An Analytical Formulation of Seismic-derived Resistivity*

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Search and Discovery Article #40245 (2007)
Posted June 30, 2007

*Adapted from extended abstract prepared for presentation at AAPG Annual Convention, Long Beach, California, April 104, 2007

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Recent developments in seismic petrophysics have shown that efforts to establish methods used for deriving water saturation (S_w) from seismic usually meet an end as empirical approaches and/or in the use of various kinds of artificial intelligence. The quest for establishing reliable methods for estimating beyond-well water saturation data has always been underway in the last one or two decades. Many investigators have spent efforts in trying to find reliable ways to use seismic data to serve the purpose. This paper presents a theoretical formulation of resistivity functions using which it becomes possible that resistivity data is extracted from seismic data. This work is basically a reformulation of the previously introduced concept of linking theory of acoustic wave propagation and water saturation models. In this theoretical work, Gassmann is still used as the acoustic velocity model while Poupon and Hossin water saturation models are used to represent formation rocks containing laminated shale and dispersed shale, respectively.

The work presented in this paper is basically to link primary wave velocity (V_p) and acoustic impedance (AI) to other parameters such as rock true resistivity (R_t), a parameter known to be very sensitive to variation in fluid saturation. Widarsono and Saptono (2003, 2004) provide laboratory verifications and first field trial with some degree of success. However, certain assumptions (i.e. constant/uniform porosity) in the theoretical formulation were still adopted in the above works, which in turn reduced the validity of the resulting formula produced and used. With this reformulation, it is hoped that a more robust model/formula of resistivity as a function of acoustic impedance ($R_t = f(AI)$) is achieved, hence a more reliable resistivity could be extracted from seismic-derived acoustic impedance.

In brief, Gassmann theory (Gassmann, 1951) is actually a modification of Hooke's theory (elastic deformation theory) and provides a theory for acoustic wave velocity in porous media. For a porous and fluid saturated elastic medium, the P-wave equation was modified into

$$V_p{}^2 = \frac{P_d + f(K_f)}{\rho_b} \tag{1}$$

where P_d is the P-wave modulus for the rock frame (or dry rock), and $f(K_f)$ is the function of the incompressibility of the fluid in the pore spaces. The P-wave modulus for the dry rock can be expressed, in turn, by

$$P_d = K_d + \frac{4}{3}\mu_b \tag{2}$$

and the function $f(K_f)$ by

$$f(K_f) = K_f \frac{(1-\frac{K_d}{K_m})^2}{(1-\frac{K_f}{K_m})\phi + (K_m - K_d)\frac{K_f}{K_m{}^2}} \tag{3}$$

The subscripts *d*, *f*, and *m* refer to the rock frame (or the dry rock,), fluid, and rock matrix. The ρ_b and $f(K_f)$ in Equations (1) and (3), respectively, contain porosity (ϕ) and water saturation (S_w). When these two equations are rearranged a water saturation function can be established. See Mendrofa (2006) for a full derivation of the water saturation function.

On the other hand, the relation between R_t and S_w is more straightforward. This is true since for brine-saturated clean sedimentary rocks the total electrical conductivity is solely governed by the amount of the brine within the pore system. The electric current simply flows through the tortuous pore system that is filled continuously by the brine and completely ignores the non-conductive hydrocarbon fraction and rock matrix.

For the purpose of this formulation work two shaly-sand models, instead of Archie model as used in Widarsono and Saptono (2003, 2004), Poupon and Hossin models have been chosen. The reason for the choice of the two models are their relative simple form and the fact that the two models were derived to accommodate the presence of laminated shale (Modified Simandoux) and dispersed shale (Hossin).

