Plane Wave Reflection and Transmission Coefficients for VTI Media

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Summary:

Presently, we obtain the reflection(*R*) and transmission (*T*) coefficients at boundary between two transverse anisotropic media with the vertical axis of symmetry (VTI) in behalf of their importance for numerical computations. Additionally, these coefficients are valuable for the full elastic wave modeling in anisotropic media. Classical *R* and *T* coefficients have been expressed as a function of the phase angle that can be computed by using the effective ray parameter. To do this, we compute a normal for each individual plane wave based on local velocity that is function of Thomson's parameter of the medium and vector cross-product of this normal with the normal to the reflector yields a ray parameter that is used here to compute the corresponding *R* and *T* coefficients for a given plane wave. Now following the Graebner's approach we obtain *R* and *T* coefficients in terms of Thomson's parameters as these parameters are essential for understanding the seismic wave's signatures in the anisotropic media. Further, as the dependency of the reflected waves amplitude on offset has proven to be a valuable exploration tool for direct hydrocarbon detection, we have demonstrated that anisotropy does have considerable influence on the *R* coefficient of seismic waves. Thus conventional AVO analysis needs to be modified in the presence of the anisotropy on either side of the reflecting boundary. A test of accuracy of Rüger's approximation is also delineated here.

Introduction:

Since the *R* coefficient play an important role in order to interpret the field records for lithology, porosity and fluid content etc (Upadhyay, 2004). Thus, the amplitude of the reflection attains more intension of Geoscientists. For isotropic media, the amplitude of the reflection is a function of the density, compressional and shear wave velocities of the two layers that make up the interface and the angle of incidence (Shearer, 1999). The velocity of isotropic media remains constant during the AVO analysis while velocity of anisotropic media varies with angle of incidence and interrupts the AVO analysis (Rüger, 2001). In order to analyze the effect of anisotropy on the *R* and *T* coefficients, VTI model is taken into account due to its simplicity among anisotropic media beyond the isotropic media. The thinly layered media with horizontal interfaces and horizontally stratified shale formations are characterized by the VTI model (Tsvankin, 2001). For VTI media the wave equation separates into a coupled pair of the equations for the P-SV waves and into a single equation for the pure SH wave (Slawinski, 2003). Further, VTI media possess z axis as axis of symmetry so there is no loss of generality in considering propagation in any plane. First we discuss about the plane wave *R* and *T* coefficients of SH wave for VTI media. Then we consider the plane wave *R* and *T* coefficients of P and SV wave as an extension of preceding work.

Theory and Method:

Plane wave *R* and *T* coefficients for SH wave: In past, *R* and *T* coefficients have been obtained in several domains according to their importance. Further, on consideration of anisotropy in seismic exploration, the *R* and *T* coefficients have been obtained in terms of the phase angle and material properties on the either side of the interface (Daley, 1977). Presently, we drive the *R* and *T* coefficients in the plane wave domain in behalf of the efficiency in terms of the computational time for Rayleigh Sommerfeld Modeling (RSM)

(Sharma, 2009). Along with this, sometimes *R* and *T* coefficients are required for use in reflectivity programs where integration over ray parameter is required (Rüger, 2001). For this case parameterization by the phase angle can be inconvenient. This inconvenience can be avoided by deriving the *R* and *T* coefficients in terms of the ray parameter. To do this, we compute the ray parameter using effective ray parameter (Sharma, 2009) that is used to compute corresponding *R* and *T* coefficients in the plane wave domain. In general, the reflected and transmitted waves are generated by an incident wave when an interface is encountered. The amplitude of the reflected and transmitted waves depends on the *R* and *T* coefficients (Krebes, 2008). In order to obtain the *R* and *T* coefficients boundary conditions, the continuity of displacement and traction, are considered at the boundary. After applying the boundary conditions *R* and *T* coefficients for anisotropic media are obtained in terms of the effective ray parameter and the elastic constant and can be written as (Slawinski, 2003)

$$R_{SH} = \frac{\rho_1 \beta_{01}^{\ 2} q_1 - \rho_2 \beta_{02} \ _2}{\rho_1 \beta_{01}^{\ 2} q_1 + \rho_2 \beta_{02}^{\ 2} q_2}, \tag{1}$$

$$T_{SH} = 2\frac{\rho_1 \beta_{01}^{\ 2} q_1}{\rho_1 \beta_{01}^{\ 2} q_1 + \rho_2 \beta_{02}^{\ 2} q_2}, \tag{2}$$

