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Forecasting Petroleum Production with a Model Based from Discharge Process*

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Abstract

This work deals with a mathematical model to forecast the production of oil and gas at both national and worldwide scales, by having as input the historical production series and the estimated ultimate resource (EUR). This volume is defined as the sum of three factors, namely, the reserves, the cumulative production and the estimate of undiscovered resource potential.

The proposed model consists of representing the depletion of a finite resource at a variable rate through time. Therefore the model allows the evolution of an increasing production up to a peak followed by a decline down to the resource exhaustion.

The graph of the proposed model is similar to a unimodal probability density function, with variable asymmetry, either negative (longer tail at initial, past years) or positive (longer tail at future years). The functional shape is a consequence of the uncountable possibilities of parameter value combinations. Symmetry, like in the logistic function, is just a very special instance, unlikely to occur. The parameters of the model are the estimated ultimate volume (EUR), the initial production and the initial rate of decline. Fitting the model to historical time series of production may be constrained by hypotheses about future demand and rate of decline.

For the sake of comparison, both the logistic function and the proposed model were fitted to the historical series of American petroleum annual production, using data published by the USGS (US Geological Survey) and the DOE/EIA (US Department of Energy/Energy Information Administration), from which three quantile estimates of EUR were taken, to quantify the uncertainty.

From a statistical perspective, the proposed model works better than the logistic function, when both are fitted to the historical data, with or without restrictions on the parameters. The logistic function underestimates the observed values after 1990, while the proposed model remains

adherent, following the positively asymmetric trend shown by the historical series over the past two decades. Therefore the proposed model is more flexible than the logistic, which is constrained to the symmetrical bell shape. The model was further applied to forecast the World Oil and Gas Production.

Introduction

Estimates of undiscovered resources, future demand and prices of oil and gas are outstanding among the factors that influence the long range planning of government agencies and petroleum companies. Based on scenarios, the evolution of reserves, production and R/P (Reserves/Production ratio) is forecasted. Strategic decisions are made either to benefit from favorable projections or to avoid the consequences of unfavorable outcomes.

This work deals with a model developed for EPE (Empresa de Pesquisa Energética, Energy Research Enterprise) to forecast the production of oil and gas at both national and worldwide scales, by having as input the historical production series and the Estimated Ultimate Recovery (EUR) volume. This is defined as the sum of four components, namely, the reserves, the reserve increments due to enhanced recovery, the cumulative production and the estimate of undiscovered resource potential.

Model

The model is based on the recognition that the development of the upstream petroleum industry in large scale aggregates a series of sequential and interconnected processes, according to the categories of uncertainty of the petroleum resource involved, from basin analysis and petroleum systems characterization to production of oil and natural gas. Exploratory prospects identification, oil and gas field discovery and pool delineation for proving reserves are the intermediate processes. The evolution of each step or process generates, or discharges, a stock or charge for the evolution of the next process. For instance, the discovery process in an exploratory play involves a petroleum potential resource, which can be seen as a stock or charge, from which discoveries are made and so new charges are provided to the evolution of the subsequent process of reserve proving.

The proposed mathematical model consists of representing the depletion of a finite resource at a variable rate through time. Therefore the model allows the evolution of an increasing production up to a peak followed by a decline down to the resource exhaustion.

In mathematical terms, the model evolved from studies developed by Petrobras (Da Silva, 1987; Da Silva and Rodrigues, 1996) and EPE (2007) to the Equation (1), which represents the cumulative production from a finite resource, assuming a constant $T = R/P$ (Resource/Production ratio).

$$V(k) = U \times \left\{ 1 - \left[(T-1)/T \right]^k \right\} \qquad (1)$$

where:

V(k) = cumulative production at end of *k*-th period;
U = EUR;
T = apparent time of exhaustion of resource *U*.

Let´s define:

$$P(k) = V(k) - V(k-1) \qquad (2)$$

where:
P(k) = production within the *k*-th period.

From Equation (1) it follows that production *P(k)* decreases with increasing *k*, and therefore Equation (1) was not designed to fit a historical production series which decreases after reaching a peak.

