with well log data and is used for inversion of seismically derived I_p and I_s. Brown and Seifer (1997) mentioned that velocity measurements in different directions using different frequencies (e.g. seismic data and well logs) require consideration of elastic scattering, intrinsic attenuation, volume effects, and path effects. Separation of these effects is important in reservoir characterization using multi-frequency measurement. Here these effects can be partially responsible for errors observed in Figure 1.

Figure 2 shows I_p and I_s obtained from seismic data and compare it with upscaled logs. This figure shows that even if we assume that the petrophysical forward function is perfectly calibrated with well logs, we should expect differences between estimated porosity, clay content, and water saturation from seismic I_p and I_s and those measured by well logs. For example, on the 2025 ms-2070 ms interval we observe that seismic I_s has large biases with respect to Backus averaged I_s log. On the same interval in Figure 1 we observe large differences between porosity and clay content obtained from seismic data and those measured in the upscaled well logs. We also observe large biases between seismic I_p and Backus averaged I_p on the 2100 ms-2150 ms interval. On this interval we observe large differences between clay content obtained from seismic data and upscaled well logs.

Figure 1 and Figure 3 show that the a posteriori uncertainty of all model parameters is large. The a posteriori uncertainty stems from uncertainty in confounding model parameters (i.e. effective pressure, bulk modulus and density of hydrocarbon), uncertainty of the petrophysical forward function, and uncertainty in the seismic I_p and I_s. In addition we invert only two data for three model parameters, and hence simply by appealing to dimensionality arguments a unique solution is impossible without strong a priori information. Analysis of Gassmann equation (Shahraeeni, 2010) shows that even when porosity and clay content values are known, for a given value of I_p and I_s the uncertainty of water saturation is high. This means that reduction in uncertainty of water saturation from petrophysical inversion is always small (Figure 1 and Figure 3).

Figure 3 shows that if this method is applied to invert 3D seismic I_p and I_s sections, the estimates of porosity, clay content, and water saturation can provide a detailed description of rock properties in a reservoir. The lateral and vertical continuity of the I_p and I_s sections, which is a result of applying kriging in the AVO inversion of seismic data, is preserved in the petrophysical parameters. A priori geological information about model parameters can be used to further constrain petrophysical inversion result.

In order to address uncertainties in the model parameters appropriately, any petrophysical inversion method must be probabilistic. The MDN method is a time and memory efficient method for probabilistic nonlinear inversion (Shahraeeni and Curtis, 2011). The nonlinear inversion

of each cross section, which included 170,322 data points and resulted in full joint posterior PDF's for φ_e, V_cl, and s_w, took 340 seconds on a regular desktop computer. The number of cross line sections is 1461 so inverting the whole 3D seismic cube with 248,840,442 data points takes only around 138 hours on the same desktop computer. Conventional sampling method to solve the same number of inverse problems will take around 27,500 hours on the same desktop computer. Note that there is a trade-off between time and memory efficiency of the MDN inversion technique, and the accuracy of the a posteriori PDF of the model parameters estimated by this technique.

As the inversion technique is independent of particular seismic attributes chosen in this case study (i.e. I_p and I_s), it can be applied to invert any set of pertinent attributes of seismic data (e.g. I_p and Poisson's ratio) for the petrophysical parameters.

Acknowledgements

We would like to thank TOTAL E&P UK for sponsoring this research.

