Fractional Fourier Transform in Coherent Noise Attenuation

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Summary

Many seismic data processing techniques are based on different forms of time- frequency representation of signals. In the fractional Fourier transform (FRFT), signals can be represented in multiple domains including time and frequency. This gives an extra degree of freedom in data processing where the traditional Fourier transform (FT) is used. The main difference between the FT and FRFT comes from the difference of their transformation kernels. In FT, the kernel is complex sinusoids, whereas in the FRFT, it is a set of linear chirps. In this paper, we took advantage of the multidomain property of the FRFT, and used it to separate spatially coherent hyperbolic events from linear events with higher level of accuracy. Based on a synthetic shotgather example, it is observed that filtering in the *f-FRFT* domain performed better in events separation than the corresponding *f-k* domain filtering. The caveat of the FRFT domain filtering is the selection of an optimum set of fractional orders. We have used the linearly-varying and second-order central-moment based techniques for fractional orders calculation. The moment based fractional order technique achieved better result than that of the constant angle (*f-k* domain) technique in separating the hyperbolic events from linear events.

Introduction

The fractional Fourier transform (FRFT) decomposes a signal into linear chirps. Thus, the FRFT is inherently suited for analysis and synthesis of signals with strong chirp-like properties. On the other hand, the traditional Fourier transform decomposes a signal into sinusoids and is used in many signal processing applications. The FRFT is a generalized integral transform represents a signal into various domains in the time-frequency plane. Two special domains of the FRFT are time and frequency. So, the FRFT can be applied to either time or frequency representation of a signal. The FRFT concept was first introduced in the 1980s by Namias (1980) and later by McBride and Kerr (1987) but its applications have not investigated until the 1990s (Almeida, 1994, 1997; Bailey and Swarztrauber, 1991; Ozaktas et al., 1996; Pei and Yeh, 1998; etc.).

The limitation of the frequency domain representation arises from the fact that if both the signal and noise overlaps in the same frequency band, then their separation from each other becomes difficult if not impossible. In that scenario, the FRFT can be useful in signal separation and noise attenuation considering its ability to represent signals in any domain in the time-frequency plane within the range of the fractional order. We have used a synthetic shotgather as an example of separating hyperbolic events from linear events using two different fractional order formulations. In the last section, we express the FRFT as forward-adjoint operator pair, which will be used to setup the FRFT filtering as an inverse problem, as a continuation of this research.

Theory of FRFT

The FRFT with a fractional order parameter α corresponds to counterclock-wise rotation by an angle in the time-frequency plane (Fig. 1). The FRFT of a time-domain signal, $x(t)$ for any fractional order can be defined as (Almeida, 1994):

$$F_x^{\alpha}(u) = X_{\alpha}(u) = \int_{-\infty}^{+\infty} K_{\alpha}(t,u)x(t)dt \tag{1}$$

where α is the fractional order, and $K_{\alpha}(t,u)$ is the transformation kernel which is defined as

$$K_{\alpha}(t,u) = \sqrt{\frac{1-j\cot\alpha}{2\pi}} \exp\left(j\frac{\cot\alpha}{2}t^2 - jut\csc\alpha + j\frac{\cot\alpha}{2}u^2 \right) \text{ for } \alpha \neq 0, \pi/2, \pi . \tag{2}$$

