Evaluation of a New 5D Seismic Volume Reconstruction Method: Tensor Completion versus Fourier Reconstruction

Nadia Kreimer* and Mauricio D. Sacchi
Department of Physics and Institute for Geophysical Research, University of Alberta, Edmonton, Alberta, Canada
kreimer@ualberta.ca

Summary

Multi-dimensional Fourier interpolators have become the industry standard for 5D seismic volume reconstruction. However, room for improvement exists and a few key aspects of seismic data reconstruction do require additional study. The latter includes stability in the presence of coherent noise and statics, recovery conditions for extremely sparse data sets and computational efficiency. This presentation addresses some of the aforementioned problems by introducing a new technique for 5D interpolation based on multilinear algebra. Prestack seismic data is organized in a tensor, which is assumed to be a low rank structure when the data is properly sampled. A practical algorithm is presented where tensor rank reduction permits to recover the missing traces and increase the signal-to-noise-ratio of the seismic volume. The technique is compared to a multi-dimensional Fourier interpolator. We have obtained encouraging results with synthetic volumes. In particular, numerical tests indicate noticeable gains in computational efficiency and reconstruction fidelity for very sparse data sets.

Introduction

Reconstruction of prestack seismic data has been an area of intense research activity during the last decade. The problem of seismic data regularization is an essential part of preconditioning strategies prior to multiple suppression, migration and detailed amplitude versus azimuth studies. The 5D volume can be organized as a tensor that contains missing entries. The complete tensor is considered to be a low-rank structure with missing observations and noise increasing its rank. Our algorithm iteratively reconstructs the seismic volume by computing a low-rank tensor data approximation to the original incomplete and noisy data set. We perform a comparison between the classical 5D Fourier reconstruction method and proposed volume reconstruction using tensor completion. For this purpose, we use one particular type of tensor decomposition named High-Order Singular Value Decomposition (HOSVD) proposed by De Lathauwer et al. (2000a).

The Problem

We consider a subvolume of data in the *f-x* domain for fixed frequency *f*. The data depends on the source-receiver positions and by transforming the data to a nominal midpoint-offset domain grid, a tensor that depends on 4 spatial indices $D^{Obs}(\omega,i,j,k,l)$ is created. For a given monochromatic frequency we identify the data by a 4th-order tensor D^{Obs}. The indices *i, j, k, l* are used to indicate *x* and *y* CMP positions and offsets in the *x* and *y* directions, respectively. It is clear that the tensor D^{Obs} contains many missing entries. It is important to stress that a typical land survey can lead to 4-dimensional grids with only 5-10% of the entries occupied by observations. Let's denote the desired tensor by the symbol D. The observations and ideal data tensor are related via $D^{Obs} = T\,D$, where T is the sampling operator and the product is in the Hadamard sense (element-wise product). Furthermore, the elements of the sampling operator are 0 if the bin is empty, and 1 if the bin contains an observation. Solving the tensor completion problem involves estimating D from D^{Obs}.

5D Reconstruction via iterative tensor completion

recovery energy environment economy
2011 CSPG CSEG CWLS Convention

The HO-SVD permits to approximate a tensor by one of lower rank (De Lathauwer et al., 2000a). The decomposition permits to approximate a tensor D by one of lower rank $\widehat{D}$ via the product of 4 unitary matrices U, V, W, Y and a small core tensor G (Kolda and Bader, 2009):

$$[\widehat{D}]_{ijkl} \approx \sum_{m=1}^{i1}\sum_{n=1}^{i2}\sum_{p=1}^{i3}\sum_{q=1}^{i4}[G]_{mnpq}[U]_{im}[V]_{jn}[W]_{pk}[Y]_{ql} \tag{1}$$

where $G, U, V, W,$ and Y are computed from D. The process of obtaining the decomposition from the data is the tensor rank reduction operator that will be written as $\widehat{D} = R\,D$. The rank-reduced tensor is obtained iteratively via an algorithm called High-Order Orthogonal Iteration (HOOI) (De Lathauwer et al., 2000b). We assume that in the ideal case, noise-free fully sampled data, the tensor is a low rank structure that can be represented via a small core tensor. Noise and missing data will increase the size of the core tensor that is required to represent the observations. Therefore, we propose to complete the data via the following iterative algorithm

$$D^{\nu} = a^{\nu} D^{Obs} + (1 - a^{\nu})TR\,D^{\nu-1} + (1-T)R\,D^{\nu-1} \tag{2}$$

The symbol ν indicates iteration and a^{ν} is an iteration dependent scalar that serves to diminish the influence of the noise in the final reconstruction. In our numerical implementation a^{ν} decays exponentially starting with $a^{0} = 0.5$ and takes between 4 and 26 iterations to converge to the reconstructed tensor in the case of the synthetic data. This depends on the level of noise and sampling density. Data synthesized via the tensor decomposition are reinserted in the empty grid points. The grid points with observations are populated by the weighted average of observations and new data at a given iteration. We used the Matlab Tensor Toolbox (Bader and Kolda, 2010) for calculating the tensor rank reduction.

