Seismic Efficiency, Overshoot and Enhanced Dynamic Weaking of Fractures Associated with Stimulation in Heavy Oil Reservoirs

Lindsey N. Meighan, Ted Urbancic and Adam Baig
ESG, Kingston, Ontario

Summary
The objective of this paper is to better understand the failure process of induced microseismic events by investigating radiated energy and seismic efficiency for two passive seismic datasets collected between April 1, 2011 and September 30th, 2013. Dataset A has a total of 4069 events (Mw=-2.5 to -0.3) recorded in a reservoir treated by steam injection. Dataset B has a total of 1763 events (Mw=-1.36 to 1.78) that occurred in a reservoir treated with water injection. Both reservoirs have similar geology. Our results show that Dataset A has low efficiency events (overshoot $<0.5$), whereas Dataset B has 1545 events with low efficiency and 216 events with high efficiency (undershoot $>0.5$). Overshoot, a measure of how large the dynamic strength $t_d$ is relative to the residual stress $t_1$ (normalized by the static stress drop) predominates in both reservoirs. Observed lower overshoot values were indicative of lower efficiency events and conversely higher observed overshoot values were suggestive of high efficiency and enhanced dynamic weakening. Utilizing these data allows for the inference of underlying variability in the source behaviour (dynamics) associated with the injection programs.

Introduction
The total energy, $E_T$, of an earthquake is prorated between radiated energy $E_r$, and the sum of energy that is dissipated or stored within the source by various processes such as frictional heating, fracture, latent heats and other processes. The total energy $E_T$ is commonly expressed as:

$$E_T = E_R + E_K + E_G$$

$E_K$ and $E_G$ are the heat and fracture energy that is dissipated or stored in the source. Increased energy dissipation of the source will decrease the amount of radiated energy. The fracture efficiency is a measure of the fracture energy times the fault area respect to the energy associated with the static stress drop, and more simply can be expressed as the ratio of fracture stress to static stress drop: $\eta_c = t_c/\Delta t_a$, where $t_c$ is the fracture stress.

Many studies show that radiated energy $E_r$ and seismic moment $M_o$, increases systematically. The seismic moment is a measure of the total amount of energy transformed during an earthquake, where only small fraction of seismic energy is converted into radiated seismic energy. The ratio of radiated energy to seismic moment ($E_r/M_o$) changes with earthquake magnitude and apparent stress. The apparent stress is the stress measure of radiated energy and corresponds to the radiated energy per unit area of slip. Thus, we expect source properties of smaller and larger earthquakes to vary. The amount of energy available to be radiated is an important physical process to understand as it ultimately represents the cause of damaging ground motion. The apparent stress and static stress drop are two common seismologically measured source parameters and are very useful for understanding source dynamics. In this paper, we adopt the commonly used ratio of apparent stress to static stress drop known as the
Salvage-Wood Efficiency [1971] to better understand the failure process of microseismic events in two similar reservoirs.

**Theory and/or Method**

The seismic moment, $M_0$, is a measure of the strength of the seismic event that is model independent and can be directly determined from the frequency spectra, $|\Omega_\omega|$, of the P and S wave:

$$M_0 = \frac{4\pi \rho c^3 R |\Omega_\omega|}{F_0}$$

$|\Omega_\omega|$ represents the spectral level of the P wave or the vector of the sum of the components of the S-wave. The more commonly used measure of earthquake strength or size is the moment magnitude, $M$:

$$M = \frac{2}{3} \log M_0 - 6.0$$

where $M$ is in Nm. The most direct measure of radiated seismic energy is of P or S waves can be estimated from the integral $J_0$, where the energy of the P or S waves is expressed as:

$$E_0 = \frac{4\pi \rho c R^2 J_0 \langle \bar{P}_0^2 \rangle}{F_0^2}$$

and assumes the loss of energy from attenuation has been accounted for in the energy flux calculation. $\langle \bar{P}_0^2 \rangle$ is the average radiation coefficient squared. We can then use the ratio of the radiated energy and seismic moment to calculate the apparent stress; a stress measure of the amount of radiated energy:

