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A low rank based seismic data interpolation via frequency-patches transform and low rank space projection

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Summary

We propose a new algorithm to improve computational efficiency for low rank based interpolation. The interpolation is carried out in the frequency spatial domain where each frequency slice is first transferred to the frequency-patches domain. A nice feature of this domain is that the number of non-zero singular values can be better related to seismic events, which favors low rank reduction. During the interpolation iteration, instead of applying singular value decomposition to limit low rank, the low rank reduction is carried out with a pre-determined low rank space projection matrix. Moreover, the technique of matrix completion is employed to avoid repeated transforms during the interpolation iteration. Compared with conventional low rank Interpolation methods, our method has the advantage of increased computational efficiency. Data examples also show that this method can be applied to complicated seismic data sets with highly satisfactory results.

Introduction

Seismic data interpolation based on low rank was first introduced to the seismic industry by Trickett et al (2010). This algorithm is based on the fact that if the original data matrix has low rank characteristics, then, with low rank constraints, the missing data points can be properly filled up. This original low rank interpolation algorithm is designed to work in the frequency domain and interpolation is carried out on each frequency slice independently. The frequency slice must first be transferred to a Hankel/Toepliz matrix and the low rank reduction is performed on this transferred matrix. We discuss later how this Hankel/Toeplitz matrix transferring turns out to be a key step for this low rank based interpolation .By realizing that this algorithm is the same as the Singular Spectrum Analysis (SSA) technique, researchers at University of Alberta have made great efforts to promote this algorithm and have extended the work to higher dimensional seismic data, e.g. Oropeza and Sacchi (2011), Gao et. al. (2012), Stanton et al. (2012). It should be noted that in order to make this kind of interpolation algorithm practical for regular usage, efficient up-to-date techniques like fast partial SVD methods, Lanczos decomposition, and randomized SVD algorithms, are required . However, the main drawback of this low rank algorithm is still computational cost that prevents it from application to large seismic data sets.

Parallel to the study of seismic data interpolation, so-called matrix completion methods for solving the similar problem of reconstructing missing data in images have been extensively developed in the past few years and tens of algorithms formulated as convex optimization that minimizes a combination of nuclear norm and L1-norm have poured in as publications for fast matrix completions, e.g. Lin et al. (2009). Yang et al. (2012) applied two algorithms of matrix completion to seismic 2D data interpolation and the results showed great speed advantages.

Ma (2013) applied a matrix completion algorithm to 3D seismic data interpolation and Ely et al. (2013) for 5D seismic data interpolation.

Further it seems that the speed problem of low rank matrix completion techniques has been solved because it is now SVD free, according to e.g. Aravkin (2013). However, when we carefully investigated the two matrix completion algorithms, APG and LMaFit (these two algorithms represent two typical methods developed for matrix completion), in the paper by Yang et al (2012), we found that, from the point of view of computation speed, the key part that is used for rank reduction in these two algorithms does not have significant speed advantages over that which was used in the original low rank seismic data reconstruction. APG actually uses partial SVD for rank reduction and LMaFit uses matrix low rank factorization so that, even if it does not explicitly use SVD, it still involves repeated matrix inverse calculations during the matrix factorization that are directly comparable to SVD computational costs.

Comparing matrix completion in image processing with the original low rank interpolation we found the big difference between these two approaches is in the preparation of the data. In image processing, the algorithms are directly applied to the data matrix of the image and in seismic low rank reduction the algorithms are applied to a Hankel/Toeplitz transformed data matrix which has been very much enlarged from the original data set. For example, if we have 100 1D data points, then, based on the Hankel matrix transform, these data points will be transferred to a 50 by 51 2D matrix that contains 2550 elements. Such data redundancy will become even larger for higher dimensions.

Can we do seismic data interpolation in the same way as that in image processing, i.e. directly perform interpolation in the original data set without a redundant data transform? Before answering the question, we would like to address the conditions required for low rank matrix completion:

(a) The data matrix should have a low rank structure;

(b) The data matrix should not contain a whole column/row of zeros because in these cases, the zero elements are projected to the null space of the matrix and rank reduction cannot work on those elements.

