

# Anisotropy-preserving 5D interpolation by hybrid Fourier transform

*Juefu Wang and Shaowu Wang, CGG*

## Summary

We present an anisotropy-preserving interpolation method based on a hybrid 5D Fourier transform, which combines a 3D FFT (Fast Fourier Transform) and 1D non-uniform DFT (Discrete Fourier Transform) in different directions. The 3D FFT is applied to each temporal frequency slice along inline, crossline, and azimuth. The 1D DFT is applied along offset within each azimuth sector for each common midpoint (CMP). The hybrid mechanism helps to achieve a balance between efficiency and geometrical accuracy, providing a flexible solution to predicting data using any desired geometry. Synthetic and real data examples show that the method is capable of handling data with significant residual moveout caused by strong anisotropy, dipping reflectors or multiples.

## Introduction

Seismic data demonstrate azimuth and/or offset dependent amplitude and travel time variations in anisotropic media. To accurately extract anisotropic parameters from seismic data, we need to make sure the input has good azimuth and offset coverage. This is especially important when we consider applications for pre-stack migrated gathers such as seismic inversion for reservoir characterization. Interpolation is often applied to improve spatial sampling to reduce migration artifacts. In recent years, multi-dimensional interpolation based on Fourier transform has become a hot topic in the geophysical community. Fourier interpolation is an effective way to remove the acquisition footprint, and to reduce migration artifacts. Depending on the manner in which the input geometry is handled, Fourier interpolation can be divided into two classes: the method of uniform DFT (Discrete Fourier Transform) and the method of NUDFT (Non-Uniform DFT). The first method (for example, Liu and Sacchi 2004; Abma and Kabir, 2005; and Trad, 2008) is usually faster since we can use the FFT as the inversion kernel. However, the FFT method requires input data to be binned on to a regular grid, which introduces geometry errors and duplicated traces in bins. The NUDFT method (for example Xu et al, 2005; Zwartjes and Sacchi, 2007) does not have this problem, but the computational effort is much higher. Another feature of NUDFT is that it is able to predict data at any arbitrary location once the Fourier spectrum has been optimized using sparse inversion.

In this paper, we try to balance the efficiency and accuracy of the two strategies by using a hybrid approach. In particular, we first apply 1D FFT to convert each trace from the time domain to the temporal frequency domain. We then perform interpolation for each frequency slice. The kernel involves a hybrid 4D transform: 3D FFT for inline, crossline, and azimuth; and 1D NUDFT for offset within each azimuth sector. In practice, the computational cost is

comparable with a conventional 5D interpolation approach that relies upon FFT in fine grids to achieve acceptable accuracy.

## Theory

Given the Fourier spectrum of a frequency slice in four dimensions, inline, crossline, azimuth, and offset, we can predict the data at an arbitrary location using the following equation:

$$d(x, y, a, h) = L_1 L_3 m(k_x, k_y, k_a, k_h), \quad (1)$$

where

$d$  is the data to predict,

$x$  and  $y$  are the crossline and inline numbers, respectively,

$a$  is the azimuth sector number,

$h$  is the offset,

$k_{x/y}$  are the corresponding crossline and inline wavenumbers,

$k_a$  is the azimuth sector wavenumber,

$k_h$  is the offset wavenumber,

$m$  is the spectrum model in the wavenumber domain,

$L_3$  is the inverse 3D FFT operator along inline, crossline, and azimuth wavenumber directions, and

$L_1$  is the inverse 1D NUDFT operator along the offset wavenumber direction.

According to Equation (1), the two FT operators are applied in a cascaded fashion.  $L_3$  converts data from the wavenumber domain  $(k_x, k_y, k_a, k_h)$  to a mixed domain  $(x, y, a, k_h)$ , and then  $L_1$  converts data to a purely space domain  $(x, y, a, h)$ .