The Poupon model (Poupon et al, 1954) is expressed in the form of

$$\frac{1}{S_w{}^{0.5n}} = R_t \left\{ \frac{V_{sh}{}^d}{\sqrt{R_{sh}}} + \frac{\phi^{0.5m}}{\sqrt{aR_w}} \right\} \tag{4}$$

while the Hossin model (taken from Dresser Atlas, 1982) is expressed in the form of

$$S_w{}^n = \frac{aR_w}{\phi^m}\left(\frac{1}{R_t} - \frac{V_{sh}{}^2}{R_{sh}} \right) \tag{5}$$

with n, V_{sh}, R_{sh}, m, a, and R_w are saturation exponent, shale fraction, shale resistivity, cementation factor, tortuosity, and formation water resistivity, respectively. The models in Equations 4 and 5 will introduce R_t into the Gassmann acoustic velocity model.
It is now the turn to underline that water saturations in any mathematical expressions should have the same general meaning physically, which is basically ‘a part of a rock’s pore space that is occupied by formation water’. Therefore, it is fundamentally correct to state that

$$S_{w(petrophysics)} = S_{w(Gassmann)} \tag{6}$$

where $S_{w(petrophysics)}$ is S_w in any water saturation models commonly known in log analysis (e.g., Modified Simandoux and Hossin models) and $S_{w(Gassmann)}$ is S_w in the Gassmann acoustic velocity model.

Following Equation (6), and after all required mathematical arrangements, a resistivity function of

$$\frac{1}{R_t} = \frac{V_{sh}}{R_{sh}} + \frac{\left(-I + \sqrt{I^2 + 4HJ}\right)^n}{\dfrac{4aR_w}{\phi^m\left(1 - V_{sh}\right)} T^n} \tag{7}$$

is obtained for porous rocks containing laminated shale, and

$$\frac{1}{R_t} = \frac{V_{sh}^{\ 2}}{R_{sh}} + \frac{\left(-I + \sqrt{I^2 + 4HJ}\right)^n}{\dfrac{4aR_w}{\phi^m} T^n} \tag{8}$$

is obtained for porous rocks containing dispersed shale. Equations (7) and (8) are essentially the solution of $R_t = f(AI)$. The variable of AI is contained in variables of I and J (i.e., I and J are functions of AI) whereas variables H and T also represent some simple mathematical expressions that are temporary established to simplify Equations (7) and (8). See Mendrofa (2006) for full derivations of the two resistivity functions.

For trial using the two equations, a set of log suites has been obtained from a Sumatran production well penetrating two gas bearing shaly-sands. Density and acoustic logs are available to derive acoustic impedance ($AI = \rho_b \times V_p$), gamma ray log for shale contents, whereas shallow and medium depth resistivity logs are available to provide environmentally corrected formation resistivity at investigation depths relevant to acoustic and density logs. Data and other information regarding the well and the two sands are obtained from laboratory reports, study reports, as well as from local knowledge. Summarily, the supporting data used in the works is:

- quartz sand density; $\rho_m = 2.648$ gm/cc
- oil density, $\rho_{hc} = 0.8$ gm/cc
- fresh water density; $\rho_w =$ = 1.0 gm/cc
- shale density; $\rho_{sh} = 2.45$ gm/cc
- shale resistivuty; $R_{sh} = 0.5$ Ohm-m
- tortuosity; a = 1.0
- cementation factor; m = 2.3
- saturation exponent; n = 1.76
- formation brine resistivity; $R_w = 0.693$ Ohm-m @ 75^{o}F or 0.27 Ohm-m @ reservoir temperature
- mud filtrate resistivity; $R_{mf} = 17.5$ Ohm-m @ 75^{o}F or 6.92 Ohm-m @ 200^{o}F (*oil base mud*)
- rock matrix bulk modulus; $K_m = 40$ Gpa
- water bulk modulus; $K_w = 2.38$ Gpa
- gas bulk modulus; $K_g = 0.021$ Gpa
- rock dry bulk modulus; $K_d = 0.24$ Gpa
- V_p/V_s ratio = 1.5 or $\sigma = 0.1$ (homogeneous and isotropic quartz sandstone, according to Gregory, 1976)

Although the two sands are gas-bearing in nature, but since the well was drilled using oil base mud and as a matter of fact that the depth of investigation of density and acoustic logs are shallow, the system is therefore oil – water (i.e., water – mud filtrate). In consequence, resistivity values from shallow and medium depth resistivity devices are taken as the 'observed R_t' in the well.