The inverse FRFT for the fractional order α can also be expressed as

$$x(t) = \int_{-\infty}^{+\infty} K_{-\alpha}(t,u)X_{\alpha}(u)du . \tag{3}$$

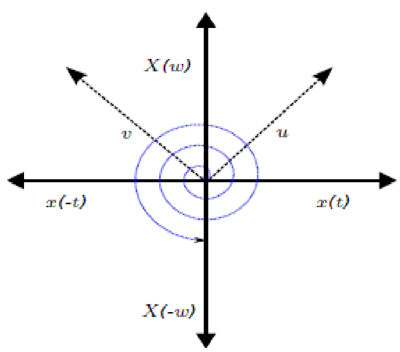


Figure 1: Rotation of FRFT in *t-ω* plane

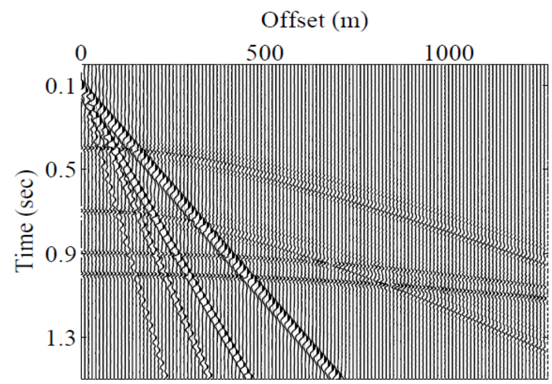
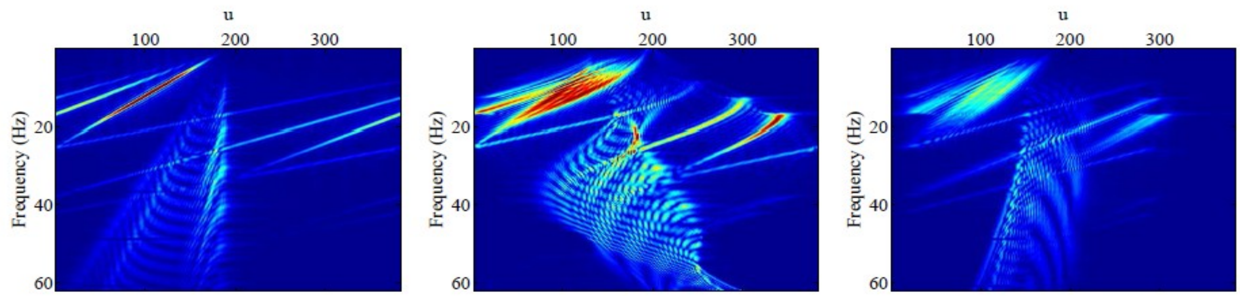


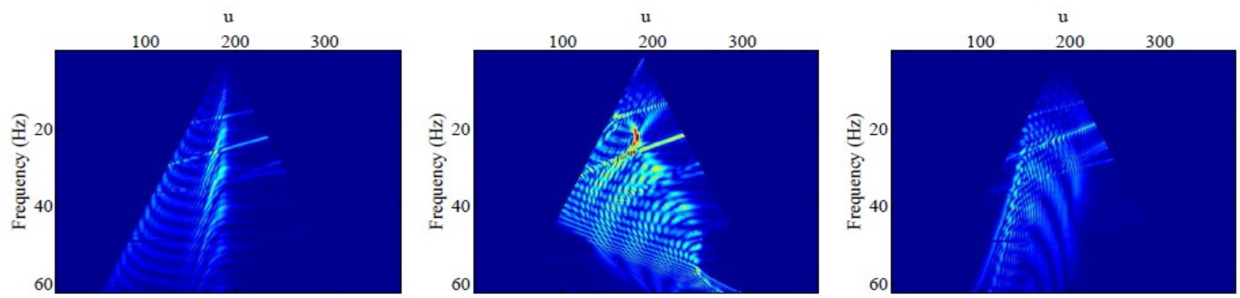
Figure 2: Synthetic shotgather with events

Example of coherent noise separation using FRFT

A synthetic shotgather (Fig. 2) with overlapping spatially coherent linear noise and hyperbolic events is used for filtering in the FRFT domain. In the shotgather, the linear events correspond to surface waves (i.e. groundroll) while the hyperbolic events correspond to primaries and multiples. First, the 2-D shotgather in *t-x* domain is converted into the *f-x* domain. In the *f-x* domain, data for each frequency across the offset direction are transformed into the FRFT domain for various fractional orders. *Constant*, *linearly-varying*, and *second-order central-moment* (SOCM) based techniques are used to select fractional orders α for filtering. For the *constan*t fractional order realization, $\alpha = \pi/2$ is used for the FRFT for every frequency. Thus, this realization corresponds to the traditional frequency-wavenumber (*f-k*) domain representation. Then a conventional fan filter is used to filter out linear events in the FRFT domain (Fig. 3). The estimated events and residual show considerable presence of linear events in the shotgather. In the *linearly-varying* technique, the fractional orders in increasing order are used for frequencies starting from low to high. The result of filtering with this technique shows improved separation of hyperbolic events from linear events than that of the *constant* α (*f-k* domain) case. For the *second-order central-moment* technique, the first derivative of the SOCM is used to find the fractional order α for each frequency. In general, the SOCM is used to measure the width of a signal. The residual shotgather shows a very little presence of linear events in compare to the *constant* and *linearly-varying* cases (Fig. 4).

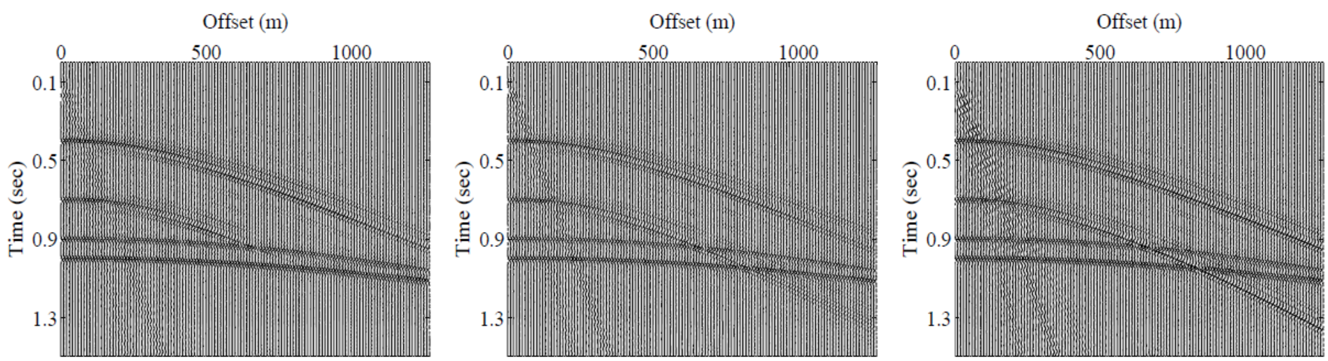


(a) *f-k* domain. (b) *f-FRFT* linear. (c) *f-FRFT* central-moment.