The task of interpolation is to find an optimal model,  $m$ , from limited samples at different locations given by a seismic survey. This can be done by minimizing the following cost function (L2 norm of prediction errors):

$$J(m) = \|d(x, y, a, h) - L_1 L_3 m(k_x, k_y, k_a, k_h)\|^2. \quad (2)$$

The Fourier spectrum  $m$  can be decomposed into amplitude and phase. Therefore Equation (2) can be recast into

$$J(z) = \|d(x, y, a, h) - L_1 L_3 A z(k_x, k_y, k_a, k_h)\|^2, \quad (3)$$

where  $A$  is the amplitude spectrum and  $z$  is the phase. Equation (3) can be minimized by the conjugate gradients (CG) methods (Hestenes and Stiefel, 1952). The amplitude spectrum  $A$  is unknown, but it can be estimated from the input using the following equation:

$$A = |L'_3 L'_1 d(x, y, a, h)|, \quad (4)$$

where  $L'_3$  is the forward 3D FFT along inline, crossline, and azimuthal directions, and  $L'_1$  is the forward non-uniform 1D transform along the offset direction for each azimuth sector at each CMP.

Typically, the amplitude spectrum  $A$  needs to be smoothed to yield a stable inversion solution. When the CG inversion is convergent, we can update the amplitude spectrum and restart the inversion to improve the solution. This is similar to the boot-strapping strategy adopted by Liu and Sacchi (2004).

Once the optimal model  $m$  is computed, we can predict data for any desired geometry using the following equation:

$$d(x, y, a, h) = L_1 L_3 A_z(k_x, k_y, k_a, k_h). \quad (5)$$

In detail, for each desired trace, we calculate the nearest inline, crossline, and azimuth sector to which it belongs, and then predict the exact offset using the spectrum within the azimuth sector. The azimuth binning error can be reduced by increasing the number of azimuth sectors. In practice, six to sixteen sectors are enough to handle data with azimuthal variation as our examples show.

## Examples

To validate the hybrid 5D interpolation, we generated a synthetic dataset with orthorhombic anisotropy using a real land survey geometry. As shown in Figure 1a, the input is quite sparse. When the offset bin size is small (40 m), the conventional interpolation method based on FFT (Liu and Sacchi, 2004) gives a similar result to the proposed hybrid 5D interpolation (see Figure 1b and Figure 1c). However, when the offset bin size is bigger, the new method better handles the event with strong orthorhombic anisotropy. The jittering result generated by

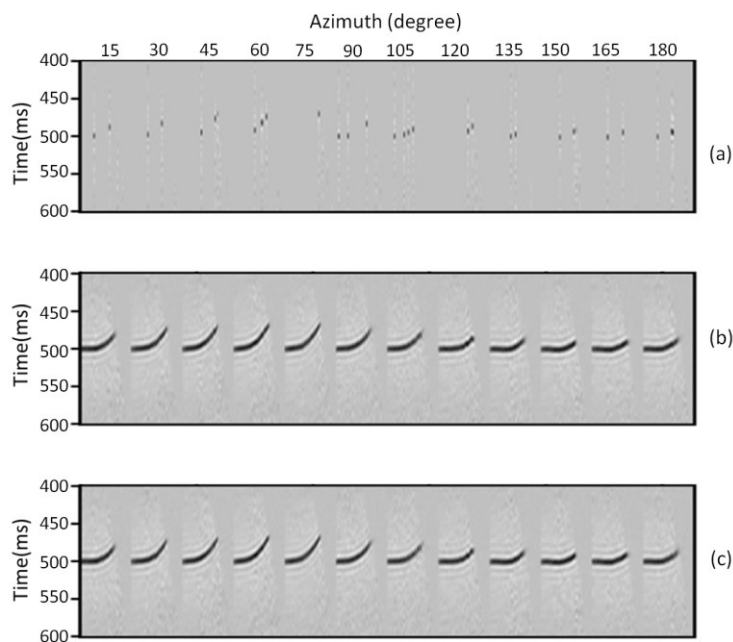


Figure 1: Comparison between different Fourier interpolation methods. (a) Binned input gather for a single CMP. Traces are sorted by azimuth then offset for the same azimuth (Common Azimuth Common Offset). The azimuth sector spacing is 15 degrees, and the offset bin spacing is 40 m. (b) Interpolated gather using MWNI (Minimum Weighted Norm Interpolation) (Liu and Sacchi, 2004). (c) Interpolated gather using our hybrid 5D method.