5D Fourier Reconstruction via sparse inversion (FRSI)

Tensor completion is compared to Fourier reconstruction. We adopted 5D Sparse Fourier Reconstruction (Sacchi et al., 1998; Zwartjes and Sacchi, 2007), a technique similar to MWNI (Liu and Sacchi, 2004; Trad 2009), which also operates in the *f-x* domain. This problem reduces to estimating the Fourier coefficients of the data by minimizing the following objective function:

$$J = \left\| TF^{H}c - D^{Obs} \right\|_2^2 + \lambda\,|c|_1 \tag{3}$$

where c are the Fourier coefficients in wave-number domain of the complete data for a fixed frequency *f*. The operator F^{H} indicates the 4D inverse Fourier transform. The cost function (3) is minimized using Iterative Reweighted Least-squares (IRLS) following the algorithm described in Hansen (1998). It is important to stress that similar results can be obtained via MWNI, a method that uses a regularization term similar to the l_1 norm (Liu and Sacchi, 2004; Trad, 2009).

Comparison

For the following example, we use synthetic 5D volumes with 3 linear events. They contain 150 time samples, and a grid size of $12 \times 12 \times 12 \times 12$. We define signal-to-noise ratio as $\sigma_{\text{Data}}/\sigma_{\text{Noise}}$, which was set to 1 and 100. The proportion of sampled traces, called sampling density, was also varied from 25 to 90%. The relative error is $\|D^{out} - D^{true}\|_2^2 / \|D^{true}\|_2^2$, where D^{out} is the output volume and D^{true} is the noise-free fully sampled synthetic volume. The performance of both algorithms is displayed in Figures 1 and 2.

recovery energy environment economy
2011 CSPG CSEG CWLS Convention

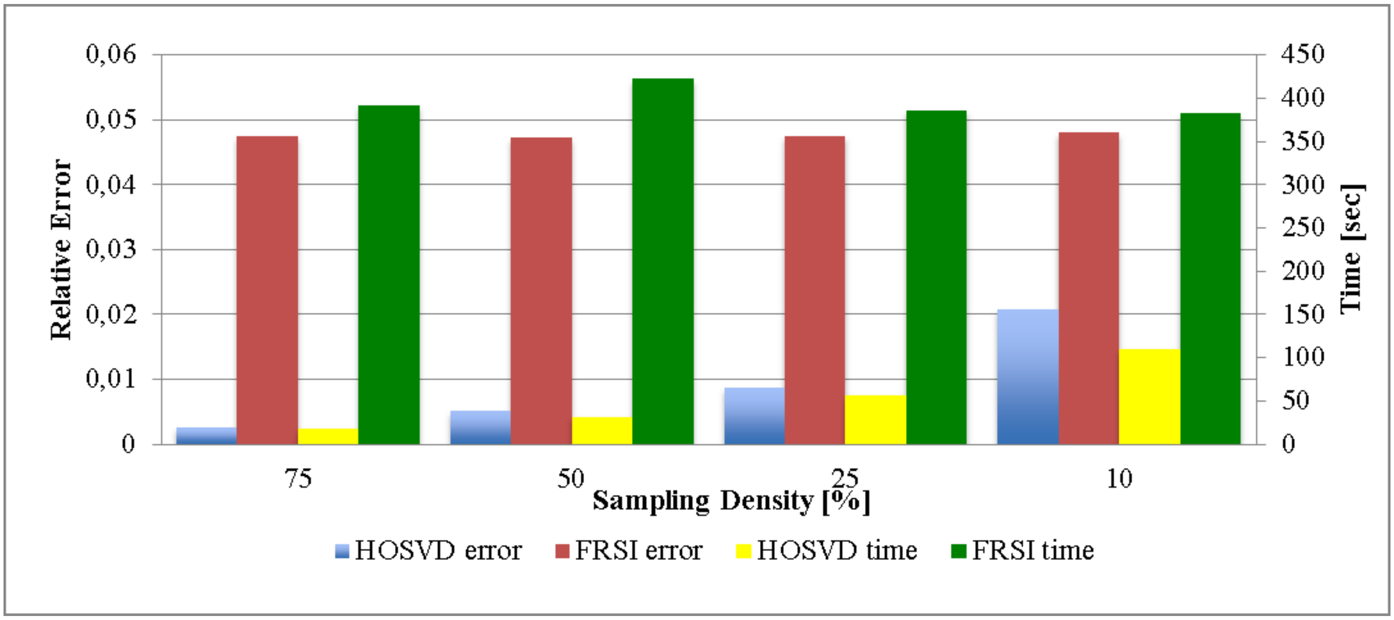

Figure 1: Comparison between Tensor Completion and Fourier reconstruction (FRSI) for a synthetic example with S/N = 100.