Figures 1-4 show comparisons between calculated resistivity values versus observed values. In general, it can be clearly seen that the two sets of resistivity show similarity in trends; i.e., variation in magnitudes with depth. The very difference between the two sets is in the respective magnitudes. The observed resistivity values tend to be significantly higher than the calculated ones. A vast number of trials have been made to modify the supporting data within the theoretically allowed ranges. Despite the trials, agreements in magnitudes are not much better between the two sets. Indeed this can be considered as a 'setback,' but nevertheless this work has started a path of research that can be followed up in the future.

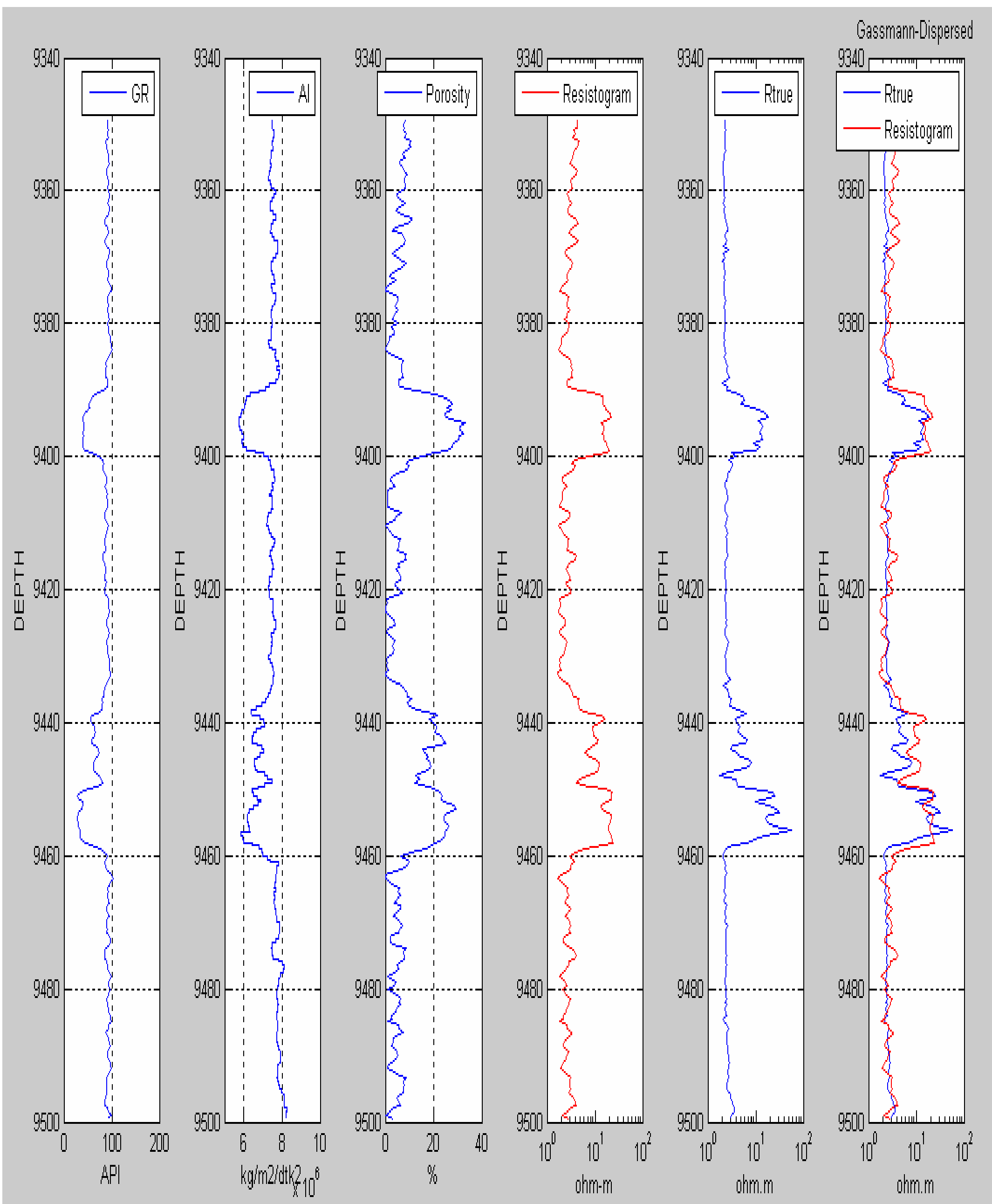

Figure 1. Acoustic impedance log and synthetic resistivity curves (red) for A-sand (dispersed shale).

