

# Simultaneous source separation via robust time variant Radon operators

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## Summary

Time invariant Radon operators are commonly used in seismic data processing for data regularization and multiple suppression. These operators use basis functions that match seismic data which enable it to focus events in the model domain. Resolution of estimated Radon models is negatively impacted by limited seismic aperture and incomplete data. Inversion using sparse priori by minimizing  $L_1$  or Cauchy model norm proved to be useful in enhancing Radon model resolution. However, sparse inversion can be susceptible for failure when erratic noise contaminates the data. In this paper we propose to add robustness to time domain Radon operators to handle erratic noise situations. A particular application for robust Radon inversion is de-blending of simultaneous source data. Robust Radon can handle incoherent (non Gaussian) blending noise that could hinder de-blending using Radon transforms.

## Introduction

High resolution, also called sparse, Radon transforms have been used to reconstruct seismic data and to remove multiple reflections. The idea of sparsity in Radon domain was first proposed by Thorson and Claerbout (1985) and was latter applied to time-invariant operators by Sacchi and Urych (1995). In essence, Radon operator is posed as an inverse problem with sparsity constraint to estimate a sparse distribution of Radon coefficients.

Firing seismic sources with time overlap (blended sources) is a technique used to reduce acquisition cost (Berkhout, 2008). If blending is considered as an operator that sum shot gathers after applying time delays, the adjoint (de-blending) operator de-shift shot gathers and separate them. However, these pseudo de-blended shot gathers are contaminated by waveforms associated with cross-talk between sources. The blending operator is non-orthogonal and single shot gathers can not be retrieved. Interferences in common receiver gather domain manifest as erratic noise (Jiang and Abma, 2010) that can be removed via a robust Radon transform (Guitton and Symes, 2003; Ji, 2012). We explore the possibility of adopting the Robust Radon transform to remove shot interferences in common receiver gathers.

## Theory

We designate  $\mathbf{m}$  as the Radon coefficients (model) and  $\mathbf{d}$  the data. Similarly,  $\mathbf{L}$  denotes the forward Radon operator. The seismic gather can be synthesized via  $\mathbf{d} = \mathbf{L}\mathbf{m}$ . We also stress that in our analysis  $\mathbf{L}$  denotes the general form of a Radon forward operator that encompasses both time variant and time-invariant operators. The coefficients  $\mathbf{m}$  are estimated by minimizing the following cost function

$$J = \|\mathbf{L}\mathbf{m} - \mathbf{d}\|_p^p + \mu \|\mathbf{m}\|_q^q. \quad (1)$$

where  $p$  and  $q$  are integers used to denote the type of norm that is adopted for the misfit and regularization terms. For  $p = q = 2$  we have the classical Radon transform with damping (Hampson, 1986). If  $p = 2$  and  $q = 1$  we have the sparse (or high resolution) Radon transform studied by Sacchi and

Ulrych (1995). The Radon coefficients  $\mathbf{m}$  are estimated by minimizing equation 1 using, for instance, Iterative Re-weighted Least Squares (IRLS) (Scales, 1987). According to the selection of  $p$  and  $q$  we define the following transforms:

- Least squares Radon Transform with damping  $p = 2$  and  $q = 2$
- High resolution (sparse) Radon Transform  $p = 2$  and  $q = 1$
- Robust Radon Transform  $p = 1$  and  $q = 2$
- Sparse and Robust Radon Transform  $p = 1$  and  $q = 1$ .

The purpose of this paper is to analyze the impact of choosing  $p$  and  $q$  in the removal of incoherent noise. This kind of noise arise from the process of seismic data blending. We will estimate  $\mathbf{m}$  and then use it to generate a clean gather. In particular, we are interested in understanding if robustness in the misfit  $p = 1$  is more important than sparsity in the model coefficients  $q = 1$ .