In most real cases, seismic data may not have a low rank embedded structure. Sorting data into a special gather domain may help the seismic data matrix to some extent in satisfying the requirement of the first condition, e.g. common offset domain (Kumar et al. 2013). However, when geological structure is very complicated, sorting may not be very helpful. Thus, for example, even in a migrated data section, the rank of the data matrix may still be high. Therefore, if we directly work on the original data set, condition (a) may be violated.

Further, unlike the image processing case, data points missed at random in seismic data are whole traces and as a result we have many whole zero columns in the seismic data matrix. Therefore, directly working on the original seismic data set may also violate condition (b) above.

As discussed earlier, seismic data needs to be pre-arranged for low rank based algorithm application. Now we can see why the Hankel transform in Trickett's orginal low rank interpolation plays such an important role in interpolation (see Oropeza and Sacchi 2011). However, while the Hankel matrix transform make seismic data satisfy the low rank matrix completion condition, it also brings a computational dilemma. By realizing the matrix completion conditions, Yang et al. (2012) and Ma (2013) adopted a patches-transform developed for texture identification in image processing (Shaeffer and Osher 2011). This patches-transform does not increase the size of data and the transformed data can satisfy the low rank matrix completion condition. However, when investigating the results carefully given in their papers, clear patches footprints appear in the interpolation results, which severely affects the quality of interpolation. The footprints show that the patches-transform acts like a small window for scanning the data and the effect of this small window impacts on the result when we transfer back to the original data matrix. Therefore, some smoothing of the window edges is necessary. Removing such edge effects can be accomplished through overlapped windows as we use in other seismic data processing applications, e.g. windowed F-X deconvolution. Actually, when we consider the overlapped patches-transform we can see that the Hankel matrix transform is a special case of the patches-transform. With partially overlapped patches-transform, we can control data redundancy.

In this paper, we focus only on low rank interpolation on a 2D data set in the frequency – spatial domain. Extending to higher dimensions is very straightforward. Our contributions to a low rank based seismic data interpolation are:

1. We use the frequency patches transform with patches overlap to replace the Hankel transform (this can much reduce data redundancy); We do not do this transform in the time domain because in the frequency domain we can deal with each frequency slice separately and hence the big matrix in the time domain can be broken into small matrices, which increases computational efficiency and avoids large computer memory issues.
2. We use a Projection matrix to low rank space which avoids the SVD like rank reduction during iterations.
3. Finally, a modified accelerated proximal gradient method Algorithm accelerates the convergence of the matrix completion.

Each of these can greatly reduce the computational costs, which makes this low rank based interpolation affordable even for very large seismic data sets.

Matrix patches transformation

Let A be an m by n matrix. The patch transform is defined by first partitioning the matrix into $\frac{n*m}{w*h}$ sub-matrices such that each of them is *w* by *h*, and then transfer each sub-matrix into a column vector of length *w**h*. The transformed matrix is formed by putting all of the vectors together as shown in figure 1 (see also in Shaeffer and Osher, 2011 and similar figure in Ma 2013 for the reduction of 3D data to 2D patches).

When the patches are non-overlapping, this transform does not increase the number of elements in the original matrix. When the patches are densely overlapped, the transform will end up exactly like the Hankel matrix. The nice feature of this transform is that the new matrix contains a low rank that represents the texture embedded in the original matrix (Shaeffer and Osher, 2011). It is very easy to verify that for the case of a diagonal matrix, where all singular values are non-zeros, the patches-transformed matrix has only one non-zero singular value. Figure 2 shows an example. In the figure, 2(a) shows three linear events with data size 512 by 256 and 2(b) shows the normalized singular values distribution for the original matrix (blue) and the transformed matrix (red) with patch size 16 by 16 to match the size of the original matrix. Even if both matrices have the same size and contain the same data, the number of non-zero singular values is dramatically reduced in the patches-transformed matrix.