$$\sigma_a = \frac{\mu E_0}{M_0}$$

where $\mu = \rho \beta^3$. The shear stress release acting at the source may be estimated by the stress drop $\Delta \sigma$, equivalent to the difference between the initial stress $\sigma_0$, and final stress $\sigma_1$, on the fault:

$$\Delta \sigma = \sigma_0 - \sigma_1$$

From dimensional analysis, the stress drop is related to the seismic moment and source radius $r_0$:

$$\Delta \sigma = \frac{7M_0}{16r_0^3}$$

Where $r_0 = K_c \beta_0/2\pi f_c$ where $K_c$ depends on the source model, in this case the Brune model is considered, $\beta_0$ is the shear wave velocity and $f_c$ is the corner frequency.

The Savage-Wood Efficiency can then be calculated by the ratio of apparent stress to static stress drop.
Low efficiency ($\eta < 0.5$) earthquakes emit less radiated energy relative to the total energy available, where higher efficiency ($\eta > 0.5$) earthquakes radiate higher amounts of the total energy available. The Radiation Efficiency, the radiated energy associated to static stress drop, is just twice the Savage-Wood Efficiency. The radiation efficiency is related to the slip velocity and thus the dynamic stress drop. From here we calculate the stress overshoot, a measure of how large the dynamic strength $t_k$, is relative to the residual stress $t_1$, normalized by the static stress drop:

$$\varepsilon = \frac{(t_k - t_1)}{\Delta t s}$$

Positive values of overshoot indicate there is high dynamic strength relative to the residual stress and thus the final stress does not exceed the overall strength of the fault. Low and negative values of overshoot indicate low dynamic strength relative to residual stress, which is suggestive of enhanced dynamic weakening. Thus, the apparent stress can be rearranged as:

$$\tau_a = \Delta \tau_s (0.5 - \varepsilon).$$

Overshoot $\varepsilon$ will occur when the seismic efficiency $\eta < 0.5$, where undershoot occurs when the overshoot value $\varepsilon$ is negative and the seismic efficiency $\eta > 0.5$. The overshoot value is a measure of "strength excess", the difference between the peak or yield stress, $t_y$, to the initial stress $t_1$. This is an important factor in determining the slip and rupture speeds during dynamic rupture. A plot of seismic efficiency versus moment magnitude will have lines of constant overshoot with zero slope representing "self similarity" of efficiency. The boundary between overshoot and undershoot is $\eta_{sw}= 0.5$. If the static stress drop remained constant, increasing apparent stress would result in a decreasing overshoot, where increasing overshoot would indicate an increase in relative dynamic strength compared to the residual stress levels.

**Examples**

Dataset A has 4069 microseismic events between April 2011 and September 2013. The ratio of radiated energy to seismic moment ($E_r/M_o$) plotted as a function of moment magnitude, Figure 1; shows a small change in the ratio $E_r/M_o$ relative to moment magnitude. The ratio of $E_r/M_o$ decreases for smaller earthquakes. Frictional stress does not directly affect the $E_r/M_o$ ratio and therefore larger values of frictional stress do not always result in smaller values $E_r/M_o$. However, in the situation that the frictional stress drops gradually over time, the dynamic stress drop will be relatively small compared to the static stress drop, and thus reducing the ratio $E_r/M_o$. Additionally, as fracture energy increases, $E_r/M_o$ becomes smaller. Thus, the small $E_r/M_o$ for this dataset, ranging between $6.15\times10^{-12}$ and $3\times10^{-7}$, may suggest a gradual drop of friction during slip or larger fracture energy.
Figure 1: Ratio of radiated energy $E_r/M_o$ plotted against moment magnitude.

The Savage-Wood Efficiency is illustrated in Figure 2:

Figure 2: Plot of apparent stress versus static stress drop. Increasing apparent stress with static stress drop and corresponds to higher efficiency of earthquakes.

There is a linear increase between the apparent stress and static stress drop. The lower apparent stress and static stress drop, corresponds to smaller magnitude events and expect lower efficiency. The relationship between the seismic efficiency and moment magnitude is illustrated in Figure 3:
Figure 3: Plot of seismic efficiency as a function of moment magnitude. The red line represents the boundary between overshoot and undershoot at 0.5 efficiency. All events recorded are low efficiency (overshoot).