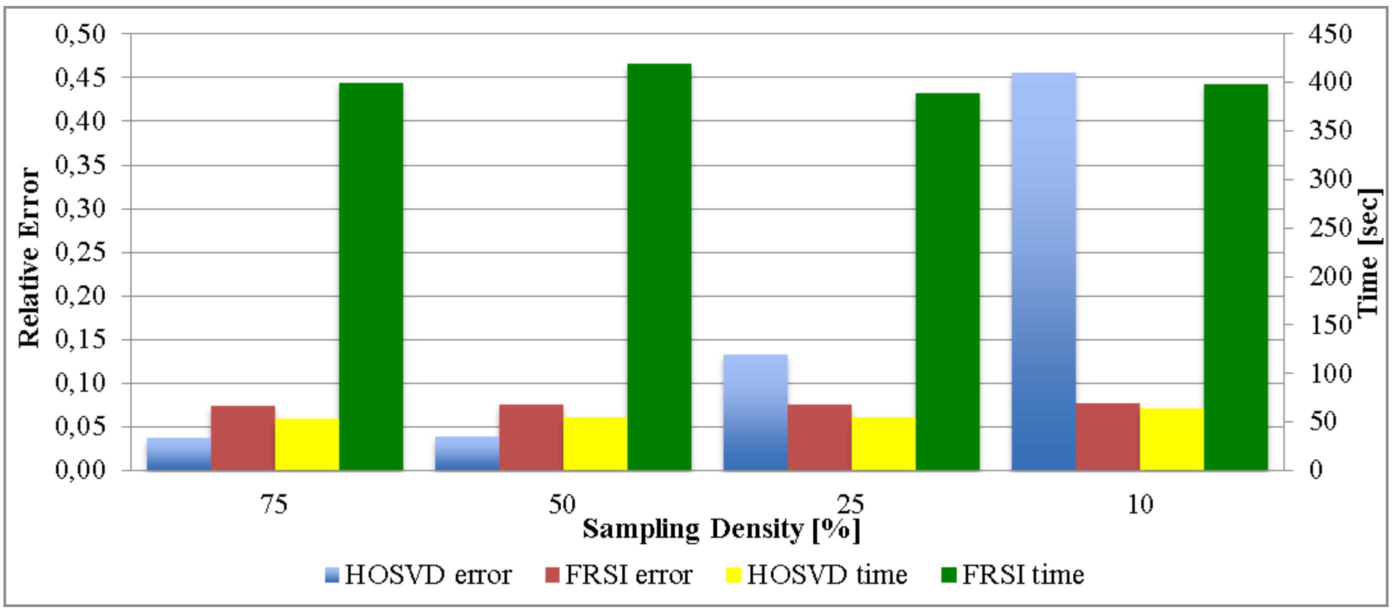

Figure 2: Comparison between Tensor Completion and Fourier Reconstruction (FRSI) for a synthetic example with S/N = 1.

Overall, the proposed tensor completion method yields smaller reconstruction errors than FRSI in addition to improving the run time. It is interesting to notice that when the S/N is not too high, the HOSVD achieves a considerably smaller error that the FRSI, even for extremely sparse surveys.

We also applied FRSI and the tensor completion to a real data set from the Western Canadian Sedimentary Basin. The orthogonal survey was transformed to a nominal geometry of size 30×30 midpoints and 12×12 offsets per midpoint bin. Only 8% of the grid is populated with traces and our task is to estimate the remaining 92% of the traces. The midpoint spacing in x and y are 25m and 50m, respectively. The minimum offset is 0m and the maximum

1400m. The tensor reconstruction used a core tensor of size $5 \times 5 \times 5 \times 5$. In Figure 3 we show a common offset section given by the two methods. The tensor completion method had a running time of 18 minutes while FRSI achieved a similar result in 10 minutes. The output data from the tensor completion method seems to adjust more the subtle amplitude variations than the output given by FRSI.