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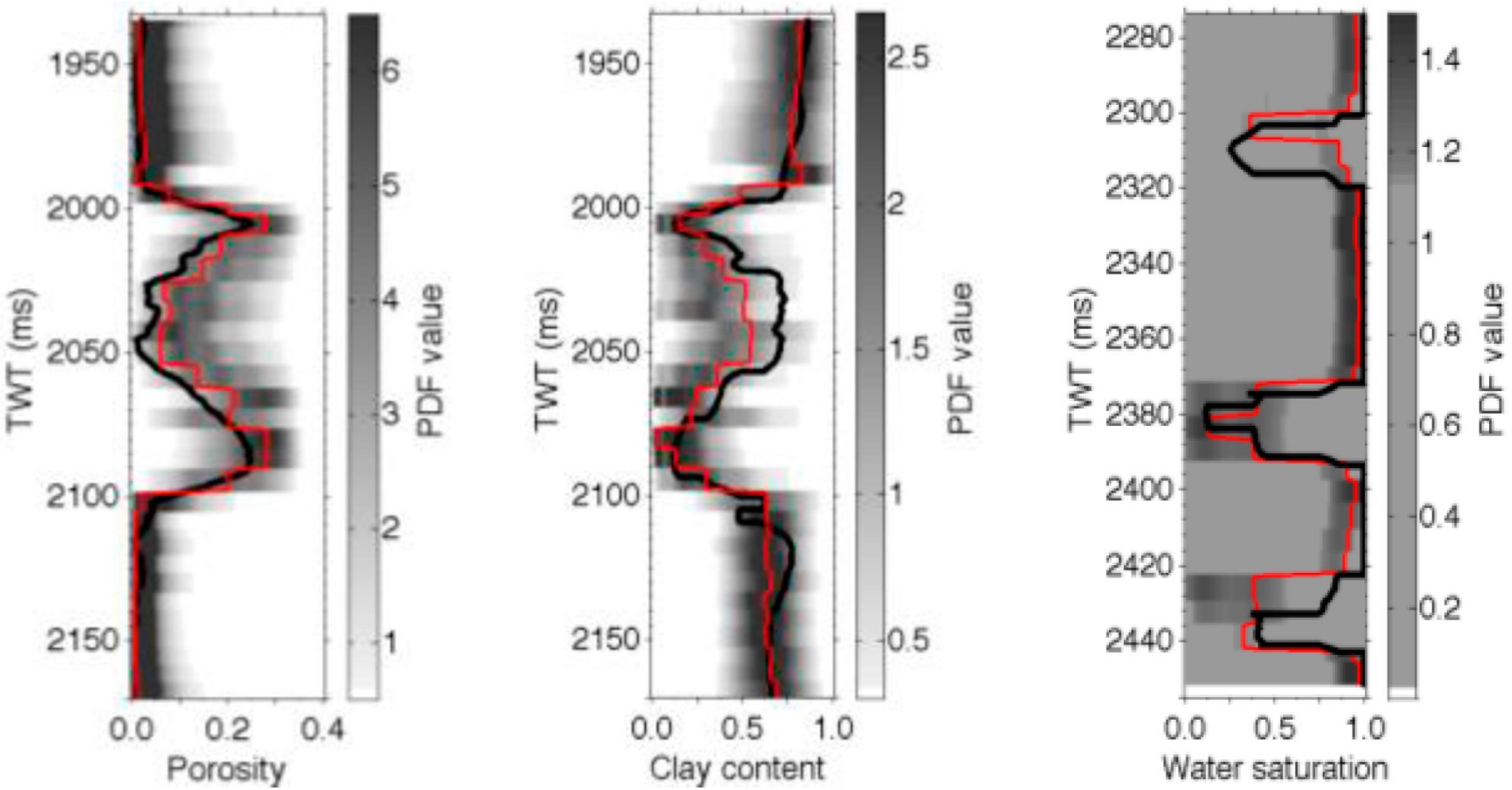

Figure 1. Marginal a posteriori PDF's of porosity (left), clay content (center), water saturation (right). Darker colors show higher probabilities. Solid black line is upscaled measured log. Solid red line is the maximum a posteriori estimate.