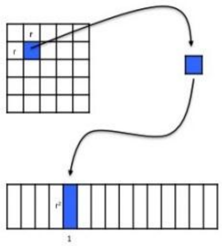

Figure 1 patch transform

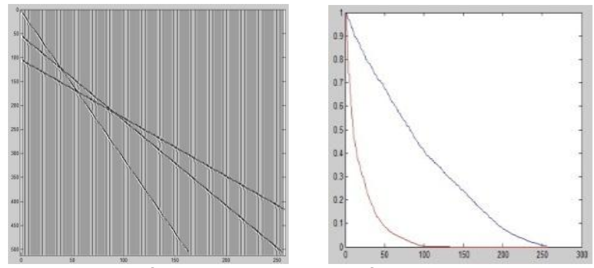

Figure 2 data matrix (left); singular values plot from original matrix (blue) and patch transformed matrix (red)

Matrix frequency-patches transformation

Directly solving the low rank completion problem for a patches-transformed matrix faces a practical problem: in seismic data, seismic events are sparse, i.e. energy is mostly concentrated at wavefronts that reflect from sub-surface reflectors, which means that if we ignore noise, most of the data are zeros. Therefore, the patches-transform from such data will lead to a very sparse matrix, which makes the matrix completion difficult. Worse still is the case when the interpolated samples located in a row or columns are all zero. Interpolating for these samples is impossible.

However if the data are transformed into the frequency domain then, within a chosen effective frequency band, the frequency traces will all have non-zero values. Then the patches transform for each frequency slice will not produce a very sparse matrix, which avoids most of the problems mentioned above for the spatial patches transform.

Further by working on frequency slice by frequency slice, the computational costs will not increase as a consequence. The reason is that the frequency domain patches-transform does not increase the total number of data points; working in each frequency slice actually reduces one dimension of the original data. For instance, dealing with original 2D data becomes a problem of how to deal with 1D frequency vectors, which means the computation is transferred from a large matrix to a series of small matrices. Furthermore, because of frequency band limits, not all frequency slices need to be considered, which further reduces the computational costs. Finally, seismic data is usually over-sampled in time and the aliasing problem is immaterial.

Low rank via projection

Because column vectors in the singular value decomposition matrix, commonly referred to as "U", represent the most energy concentrated directions. Usually none Gaussian noise, e.g. outliers, missing data points may affect accuracy of U estimation (e.g. Aravkin et al. 2013), we apply regularized low rank approximation for obtaining a robust U estimation and then, like principal components analysis, make a low rank space projection matrix, i.e. $P = U * U'$. With this projection matrix, the rank reduction at each iteration can be simply carried out via the matrix multiplication.

Matrix completion via modified APG algorithm

After the frequency-patches transform, the problem becomes that of matrix completion, i.e. fill in missing data elements in the matrix along the most energy concentrated direction.

The matrix completion problem can be written as an optimization problem:

Let A be an *m* by *n* matrix that is only known on a sub set Ω of the complete set of elements inside the matrix. The low-rank matrix completion problem consists of finding the matrix X with lowest rank that agrees with A on Ω:

$$\min\ rank(X), \quad subject\ to\ P_\Omega(X) = P_\Omega(A)$$

Where P_Ω denotes the orthogonal project onto Ω, i.e.

$$P_\Omega:\ X_{i,j} = \begin{cases} X_{i,j} & if\ (i,j)\ within\ \Omega \\ 0 & otherwise \end{cases}$$

We adapt the accelerated proximal gradient method (APG) for solving matrix completion as it has been shown to have fast convergence and produce high quality results (e.g. Yang, 2012). The APG is an iterative updating algorithm and at the k'th stage, it can be written as follows:

$$Y^k = X^k + \frac{t^{k-1} - 1}{t^k}\left(X^k - X^{k-1}\right)$$

$$G^k = Y^k - \tau\left(P_\Omega(Y^k) - P_\Omega(A)\right)$$

$$X^{k+1} = S_\tau(G^k)$$

$$t^{k+1} = \frac{1}{2}(1 + \sqrt{1 + 4(t^k)^2}$$

Where $S_\tau(G) = Udiag((\sigma - \mu\tau))_+ V^*$ i.e. singular value decomposition and + means exclude values that are less than zero. With this procedure, the components related to small singular values are excluded.