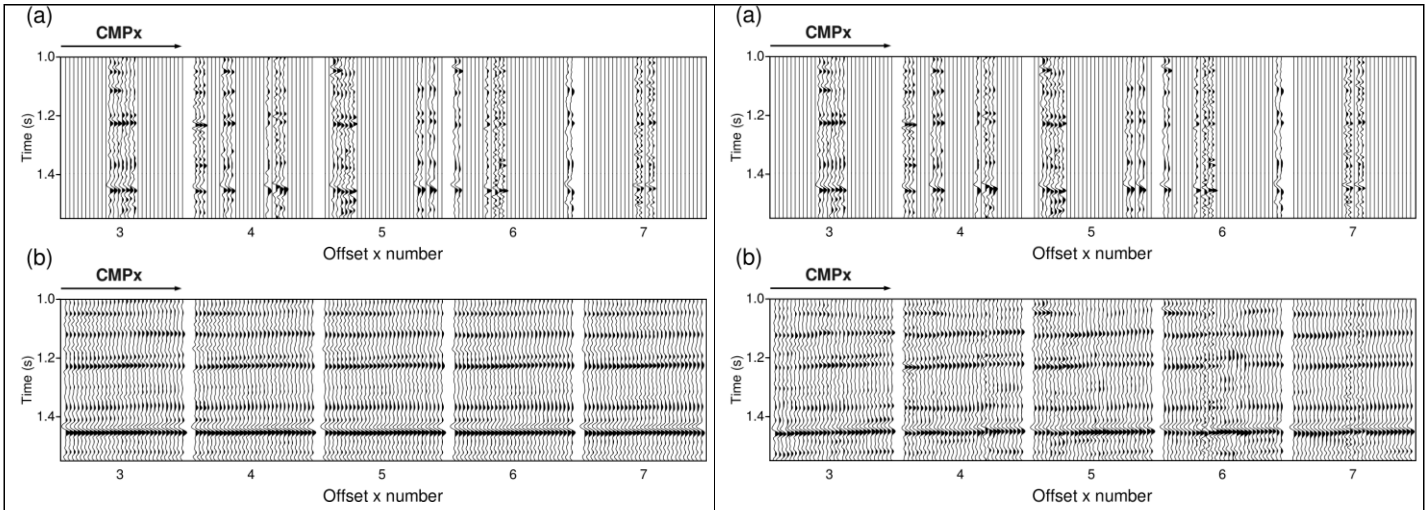

Figure 3: A small part of the real 5D volume used to test the proposed tensor completion method. Common Offset section reconstructed with FRSI (left) and Tensor completion (right). (a) Part of the 5D input and (b) is part of the 5D reconstructed data.

Conclusions

Our research shows that tensor analysis is a useful tool for seismic data reconstruction. Synthetic examples have shown that very sparse data sets can be interpolated with a high degree of accuracy. In this regard, the technique appears to provide results similar to multidimensional Fourier interpolation (FRSI). However, our results with real data tend to be smoother with FRSI because the underlying model is the classical superposition of exponentials that might not adapt well to subtle changes in the character of the signal. On the other hand, tensor completion is data-driven and seems to adapt better to small variations in amplitude in the signal. It is clear that with real data, i.e. not knowing the *true* solution, it is quite difficult to assess the quality of the interpolation. This is why it is so important to drive our research with a combination of synthetics tests and field data experiments.

References

Abma, R. and Kabir, N., 2006, 3D interpolation of irregular data with a POCS algorithm, Geophysics, 71(6), E91-E97.

Bader, B. and Kolda, T. G., March 2010, *MATLAB Tensor Toolbox Version 2.4*, http://csmr.ca.sandia.gov/~tgkolda/TensorToolbox/

De Lathauwer, L., De Moor, B. and Vandewalle, J., 2000a, A multilinear singular value decomposition, SIAM J. Matrix Anal. Appl., 21, 1253-1278

De Lathauwer, L., De Moor, B. and Vandewalle, J., 2000b, On the Best Rank-1 and Rank-(R_1,R_2,. . .,R_N) Approximation of Higher-Order Tensors, *SIAM J. Matrix Anal. Appl.* 21, 4

Hunt, L. S. Reynolds, M. Hadley, J. Downton, D. Trad and B. Durrani, 2008, Interpolation, PSTM and AVO for the Viking and Nisku targets in the West Central Alberta, CSEG Recorder, 33, 7-19

Kolda, T.G. and B.W. Bader, 2009, Tensor decomposition and applications, SIAM Review, 51, 455-500.

Liu, B. and M.D. Sacchi, 2004,Minimum Weighted Norm Interpolation of seismic records, Geophysics 69, 1560-1568.

Sacchi, M. D., Ulrych, T.J. and Walker, C.J., 1998, Interpolation and extrapolation using a high-resolution discrete Fourier transform, IEEE Transaction on Signal Processing, 31–38.

Spitz, S., 1991, Seismic trace interpolation in the f-x domain, Geophysics, 56, 7850-794.

Trad, D., 2009, Five-dimensional interpolation: Recovering from acquisition constraints, Geophysics 74, V123

Zwartjes, P.M. and Sacchi, M.D, 2007, Fourier reconstruction of nonuniformly sampled, aliased seismic data, Geophysics 72, V21.

Hansen, P. C., 1998, Rank-deficient and discrete ill-posed problems: Numerical aspects of linear inversion: SIAM Monographs on Mathematical Modeling and Computation v. 4.