Where ρ, β_0 and q represent density, vertical velocity and slowness of shear wave, respectively. Upper and lower mediums are represented by subscript 1 and 2, respectively. Vertical slowness of upper and lower medium can be represented as $q_i = \sqrt{\beta_{0i}^{\ -2} - p_I^{\ 2}(2\gamma_i + 1)}$ with i=1, 2 and γ indicates Thomson's parameter. $_I$ is the effective ray parameter and can be computed as

$$p_I = |\hat{\boldsymbol{p}} \times \hat{\boldsymbol{a}}|\sqrt{p_1^2 + \ _2^2 + q^2}. \tag{3}$$

Where p_1 and p_2 are the horizontal components of the slowness vector. The slowness vector $\hat{\boldsymbol{p}}$ characterizes the direction of incident wavefield and $\hat{\boldsymbol{a}}$ is unit normal vector to interface (see Sharma, 2009 for analytic expressions).

Plane wave *R* and *T* coefficients for P-SV waves:

Historically, the *R* and *T* coefficients of P-SV waves for an isotropic media have been studied by numerous authors (Aki, 1980, and Kennett, 2001). Further, Daley and Horn has extended this study for the anisotropic media (Daley, 1977). Graebner (Graebner, 1992) has published the *R* and *T* coefficient in terms of the elastic coefficients and the horizontal and the vertical components of the slowness vector. Since Thomson's parameters for an anisotropic medium play an important role in order to reduce the non uniqueness of the inverse problem, where it is needed to model the data in a given geologic environment (Grechka, 2009), we derive the *R* and *T* coefficients in terms of Thomson's parameters for seeking the effect of Thomson's parameters (δ, ε) on these coefficients. To do this, we develop a relationship between the elastic constants used by Graebner and Thomson's parameters (Thomson, 1986) that is represented as

$$A = \rho\alpha_0^{\ 2}(1+2\varepsilon), C = \rho\alpha_0^{\ 2}, L = \rho\beta_0^{\ 2}, N = \rho\beta_0^{\ 2},$$

and

$$F = \rho\sqrt{\left(\alpha_0^{\ 2} - \beta_0^{\ 2}\right)\left((2\delta+1)\alpha_0^{\ 2} - \beta_0^{\ 2}\right)} - \rho\beta_0^{\ 2}. \tag{4}$$

Further, by using the effective ray parameter (p_I) and following Graebner's approach, we obtain 3D *R* and *T* coefficients for VTI media by solving the equation $Sx = b$. For this case the matrix $S$is given by

$$S=\begin{bmatrix} l_{\alpha 1} & m_{\beta 1} & -l_{\alpha 2} & -m_{\beta 2} \\ m_{\alpha 1} & -l_{\beta 1} & m_{\alpha 2} & l_{\beta 2} \\ a_1 & b_1 & a_2 & b_2 \\ c_1 & d_1 & -c_2 & -d_2 \end{bmatrix},\quad x=\begin{bmatrix} r_{pp} & r_{ps} & t_{pp} & t_{ps} \end{bmatrix}',\ \text{and}\quad b=\begin{bmatrix} -l_{\alpha 1} \\ m_{\alpha 1} \\ L_1(q_1 l_{\alpha 1}+p_I m_{\alpha 1} \\ -p_I l_{\alpha 1} F_1 - q_{\alpha 1} m_{\alpha 1} \end{bmatrix}. \qquad (5)$$

Where l_{ij} and m_{ij} are the eigenvectors of the P wave and SV wave, $i=\alpha,\beta$ are corresponding to P and SV wave, respectively and $a_j = L_j(q_{\alpha j} l_{\alpha j} + p_I m_{\alpha j})$, $b_j = L_j(q_{\beta j} m_{\beta j} - p_I l_{\beta j})$, $c_j = p_I l_{\alpha j} F_j + q_{\alpha j} m_{\alpha j} C_j$, $d_j = p_I m_{\beta j} F_j - q_{\beta j} l_{\beta j} C_j$. $j = 1,2$ characterize the upper and lower medium, respectively(see Sharma 2010 for more detail).