Let´s then define a variable ratio between the decreasing Remaining Resources and the Production within each *k*-th period, namely,

$$T(k) = R(k)/P(k) \qquad (3)$$

where:
T(k) = apparent time of exhaustion of resource *R(k);*
P(k) = production during *k*-th period;
R(k) = remaining resource at the start of *k*-th period;

Defining:

$$T(k) = T_{\min} + (T_{\max} - T_{\min}) \times (1-t)^{k-1} \qquad (4)$$

where:
$T_{max} = U / P(1)$
$T_{min} = R(n)/P(n)$, where $n = \max(k)$
t = rate of decline of the apparent time of exhaustion $(T_{max} - T_{min})$

and replacing *T* of Equation (1) with *T(k)* of Equation (4) and making algebraic arrangements yields:

$$V(k) = U \times \left\{ 1 - \left[1 - 1\Big/\left(T_{\min} + (T_{\max} - T_{\min}) \times (1-t)^{k-1} \right) \right]^{k} \right\} \qquad (5)$$

The graph of Equation (5) is similar to that of a single mode density function, with variable skewness, either negative (longer tail at initial past years) or positive (longer tail at final future years). The functional shape is a consequence of the uncountable possibilities of parameter value combinations. Perfect symmetry, like in the logistic function, is just a very special instance, unlikely to occur.

The essential parameters of the model are the original volume of the resource U = EUR, the apparent times T_{min} and T_{max} and the rate of decline t. Although auxiliary parameters have been added to account for lacking records such as actual year and volume of initial production. Fitting the model to historical series of production should be constrained by the ranges of parameters, and may also be constrained by hypotheses about future demand and rate of decline. Curve fitting is performed by minimizing the sum of squares of relative residuals. The goodness of fit is usually excellent when fitting Equation (5) to cumulative production data. However, when the fitted model and the historical series are transformed into the corresponding annual production series, the goodness of fit degrades. The solution to this problem consists of a simultaneous fit, with a composite sum of squares.

Examples

For the sake of comparison, both the logistic function and the proposed model were fitted to the historical series of American petroleum yearly production, using data published by EIA (2009). The result is shown in Figure 1. From a statistical perspective, the proposed model works better than the logistic function, when both are fitted to the historical data. The logistic function underestimates the observed values after 1990, while the proposed model remains adherent, following the positively asymmetric trend shown by the historical series over the past two decades. Therefore the proposed model is more flexible than the logistic, which is limited to the symmetrical bell shape.

To illustrate the application of the model at the worldwide scale, a historical series of natural gas production published by BP (2009) was chosen, comprising the period 1970-2008. Three fits were performed, each having as constraint one estimate of EUR based on quantiles published by the USGS (2009).

The following expected value and quantiles were used: $F_{95} = 310 \times 10^{12}\ m^3$; $EV = 482 \times 10^{12}\ m^3$; $F_{05} = 612 \times 10^{12}\ m^3$. The goodness of fit is excellent for all three hypotheses of EUR. The maximum projected yearly production for the three EUR hypotheses are as follows: $3.4 \times 10^{12}\ m^3$ in 2026, $3.9 \times 10^{12}\ m^3$ in 2037 and $4.2 \times 10^{12}\ m^3$ in 2043.

These projections were further revised by constraining the fitting of Equation (5) by the forecast demand volume of gas in 2030 from the reference scenario by IEA (2008). The forecast value of $4.4 \times 10^{12}\ m^3$ of gas demand in 2030 is circled in Figure 2 and Figure 3.

The quantiles and the expected value of EUR were also revised as follows: $F_{95} = 312 \times 10^{12}\ m^3$; $EV = 448 \times 10^{12}\ m^3$; $F_{05} = 618 \times 10^{12}\ m^3$. According to Figure 3, the peak production forecasts are: $4.4 \times 10^{12}\ m^3$ in 2034, $4.7 \times 10^{12}\ m^3$ in 2041 and $5.1 \times 10^{12}\ m^3$ in 2051. It should be noted that the three curves have a common point in 2030, are very consistent up to this point and have distinct degrees of asymmetry.