MWNI at large offsets in Figure 2b is due to binning errors within the input, a common problem to FFT-based interpolation methods. In contrast, the hybrid 5D result (Figure 2c) looks more accurate for the same offsets since the algorithm uses true offsets of all input traces, no offset binning is required prior to interpolation.

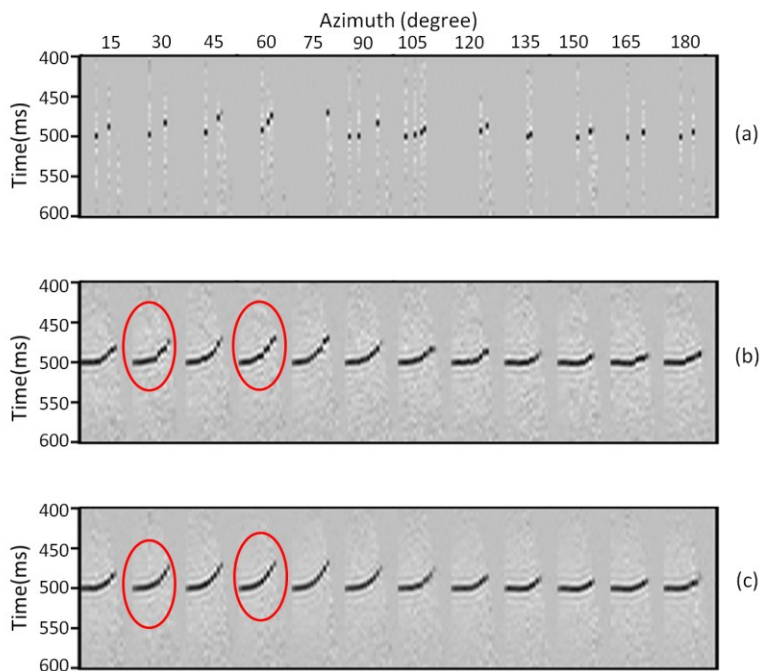


Figure 2: Comparison between different Fourier interpolation methods. (a) Binned input gather for a single CMP. Traces are sorted by azimuth, then offset, for the same azimuth. The azimuth sector spacing is 15 degrees, and the offset bin spacing is 100 m. (b) Interpolated gather using MWNI (Liu and Sacchi, 2004). (c) Interpolated gather using our hybrid 5D method. In the circled area the result of the new method looks more continuous.

We also compare the two algorithms using a real land dataset. In this test (see Figure 3), we predict data with an offset spacing of 100 m. Similar to what we observed in the synthetic tests, the new method is capable of preserving travel time accuracy in the offset direction. Careful comparison of the recovered far-offset data (see the enclosed areas in Figure 3b and Figure 3c) further confirms the advantage of the new method. The events with large curvature interpolated by the new method look more continuous than those interpolated by the MWNI method.

We note that the MWNI can be improved by preconditioning the input data by applying orthorhombic NMO correction; however, that would need extra effort in analyzing orthorhombic velocity parameters before applying the interpolation. This new method does not require this effort. Furthermore, multiples or dipping events are usually not flat after NMO correction, which may also pose a challenge to the conventional interpolation method.

In this paper, we have partially solved the data binning problem, since only a 1D non-uniform DFT along the offset direction is used. Errors related to azimuth sectoring still remain. This problem may be alleviated by increasing the number of azimuth sectors. A more accurate solution is to use 2D non-uniform DFT along offset-x and offset-y directions. In this case, there

will be no azimuth and offset binning errors, but the cost will be higher than the current method.