Examples:

Now following the equations from 1 to 3 as discussed above, we obtain the *R* and *T* coefficients of SH wave in plane wave domain for VTI media. To authenticate the proposed approach we obtain these coefficients for an isotropic medium by employing a constraint on the γ (γ=0) in equations 1 and 2. Figure 1a and 1b show the in-line and cross-line slices of the *R* and *T* coefficients. The obtained results by following the anisotropic and isotropic algorithms are denoted by the red and the green colors, respectively, and the overlapping of these results ensures the efficacy of the proposed approach of obtaining *R* and *T* coefficients. On consideration AVO analysis for pre-critical propagation, we analyze the influence of Thomson's parameter on it. Figure 2 shows the R coefficient curves as a function of the horizontal slowness for the situation where the vertical velocity of the upper medium exceeds the vertical velocity of the lower medium. Further, four sub-cases shown in legend, have been taken into account. The change between the *R* coefficients values, as well as the change of the slope of it, is significant among the individual sub-cases. Further, following the Graebner's approach and the relationship that has been build above, we have computed the *R* and *T* coefficients for P and SV waves. In order to test the accuracy of the plane wave domain *R* coefficient the three models characterized by the class 1, 2 and 3 type of Gas-sand anomaly, respectively are considered. The model parameters used presently are taken from Rüger (Rüger, 2001). Further, the accuracy of the popular approximation given by Rüger is also tested here. Figure 3 shows the P-wave reflectivity with horizontal slowness for an isotropic media. It is indicated from this figure that curves obtained by the exact algorithms of the isotropic media and VTI media are analogous to each other while approximation of Rüger provides a close match to the exact solutions near to the zero horizontal slowness and deviates from the exact solution as slowness increases. The overlapping of the plane wave *R* coefficients obtained by the exact isotropic and degenerated anisotropic algorithms establishes the accuracy of the approach followed by the author. Further, the overlay of obtained exact reflection coefficient with the reflection coefficient obtained by applying Rüger's approximation near to the horizontal slowness can be treated as supportive result in favour of the accuracy of the exact plane wave *R* coefficient. To illustrate the effect of the anisotropy on the P-P reflectivity and the accuracy of the Rüger's approximation's, we show the P-wave *R* coefficients for the same three models as used previously but now the VTI symmetry has been introduced into overburden shale by considering the anisotropic parameters ($\varepsilon = 0.133, \delta = 0.12$). Figure 4a illustrates the effect of the anisotropy on the P-P reflection coefficient and accuracy of Rüger's approximation as exact VTI *R* coefficient are compared with the corresponding isotropic ($\varepsilon, \delta = 0$) and VTI approximated *R* coefficients. Further, the examples are repeated for two different value of anisotropy parameter (δ=0 and δ= 0.133) in Figures 4b and 4c, respectively. By examining these Figures, it is observed that the difference between the curves are restricted to the large values of slowness but the accuracy of the approximation remains unchanged near to and at the horizontal slowness.

Conclusions:

We have presented the plane wave *R* coefficient of the SH- and P-SV-waves for anisotropic media by following the Graebner's approach and using effective ray parameter. It has been demonstrated that ignoring the presence of anisotropy in VTI media has the potential of severely distorting the AVO analysis. The authentication of the obtained plane wave *R* coefficient of P-wave has been described in reference to isotropic and Rüger's approximated *R* coefficients. Since there is considerable difference between the *R* coefficient curve obtained from the exact and approximated algorithms at the large value of the horizontal slowness which may also be noticeable near to zero slowness in the presence of strong anisotropy, we should deal with the more exact algorithm so that the scanty of the accuracy could be avoided.

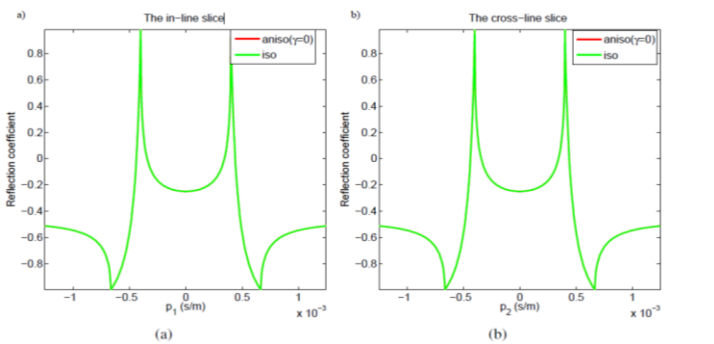

FIG.1: (a) In-line (b) Cross-line slices of the SH wave R coefficients. The red and green lines indicate the R coefficients obtained by anisotropic and isotropic algorithms.