The conclusion is that Equation (5) is useful for the production forecasting of a finite resource like oil and gas, which reaches a peak production and declines with a variable rate, not entirely dependent on past history, but flexible enough to incorporate alternative scenarios.

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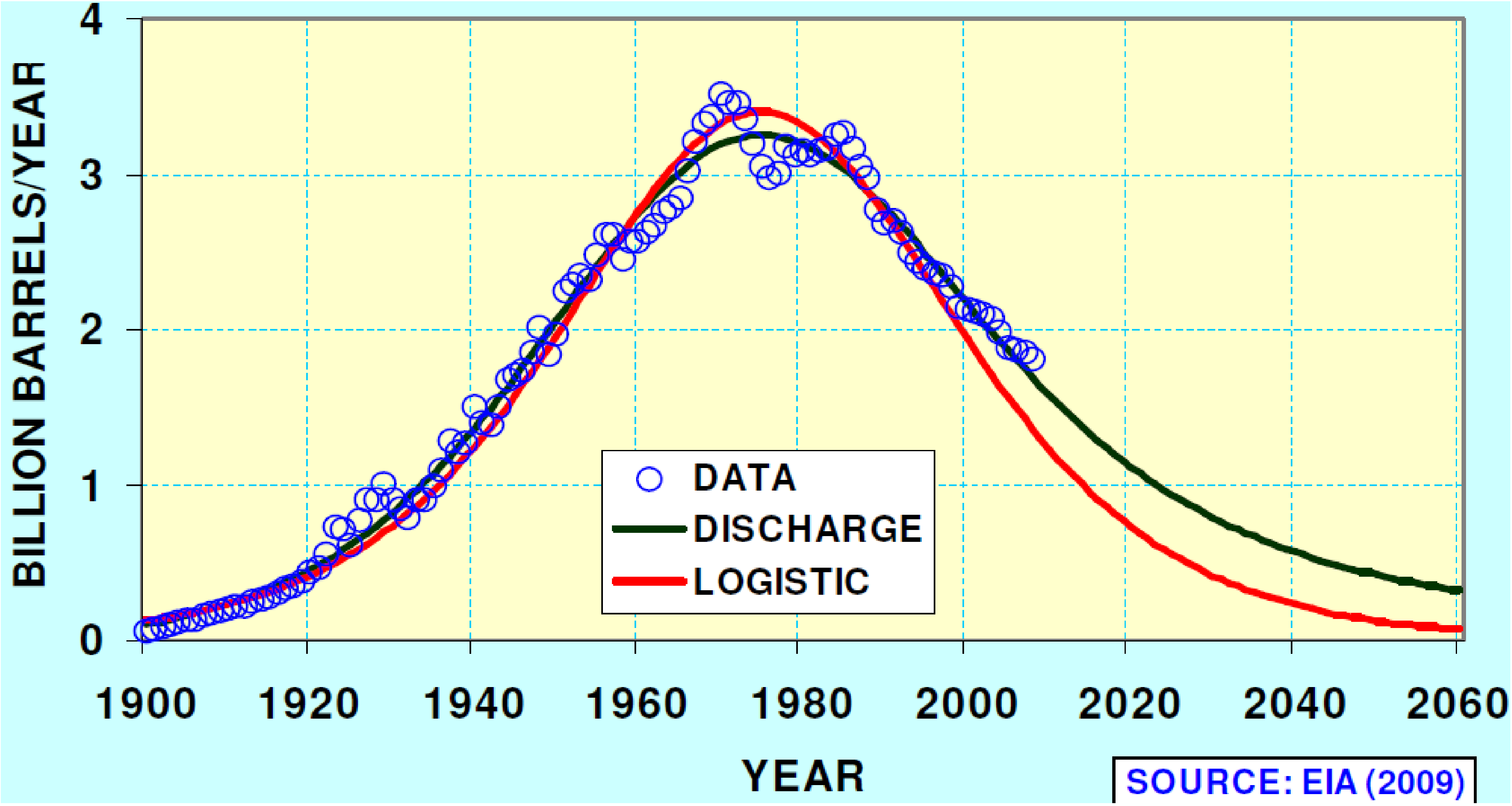

Figure 1. Fitting logistic and proposed model to historical series of American annual production of oil. Source of data: EIA (2009).