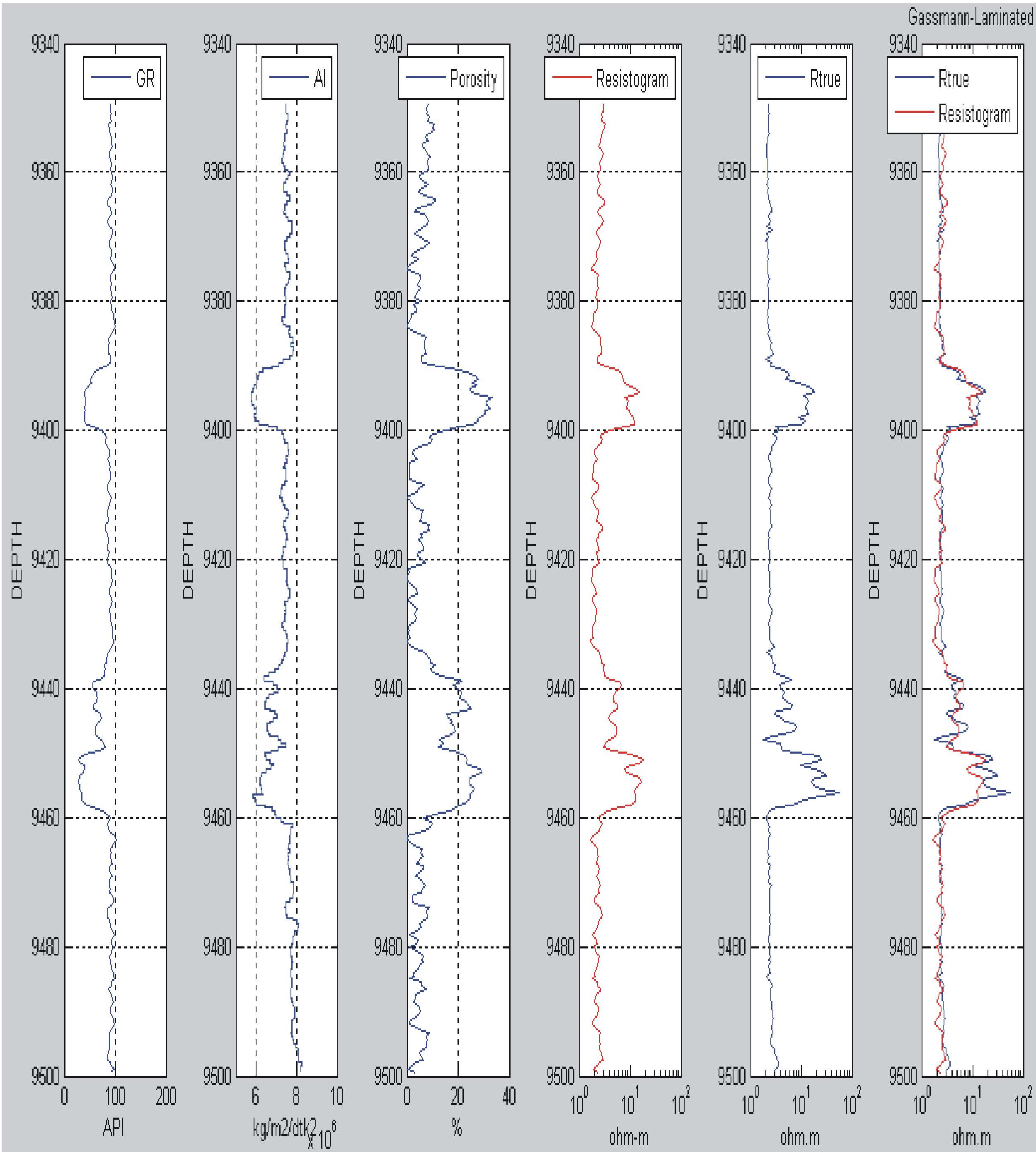

Figure 2. Acoustic impedance log and synthetic resistivity curves (red) for A-sand (laminated shale).