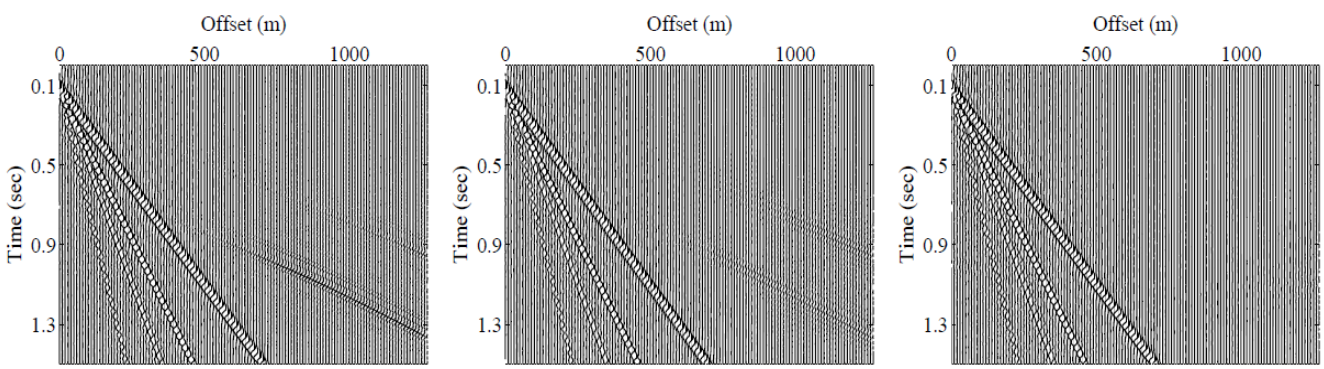


(d) Filtered *f-k* domain. (e) Filtered *f-FRFT* linear. (f) Filtered *f-FRFT* central-moment.

Figure 3: Magnitudes of *f-FRFT* and filtered *f-FRFT* for different realizations.



(a) Estimated events: *f-k*. (b) Estimated events: *f-FRFT* linear. (c) Estimated events: *f-FRFT* central-moment.



(d) Residual: (*f-k*). (e) Residual: *f-FRFT* linear. (f) Residual: *f-FRFT* central-moment.

Figure 4: Estimated and residual shotgathers for different *f-FRFT* realizations.

FRFT forward-adjoint operator pair

In this section, we express the FRFT of a 2D image or shotgather $d(t,x)$ as forward-adjoint operator pair.

$$D(\alpha,u,\omega)=\iint K^{\zeta}(\alpha,x,u)\,d(t,x)\,e^{-j\omega t}\;dt\,dx\,, \tag{4}$$

$$d(t,x)=\frac{1}{2\pi}\iiint K(\alpha,x,u)\,D(\alpha,u,\omega)\,e^{j\omega t}\;d\omega\,du\,d\alpha\,, \tag{5}$$

where $K^{\zeta}(\alpha,x,u)$ and $K(\alpha,x,u)$are the adjoint and forward FRFT operator for $\alpha \neq 0, \pi/2, \pi$ respectively.

$$K^{\zeta}(\alpha,x,u)=\sqrt{\frac{1-j\cot\alpha}{2\pi}}\exp\left(j\frac{\cot\alpha}{2}x^2-jux\csc\alpha+j\frac{\cot\alpha}{2}u^2\right). \tag{6}$$

$$K(\alpha,x,u)=\sqrt{\frac{1+j\cot\alpha}{2\pi}}\exp\left(-j\frac{\cot\alpha}{2}x^2+jux\csc\alpha-j\frac{\cot\alpha}{2}u^2\right). \tag{7}$$

$$K^{\zeta}(\alpha,x,u)=K(-\alpha,x,u). \tag{8}$$

Conclusions

Multiple representations of a time or frequency domain signal in the fractional Fourier transform gives an edge in signal processing over the traditional Fourier transform. The synthetic shotgather example showed the effectiveness of the FRFT in attenuating coherent noise and separating useful seismic events. Determination of a set of optimum fractional orders for a particular application is the key in using FRFT over the traditional Fourier transform. For general seismic data processing, the FRFT can be expressed as forward-adjoint operator pair, and filtering for events estimation can be setup as solving a regularized inverse problem.

Acknowledgements

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