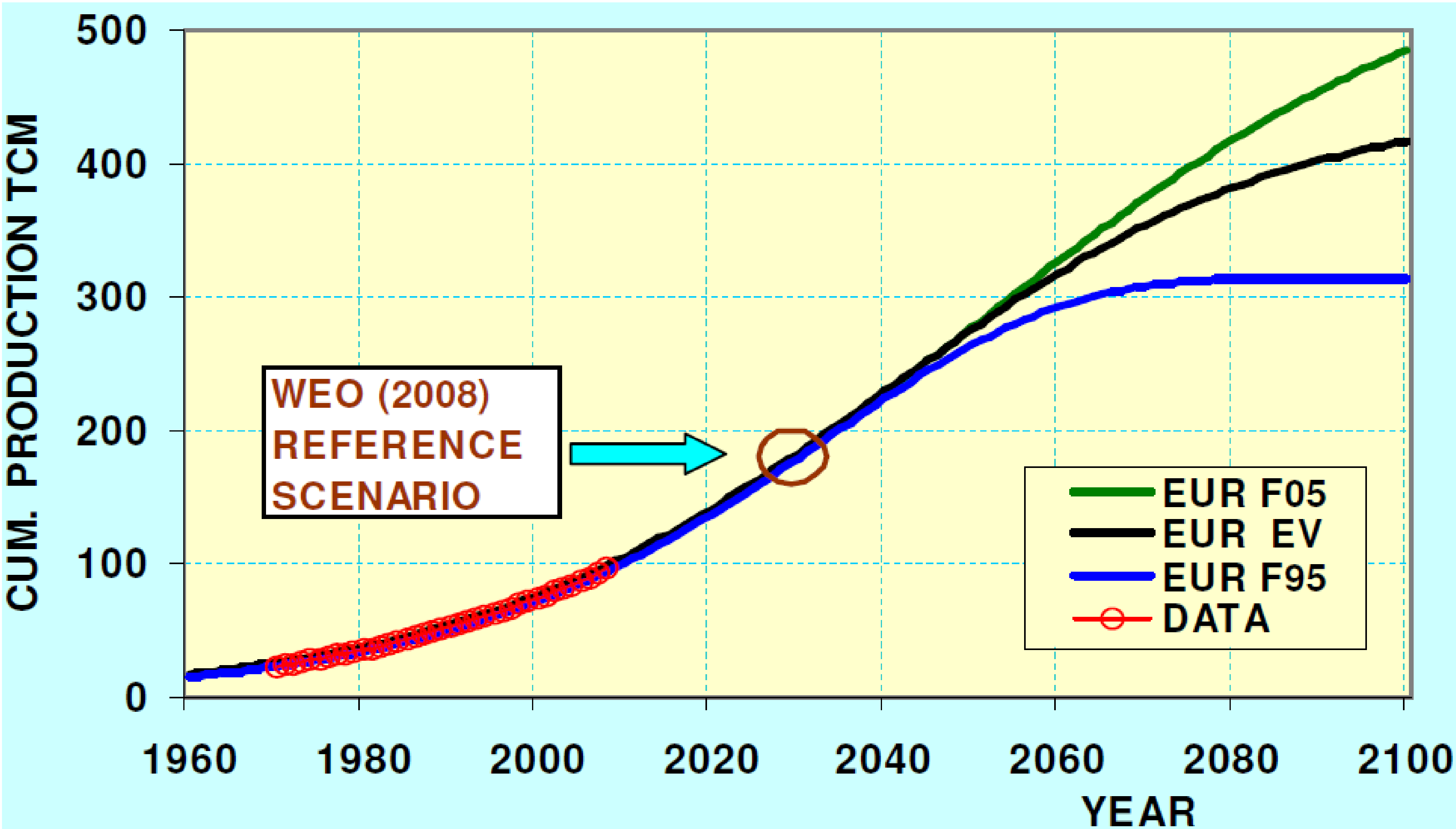

Figure 2. Fitting proposed model to World natural gas cumulative production data published by BP (2009).