Details of this algorithm can be found at Yang, 2012.

Note that the highest cost of this algorithm is the $S_\tau(G)$ solution. Thus we modify APG algorithm by using the low rank projection as mentioned above.

Summary of our algorithm

Our algorithm can thus be summarized as: for each frequency slice

1. Frequency-patches transform to matrix ;
2. Calculate low-rank space projection matrix;
3. Apply modified APG for matrix completion;
4. Transform back to frequency slice.

Synthetic data examples

The first example compares our algorithm with the time spatial patch transform and the original Hankel matrix transform, figure 3. Three dipping events with 30% randomly missing traces are used as input data for the simulations. In the figure from left to right are the input data, the F-K spectrum of input data, and the results with APG with time spatial patch transform, Hankel transformed and our frequency-patches transform with half window overlap. The result of the time spatial patch transform does not reconstruct missing traces very well (note that with less aliasing a better result can be obtained). The results from conventional SSA and our algorithm are almost identical but our algorithm is nearly 50 times faster! Two things are worthwhile mentioning: the first is that the spectrum shows the data events are strongly aliased and this low rank based interpolation is very robust for aliased events interpolation; secondly the

Hankel matrix transform makes the data about 15 times larger than that with a half window overlapped frequency-patches transform.
The second example will show how this low rank reconstruction works on curved events. The input data contains hyperbolic curved events. Again, 30% traces are randomly removed to create the input data. The result shows that our algorithm can reconstruct missing traces very well as shown in Figure 4.

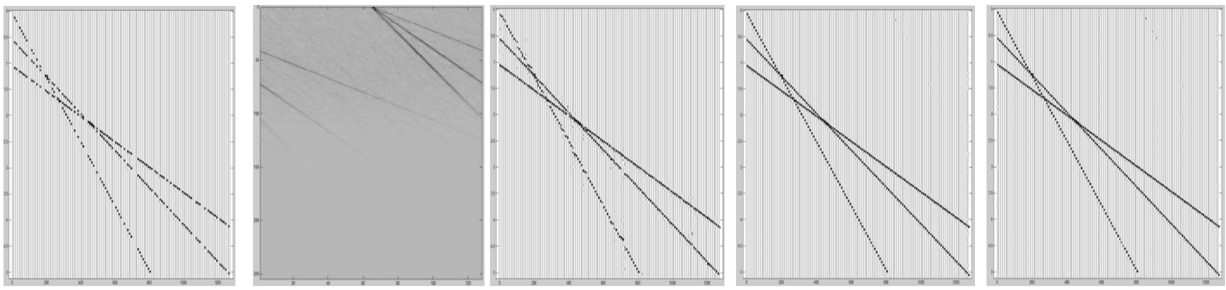

Figure 3. Input data, spectrum of input, t-x domain result, with Hankel transform result and new algorithm result, respectively from left to right

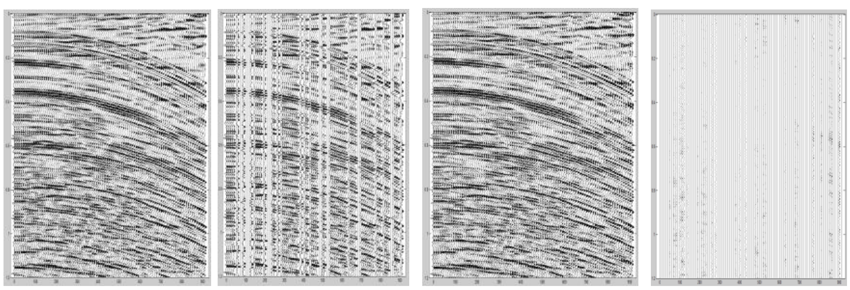

Figure 4. Original data, input data, reconstructed data with our algorithm and its misfit.