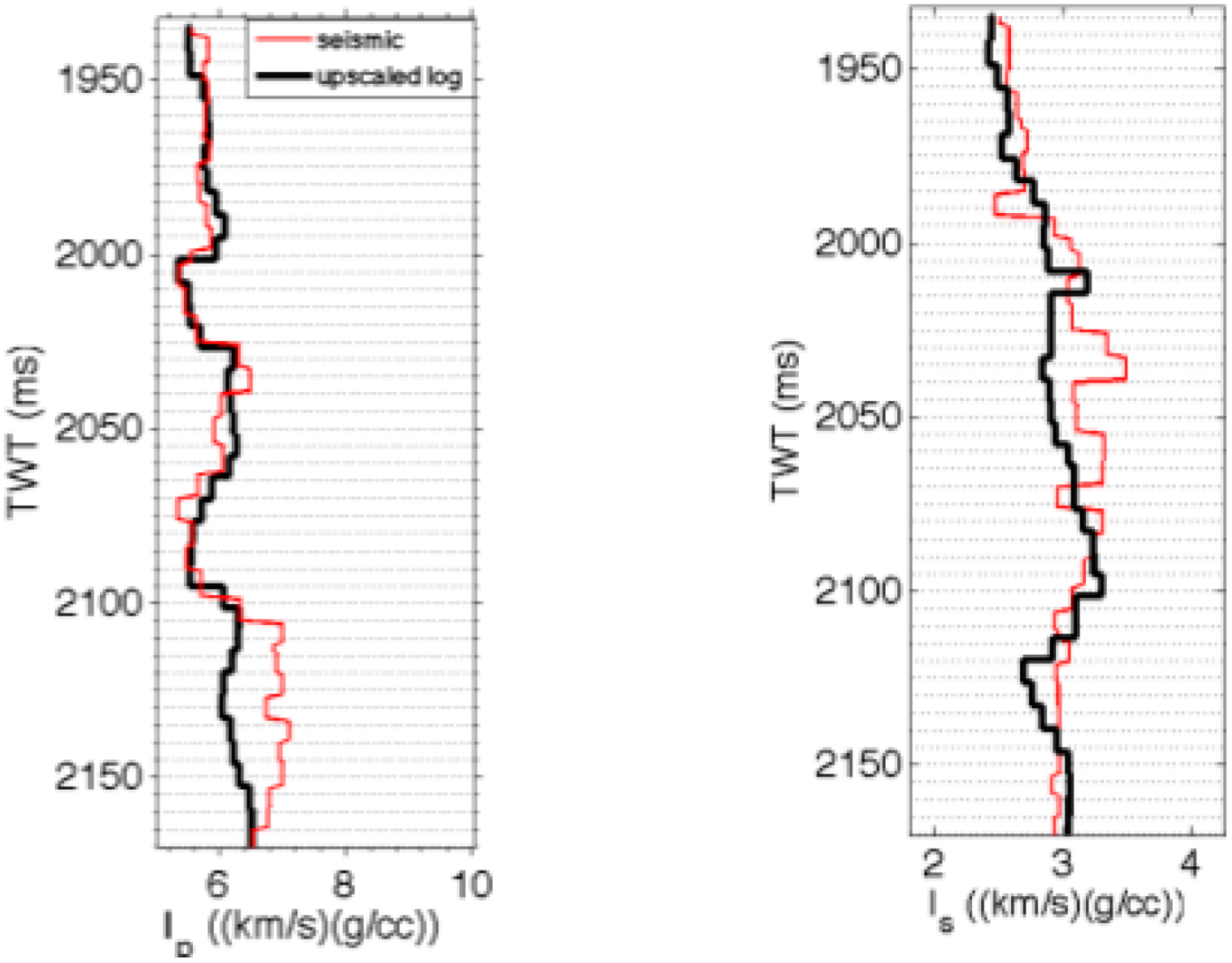

Figure 2. Comparison between seismic I_p and I_s (red) and Backus averaged I_p and I_s measured logs (black).

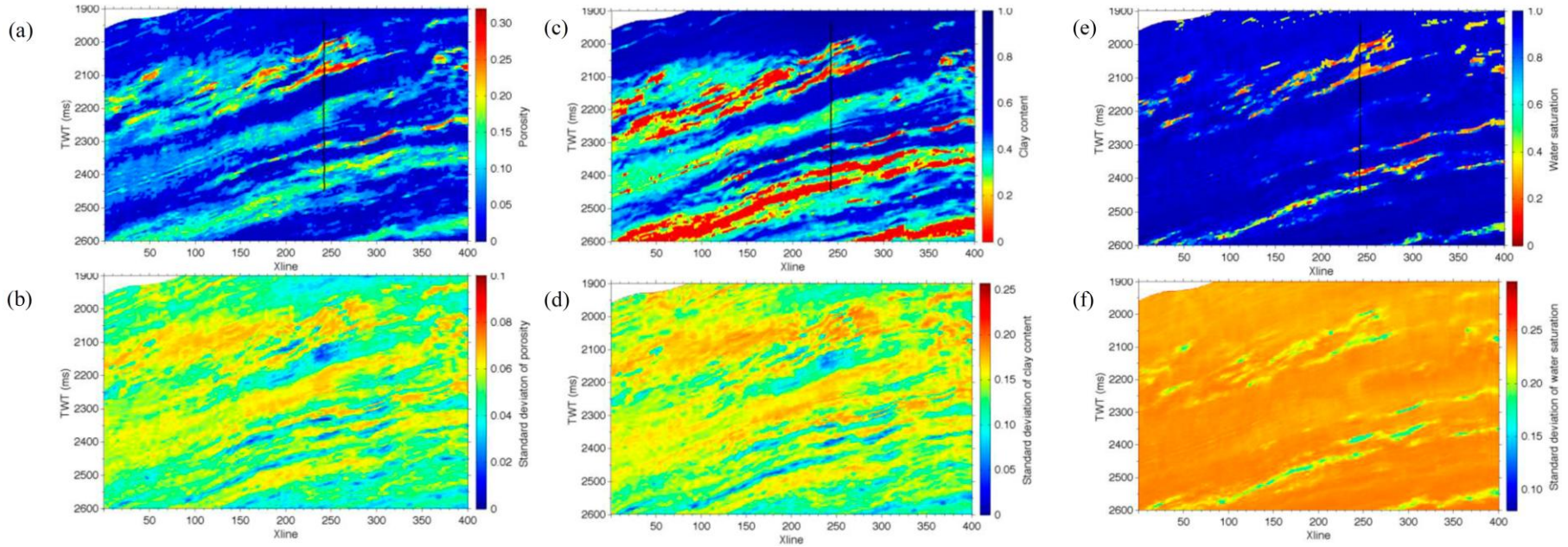

Figure 3. (a) Maximum a posteriori (MAP) estimate of φ_e. (b) Standard deviation of φ_e. (c) MAP estimate of V_{cl}. (d) Standard deviation of V_{cl}. (e) MAP estimate of s_w. (f) Standard deviation of s_w. The scale of the TWT and Xline axes are from real data but the numbers are fictitious.