## Examples

Blended data can be de-blended using the source firing information such as,

$$\tilde{\mathbf{d}} = \Gamma^T \mathbf{b} \quad (2)$$

where  $\mathbf{b}$  is the blended data,  $\tilde{\mathbf{d}}$  is pseudo de-blended data and  $\Gamma$  is blending operator. If the blending operator in the frequency space domain is given by

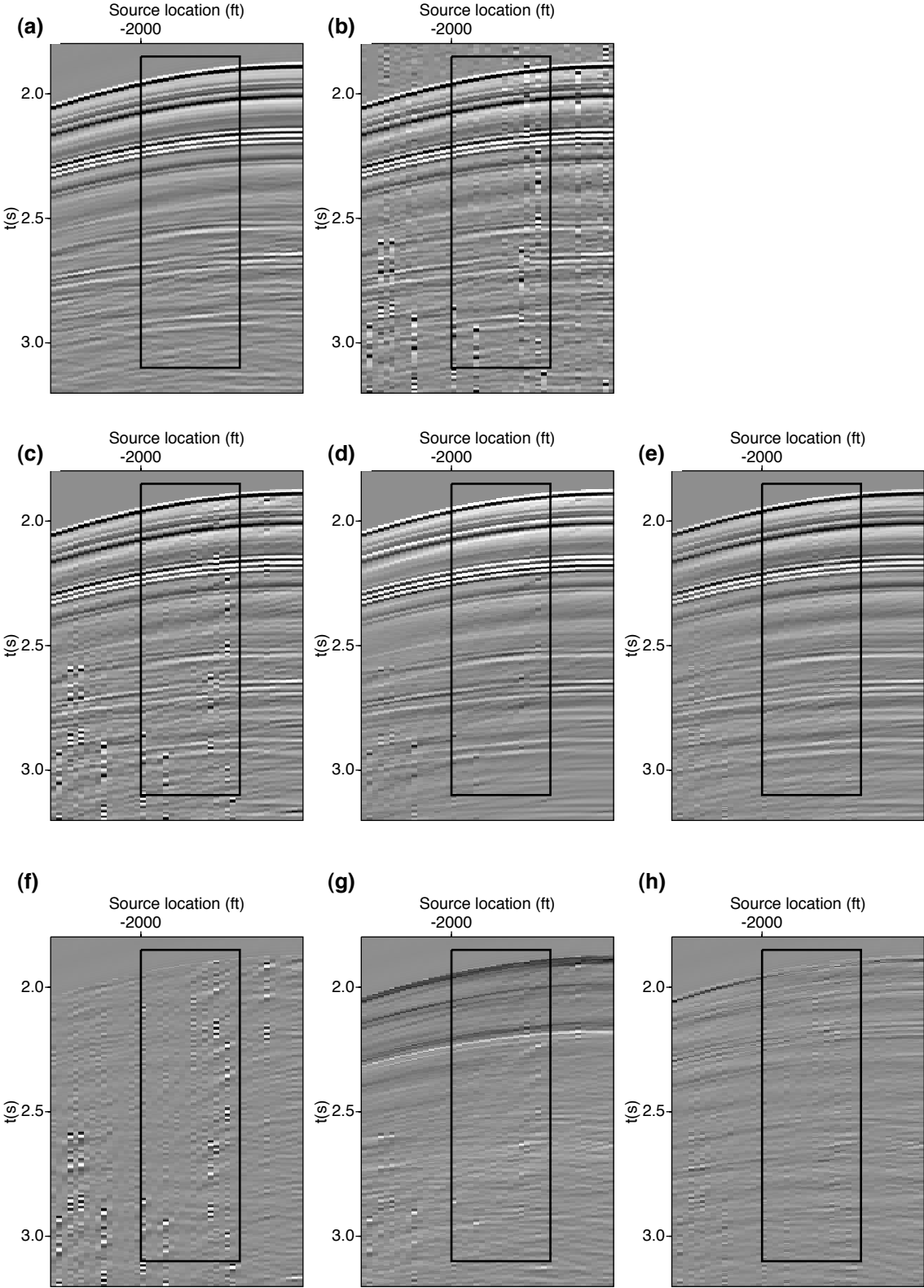
$$[\Gamma]_{ij} = e^{i\omega\tau_{ij}} \quad (3)$$