The data for our last example is from a marine CDP stacked dataset, Figure 5. The geology structure in this data set is complicated and our algorithm successfully reconstructs the missing traces.

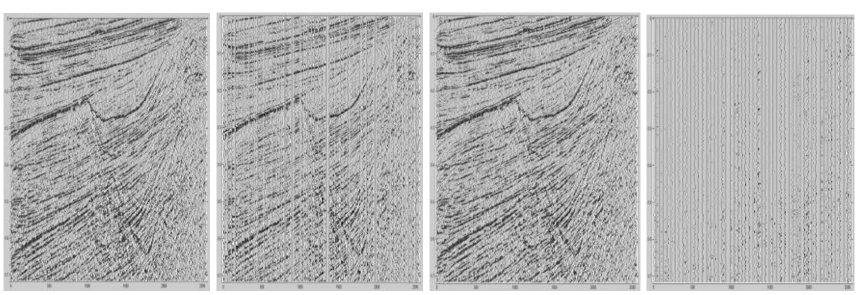

Figure 5. Original data, input data, reconstructed data with our algorithm and its misfit.

Conclusions

We have presented a new seismic interpolation method that can reduce computational requirements drastically from conventional low rank reduction based seismic data interpolation.

This new kind of interpolation is very suitable for handling large seismic data sets. Speed increases can be expected to increase exponentially with the size of the data set. Although we focused only on 2D data, the extension to higher dimensions is very straightforward. The idea of low rank space with projection matrix can be implemented in most other advanced matrix completion algorithms.

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References

Aravkin,A., R. Kumar, H. Mansour, B. Recht and F. Hermann, 2013, A robust SVD-free approach to matrix completion, with applications to interpolation of large scale data, http://arxiv.org/pdf/1302.4886.pdf

Ely, G.,S. Aeron, N. Hao and M. E. Kilmer, 5D and 4D pre-stack seismic data completion using tensor nuclear norm (TNN), Submitted to SEG 2013, Houston, TX, USA.

Gao, J. 2013, A fast reduced-rank interpolation method for prestack seismic volumes that depend on four spatial dimensions, Geophysics, Vol.78, No.1.

Kumar, R., A. Aravkin, H. Mansour, B. Recht and F. Hermann,, 2013, Seismic data interpolation and denoising using SVD-free low-rank matrix factorization, 75th EAGE Conference & Exhibition incorporating SPE EUROPEC 2013

Kumar, R., A. Aravkin, H. Mansour, B. Recht and F. Hermann, 2013, Reconstruction of seismic wavefields via low-rank matrix factorization in the hierarchical-separable matrix representation. SEG2013.

Lin, Z., A. Ganesh, J. Wright, L. Wu, M. Chen, and Y. Ma, Fast convex optimization algorithms for exact recovery of a corrupted low-rank matrix, International Workshop on Computational Advances in Multi-Sensor Adaptive Processing, December 2009.

Ma, J. 2013, Three-dimensional irregular seismic data reconstruction via low rank matrix completion, Geophysics, Vol. 78, No.5.

Oropeza and Sacchi, “Simultaneous seismic data denoising and reconstruction via multichannel singular spectrum analysis,” Geophysics, vol. 76, no. 3, pp. V25–V32, 2011.

Shaeffer and S. Osher, 2011, A low patch-rank interpretation of texture, UCLA CAM report.

Stanton, A., N. Kreimer, D. Bonar, M. Naghizadeh and M. Sacchi, A comparison of 5D reconstruction methods, 82th Annual meeting, SEG, Expanded Abstracts, 2012.

Trickett et al., 2010, Rank-reduction-based trace interpolation, 80th Annual meeting, SEG, Expanded Abstracts, 2010.

Yang, Y., et al., 2012, Seismic data reconstruction via matrix completion, UCLA CAM report.