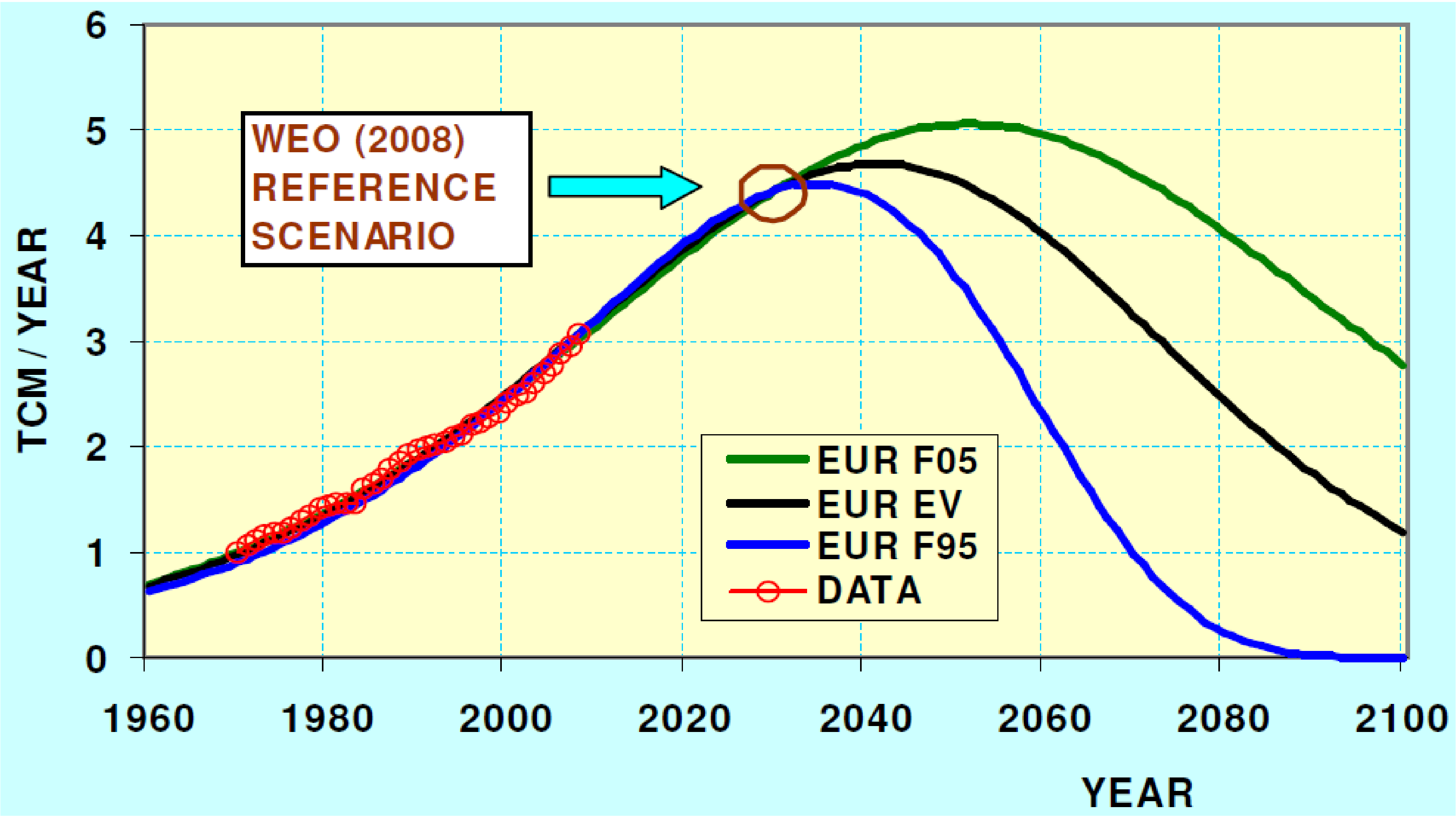

Figure 3. Fitting proposed model to World natural gas annual production data published by BP (2009).