where  $\tau_{ij}$  is the delay of source  $i$  firing time with respect to detector  $j$ . Therefore, the adjoint operator simply shifts data according to delays between the source and detector. This process is known as pseudo de-blending Berkhout (2008). In common detector domain the signal is coherent over hyperbolic paths and blending noise manifests as erratic inferences. Moore et al. (2008) and Akerberg et al. (2008) proposed using sparse inversion of the time-invariant Radon operator to remove blended noise. Mahdad et al. (2012) proposed using the Radon operators to iteratively estimate blending noise by filtering in the Radon domain. In this work we test the Robust Radon operator with marine data from the Gulf of Mexico. The data were blended numerically with 50% time reduction compared to the conventional acquisition. Four moving sources were used to simulate a practical blended acquisition survey. Each source fires while it is moving along the same line and in the same direction as other sources.

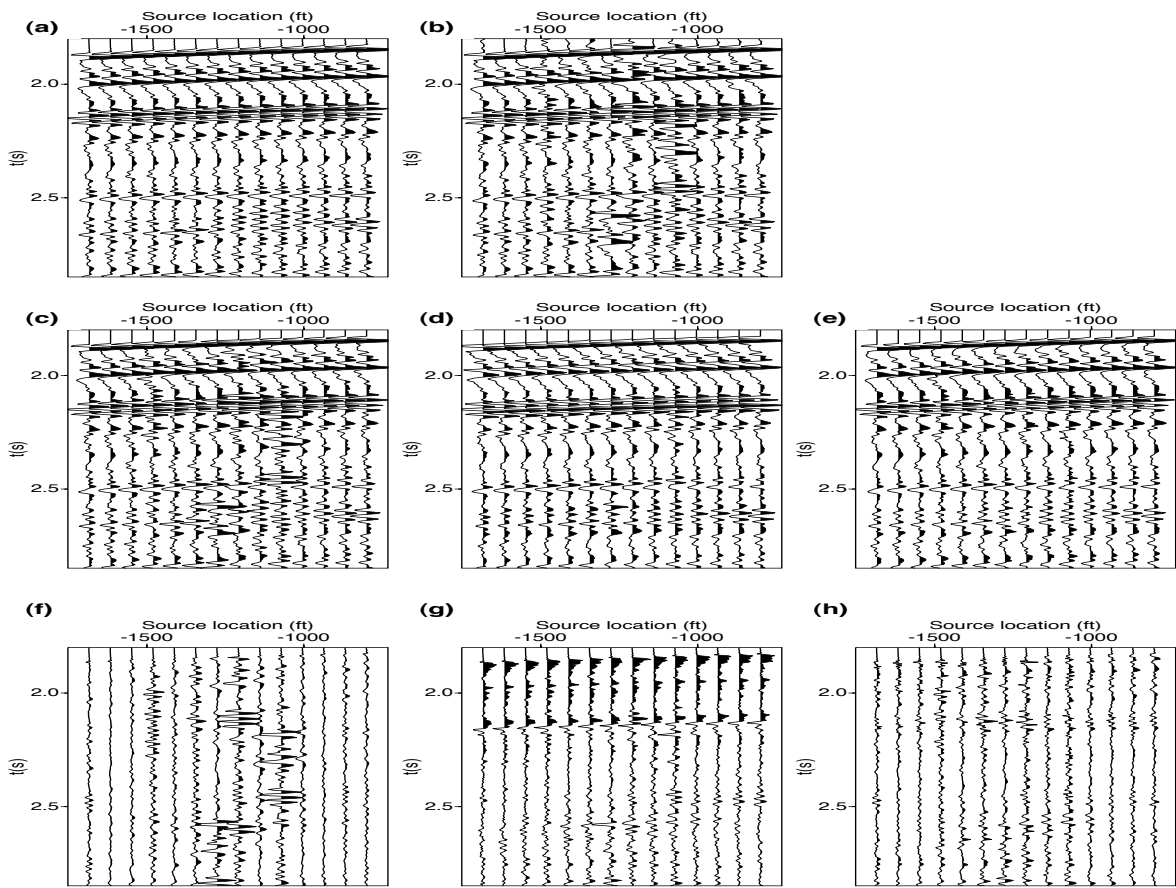
Figure(1) shows data estimate for common detector gather using least squares, sparse and robust inversion . To quantitatively determine the accuracy of data retrieved by inversion type let us define,

$$Q = 10\text{Log} \frac{\|\mathbf{d}_{original}\|_2^2}{\|\mathbf{d}_{original} - \mathbf{d}_{retrieved}\|_2^2}. \quad (4)$$