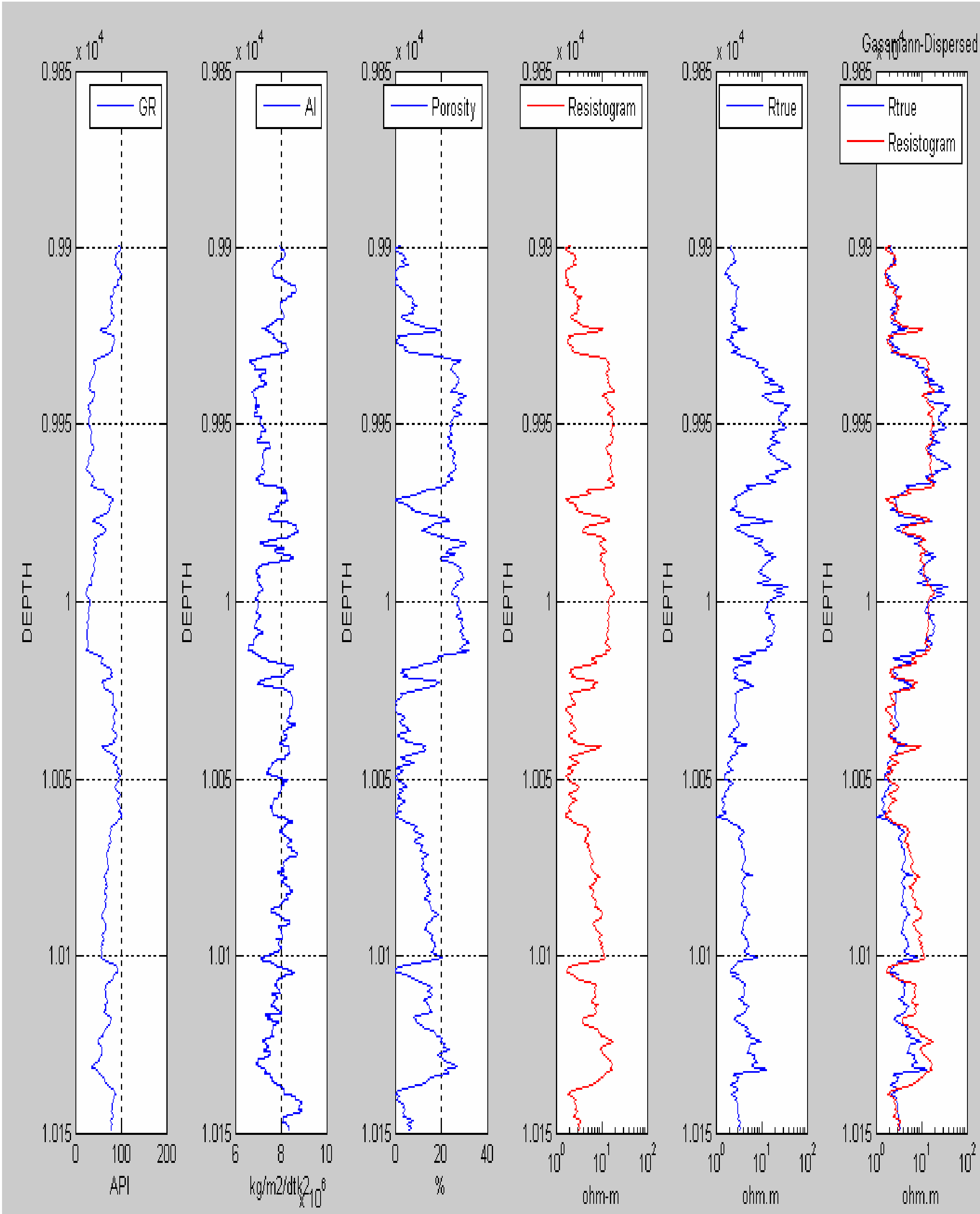

Figure 3. Acoustic impedance log and synthetic resistivity curves (red) for B-sand (dispersed shale).