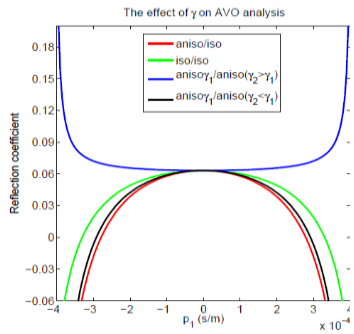

FIG.2: The influence of anisotropy on AVO analysis.

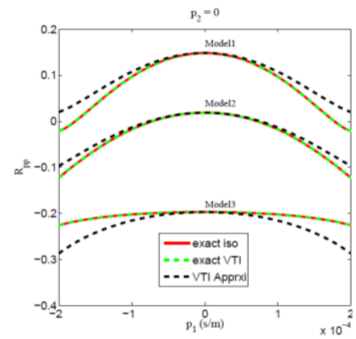

FIG.3: P wave R coefficients for three shale/gas-sand interface. The solid red line, dashed green and black indicate the exact isotropic, exact VTI and approximated R coefficients.

a) b) c)

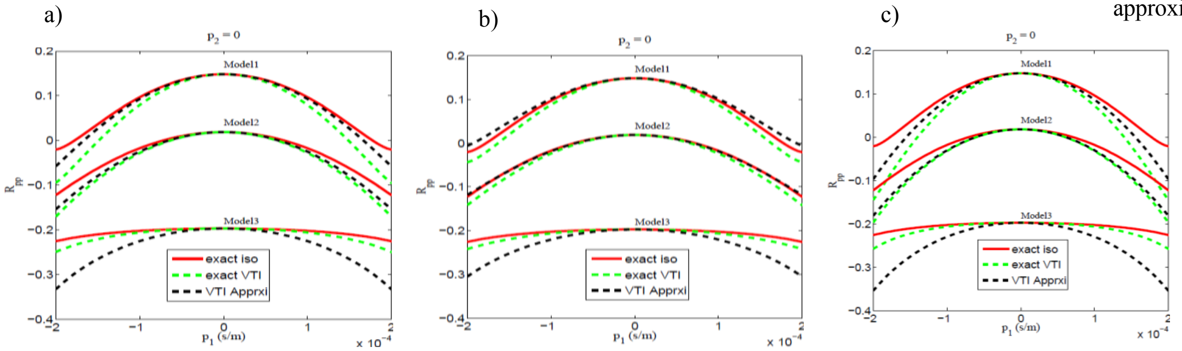

FIG.4: R coefficients curve of P wave for three models after introducing VTI into the shale overburden with anisotropic parameters (a) $\boldsymbol{\varepsilon = 0.133, \delta = 0.12}$ (b) $\boldsymbol{\varepsilon = 0.133, \delta = 0}$ (c) $\boldsymbol{\varepsilon = 0.133, \delta = -0.24}$.

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References

Aki, K., and Richards, P.G.,1980, Quantitative Seismology Theory and Methods: W.H. Freeman and Co.,San Francisco.

Daley, P., and Horn, F., 1977, R and T coefficients for transversely isotropic media: Bulletin of the Seismological Society of America, **67**.

Graebner, M., 1992, Plane-wave reflection and transmission coefficients for a transversely isotropic solid: Geophysics, **57**.

Grechka, V., 2009, Application of Seismic Anisotropy in the Oil and Gas Industry: EAGE.

Kennett, B. L. N., 2001, The Seismic Wavefield: Cambridge.

Krebes, E. S., 2008, Seismic Theory and Methods: Course Notes.

Rüger, A., 2001, Reflection Coefficient and Azimuthal AVO analysis in Anisotropic Media: Society of Exploration Geophysicists.

Sharma, R. K., and Ferguson, R. J., 2009, R and t coefficients for sh wave in plane wave domain: CREWES Research Report, **21**.

Sharna, R.K, and Ferguson , R.J.,2010,R and T coefficients for VTI media: CREWES Research Report, **22**.

Shearer, P. M., 1999, Introduction to Seismology: Elesvier.

Slawinski, M. A., 2003, Seismic waves and Rays in Elastic Media.: Pergamon SecondEdition.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, No. 10, 1954–1966

Tsvankin, I., 2001, Seismic Signatures and Analysis of Reflection data in Anisotropic Media.:Handbook of Geophysical Exploration Series.

Upadhyay, S. K., 2004, Seismic Reflection Processing: Springer.