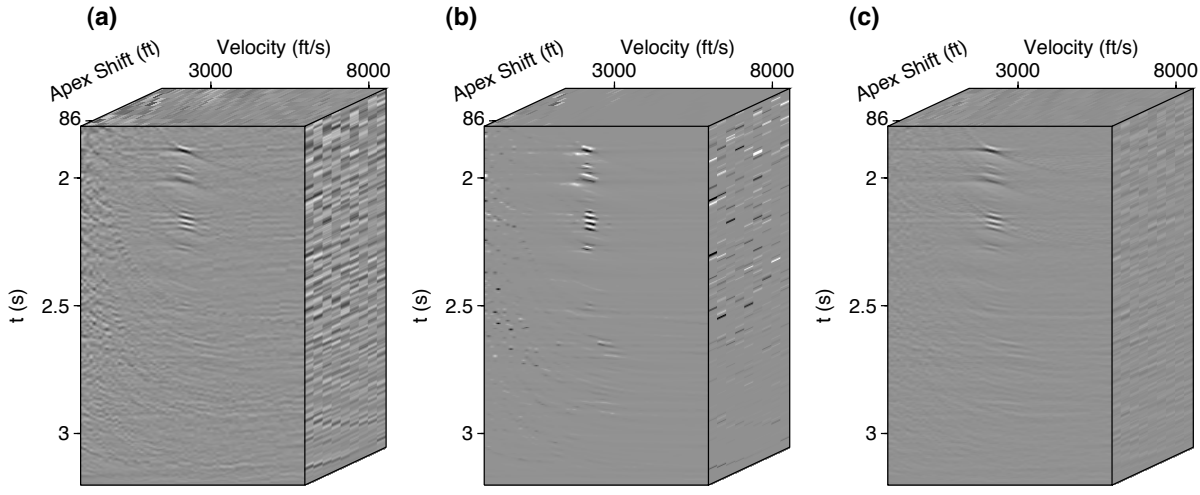
Where  $Q$  represents a scale for retrieved data quality. The  $Q$  value for this common detector gather least square, sparse and robust model estimates are 10.71, 7.18 and 14.28 respectively.



**Figure 1** Common detector gather of real data. (a) Original unblended. (b) Blended data. (c) Retrieved using  $p = 2$  and  $q = 2$ . (d) Retrieved using  $p = 2$  and  $q = 1$ . (e) Retrieved using  $p = 1$  and  $q = 2$ . (f) Error in  $p = 2$  and  $q = 2$  data. (g) Error in  $p = 2$  and  $q = 1$  data. (h) Error in  $p = 1$  and  $q = 2$  data.



**Figure 2** Close up of the real data example. (a) Original unblended data. (b) Blended data. (c) Retrieved data using  $p = 2$  and  $q = 2$ . (d) Retrieved data using  $p = 2$  and  $q = 1$  (e) Retrieved using  $p = 1$  and  $q = 1$ . (f) Error in  $p = 2$  and  $q = 2$  data. (g) Error in  $p = 2$  and  $q = 1$  data. (h) Error in  $p = 1$  and  $q = 1$  data.



**Figure 3** Radon model for real the data using inversion with (a)  $p = 2$  and  $q = 2$ , (b)  $p = 2$  and  $q = 1$  and, (c)  $p = 1$  and  $q = 2$

## Conclusion

Incoherent de-blending noise in common detector gathers can be removed using Robust Hyperbolic or Apex Shifted Hyperbolic Radon operators. Sparse constraints for Radon operators produce sparse model which facilitate separation of waveforms with similar move-out for multiple suppression. However, in cases where one needs to separate coherent reflections from incoherent noise (the de-blending case), equipping the misfit function with an  $L_1$  norm is more important than adding sparsity in the regularization term of the cost function of the problem.

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