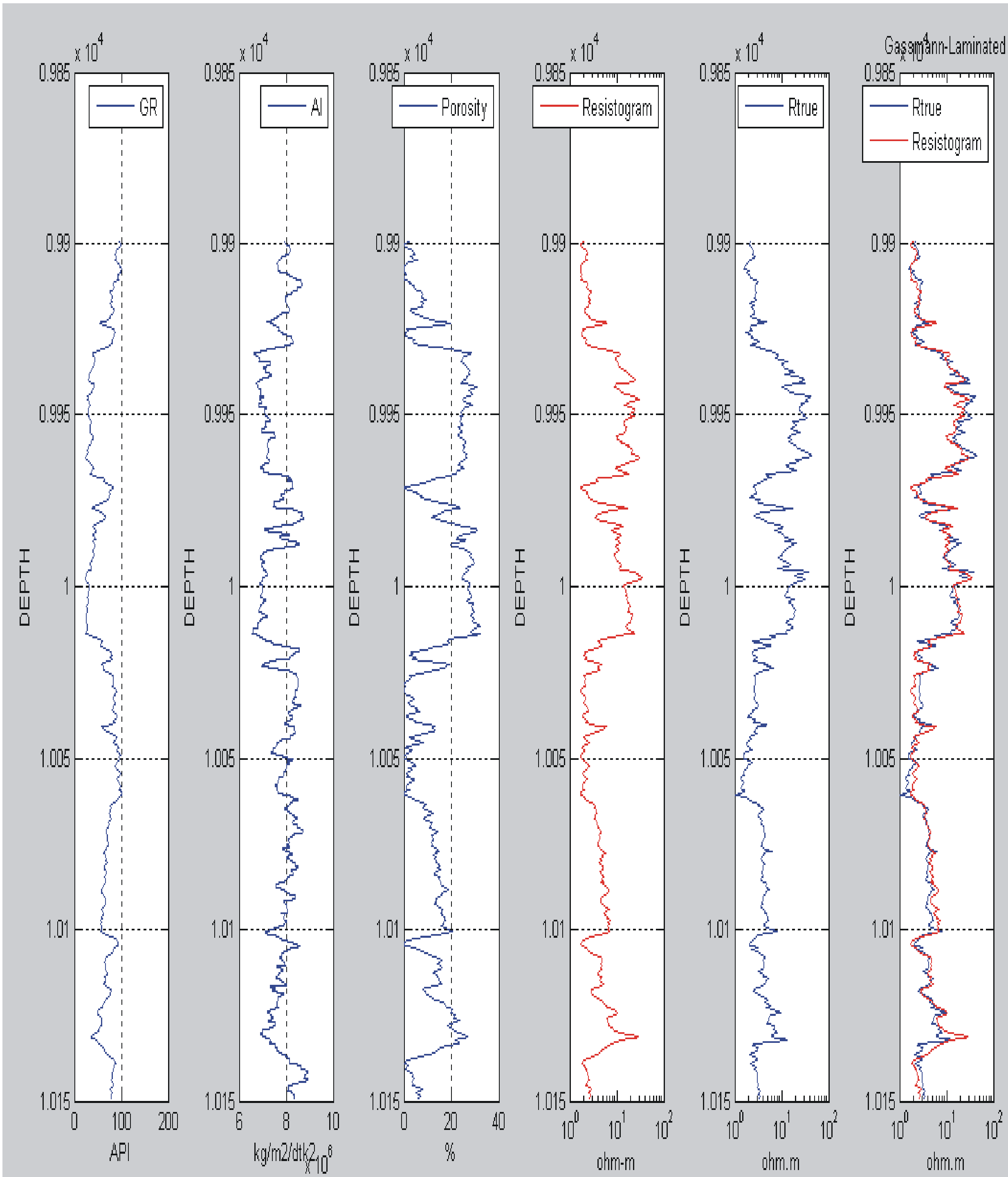

Figure 4. Acoustic impedance log and synthetic resistivity curves (red) for B-sand (laminated shale).

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