Overshoot for this dataset are all below 0.5 ranging between $\eta=0.01$ to 0.49. This is an indication of lower seismic efficiency ($\eta<0.5$) and thus higher “strength excess”. The microseismic events for this dataset suggest that the average resisting stress is higher than the final stress. This is conducive to low rupture velocities or a high velocity encounter at a high strength barrier. The wide range of overshoot corresponds to the larger range of efficiency and apparent stress. The lower values of apparent stress also suggest lower rupture velocity and longer event duration, corresponding to lower values of slip velocity. The size and range in magnitudes for this dataset is small ($M_w=-2.2$ to -0.2) and thus we expect a relatively small difference in seismic efficiency across this magnitude range.

Dataset B has a total of 1763 event locations between April 2011 and September 2013. The ratio $E/M_o$ plotted as a function of moment magnitude is illustrated in Figure 4:
Figure 4: Ratio of radiated energy $E_r/M_o$ plotted against moment magnitude for Dataset B.

For this dataset, the larger range in magnitudes ($M_w = -1.36$ to 1.78), resulted in a larger range in $E_r/M_o$, $2.2 \times 10^{-11}$ to $4.4 \times 10^3$. On average, the higher $E_r/M_o$ suggests a higher static stress drop and apparent stress and therefore greater seismic efficiency. A plot of seismic efficiency, Figure 5, shows higher apparent stress and static stress drops than compared to the previous dataset:

Figure 5: Plot of apparent stress versus static stress drop for Dataset B.

The relationship between seismic efficiency and moment magnitude, Figure 6, illustrates the higher seismic efficiency across the dataset:
Figure 6: Plot of seismic efficiency as a function of moment magnitude. The red line represents the boundary between overshoot and undershoot at 0.5 efficiency. 1545 events are recorded with low efficiency and 213 events recorded with high efficiency.

There are 218 of the 1763 events are recorded with conditions of undershoot, indicating higher efficiency and suggestive of enhanced dynamic weakening. Since the apparent stress is expected to increase with magnitude, a constant static stress drop and increasing apparent stress would result in decreasing overshoot, and thus increasing seismic efficiency. The remaining 1545 events of this dataset are in conditions of overshoot, suggesting lower efficiency events and greater dynamic strength relative to the present stress levels.

Beeler et al. [2003] suggests that seismic efficiency greater than 0.3 is a more than typical indication of dynamic weakening. Typical efficiencies are expected to be in the range of 0.1 to 0.3. A wide range of seismic efficiencies and other source parameters may be an indication of varying source dynamics. In order to determine source dynamics with earthquake magnitude, we can assess the changes in static stress drop and efficiencies.
Conclusions

In this study, we investigate the radiated energy and Savage-Wood Efficiency of two datasets of microseismic events recorded in steam and water injection reservoirs, to better understand the source dynamics. We find under similar geological conditions and time period, the seismic efficiency and thus radiation efficiency are unique. Dataset A with 4069 events ($M_w = -2.2$ to -0.2) have low efficiency events ($\eta = 0.01$ to 0.48) and considered overshoot ($\epsilon = 0.1$ to 0.5). Dataset B has much fewer events but greater range of magnitudes ($M_w = -1.4$ to 1.8) where 1545 of the total 1763 events exhibit low efficiency ($\eta = 0.004$ to 0.49) and thus in overshoot ($\epsilon = 0.002$ to 0.49). The remaining 213 events are considered to be high efficiency ($\eta = 0.007$ to 416) and considered undershoot ($\epsilon = -415$ to 0.49) suggesting enhanced dynamic weakening. It is clear from Dataset B that the larger range in magnitudes correlates to a larger range in seismic efficiencies and inferred varying source dynamics.

Further to this study, we propose to explore in the effects of fracture efficiency in the failure process and infer changes in fracture energy with other source parameters with event size. Additionally, exploring the dynamic stress drop by determining the peak acceleration and velocity value to calculate the radius of the most energetic asperity and thus maximum stress release; for which we can calculate rupture velocity and interpret changes in the failure process.