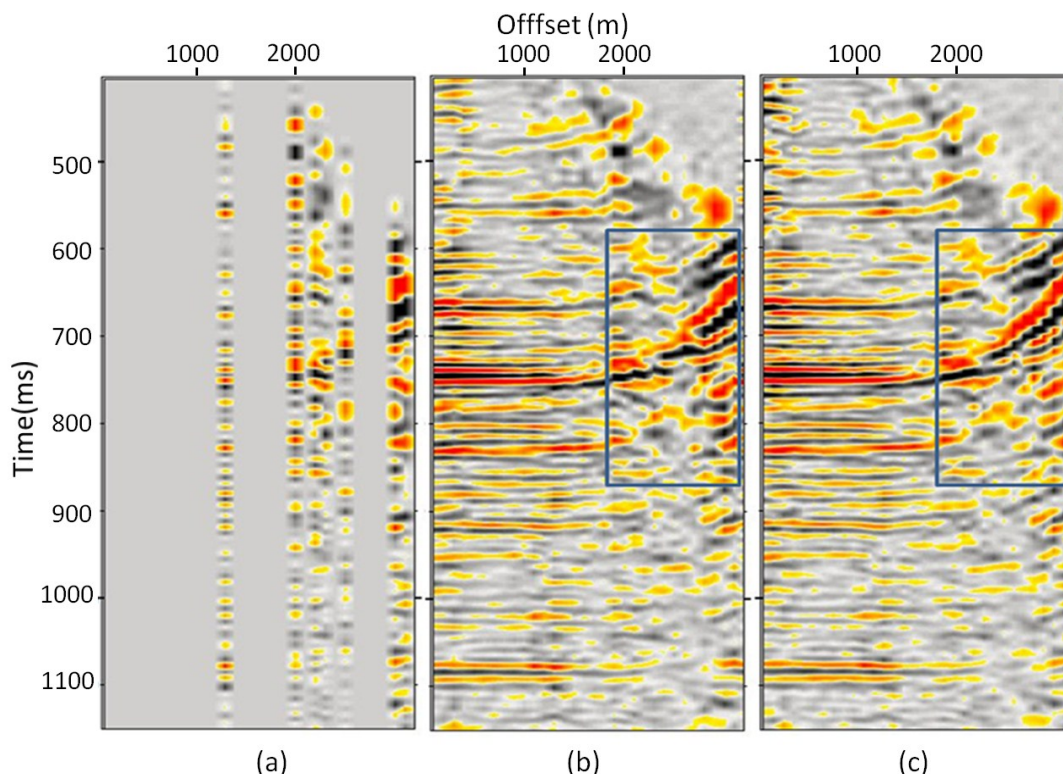


Figure 3: Comparison between different Fourier interpolation methods. (a) Binned input gather for a single CMP. Traces for a single 30 degree azimuth sector are shown sorted by offset. The offset bin spacing is 100 m. (b) Interpolated gather using MWNI (Liu and Sacchi, 2004). (c) Interpolated gather using our hybrid 5D method.

## Conclusions

We have proposed a hybrid 5D Fourier interpolation method that uses actual offset information of input traces to eliminate errors due to offset binning. The advantage of the method is more obvious when coarser offset binning is used in the conventional FFT-based method. More importantly, the algorithm is flexible in predicting seismic data for any user-defined output geometry without losing offset accuracy. This is an important feature because the output geometry is often different from the geometry (or binning) used in the interpolation kernel.

## Acknowledgements

We would like to thank CGG for allowing us to publish this paper. We also appreciate TC Hollis and Shuang Sun for their assistance in processing the real dataset.

## References

- Abma, R. and N. Kabir, 2005, 3D interpolation of irregular data with a POCS algorithm: 75th Annual International Meeting, SEG, Expanded Abstracts, 2150-2153.
- Hestenes, M. R. and E. Stiefel, 1952, Method of conjugate gradients for solving linear systems: Journal of Research of the National Bureau of Standards, 49, 406-436.
- Liu, B. and M. D. Sacchi, 2004, Minimum weighted norm interpolation of seismic records: Geophysics, 69, 1560-1568.
- Trad, D., 2008, Five dimensional seismic data interpolation: 78th Annual International Meeting, SEG, Expanded Abstracts, 978-981.

Xu, S., Y. Zhang, D. Pham and G. Lambare, 2005, Antileakage Fourier transform for seismic data regularization: *Geophysics*, 70, V87-V95.

Zwartjes, P. M. and M. D. Sacchi, 2007, Fourier reconstruction of nonuniformly sampled, aliased seismic data: *Geophysics*, 72, V21-V32.