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Hydraulic Fracturing as a Global Cascade in Networked Systems

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Summary

Networked systems require the consideration of the interactions between component parts as well as the parts themselves in understanding the properties of the system. In the case of hydraulic fracturing, the process can be regarded as the spread of a fractured state through an initially unfractured network of rock elements. In this study, we implement a spreading model in networks to evaluate the dynamics of the hydraulic fracturing process in various rock types. The corresponding results regarding the stresses that must be overcome, the areal extent and energy release in hydraulic fracturing were in qualitative agreement with empirical observations.

Introduction

Many macroscopic phenomena manifest as the result of a network of interacting agents and often exhibit dynamics that are reciprocal to its network structure, resulting in behavior that can exhibit nonlinearities. These network interactions govern phenomena ranging from collective behavior in schooling fish to the spreading of viruses in human networks. In these networked systems, the interactions between component parts are just as important as the parts themselves in defining the properties of the system (Motter and Albert, 2012). In addition, it is widely accepted that macroscopic phenomena do not depend on the microscopic details of the process, as in effective field theories that are applicable at some chosen length scale and ignores the substructure and degrees of freedom at shorter distances. Therefore, the description of seemingly complex phenomena can be greatly reduced in complexity by application of the above paradigms. In this study, we apply these concepts to hydraulic fracturing to investigate the dynamical process under which hydraulically induced fractures propagate in various rock types. These simplifications allow us to discard the complex fluid flow and fracture mechanics in modeling the dynamic response of hydraulic fracturing (i.e. Lutz, 1991). It should be noted that the simplified approach only provides qualitative descriptions and lacks the rigor in understanding the phenomenon at a fundamental level. It does however provide an alternative conceptual view of the problem.

Many observations concerning fracture propagation in so called brittle or ductile rocks have been well established with the aid of empirical data, where brittleness is often associated with higher quartz content and a relative low for the Poisson’s ratio. For example, engineering data such as the instantaneous shut in pressure (ISIP), which provides an indication for the stress that must be overcome for fracture propagation, is found to correlate with Poisson’s ratio (i.e. Maxwell et al., 2011). It is also generally observed that fractures propagate further in brittle rocks while propagation is more localized in ductile rocks, as suggested by microseismic event locations. In addition, microseismic moment densities are observed to decrease with increasing brittleness.

In the following, we implement a simple network spreading model in an attempt to model the dynamics of the hydraulic fracturing process and evaluate the corresponding fracture propagation response for various rock types.

Fracture Spreading Model

Here, we adopt the abstraction of a network to represent a rock mass with interacting elements, where the network consists of a set of nodes connected by lines or edges. The network is a purely theoretical object but provides an extremely useful representation of complex systems with interacting components. To investigate the dynamics of fracture propagation through a network of rock elements, we implement a spreading model proposed by Watts (2002) for the description of global cascades on random networks. In the model, a binary decision process with externalities is considered. For a given network, each individual in the population, represented by a node, must decide between two alternative actions, where their decisions are based solely on the actions of other members in the population. In the case of hydraulic fracturing, the process can be regarded as the spread of the fractured state in a network of initially unfractured rock elements. For the model specification, we consider a population where an individual agent observes the states (0 for unfractured or 1 for fractured) of its connected neighbors, where the range of connections is known as the degree, and if a certain threshold fraction, defined on the unit interval, is achieved, it adopts state 1, else it remains in state 0. To initiate the system, a set of seed nodes are placed in the network and the process is subsequently iterated through a series of time steps. A successful hydraulic fracture treatment is then defined by a cascade event, where if a cascade is triggered, state 1 spreads throughout the network and if a cascade is not triggered, the network remains in its initial state.

To calculate the threshold, we implement the uniaxial strain condition for loading of an elastic solid given by

$$\sigma_3 = \frac{\nu}{1-\nu}\sigma_1, \qquad (1)$$

where ν is the Poisson's ratio and σ_1 and σ_3 are the maximum and minimum principle stress magnitudes respectively. For a given value of σ_1, a lower value of σ_3 can be achieved through lowering the value of ν, and according to the Mohr-Coulombe failure criterion (Coulomb, 1773), results in a larger Mohr circle and hence is more easily fractured. Since the quantity $\nu/(1-\nu)$ is defined on the unit interval for all possible values of ν between 0 and 0.5, it can readily be used for the threshold condition. Therefore, a material with a lower Poisson's ratio is more easily fractured and thus requires less influence to achieve failure.

To calculate the degree, we consider how information is transferred in an elastic solid. Upon the application of a stress, particle motion is excited through strain waves and propagates throughout the medium. Therefore, we associate the transfer of information regarding the state of stress through the mechanics of wave propagation. The wave equation can then be used to evaluate how energy propagates through an elastic solid and provide an indication for the network of connected nodes. In a 3D homogeneous medium, the Green's function for the scalar wave equation is given by

$$G = \frac{\delta(ct-r)}{4\pi cr}, \qquad (2)$$

where c is the P-wave velocity, t is time, r is the radial distance from the source location and δ represents an impulse function. According to equation 2, the rate at which the amplitude decays is inversely proportional to r and is scaled by the inverse of the P-wave velocity. Therefore, a material with a higher value of c corresponding to the effective medium, experiences more amplitude decay at a given radial distance r from the source point and results in a more localized connectivity.

Rock Model

For the evaluation of the fracture propagation response in different rock types, we take the mean of the Hashin-Shtrikman bounds (1963) for a two-phase material consisting of quartz and clay with varying mineral fractions. With this approach, we avoid the ill-defined concept of brittleness which is not a fundamental property of an elastic solid. Figure 1 shows the upper and lower bounds and mean for the P- and S-wave velocities of the two phase material calculated using the values in Table 1.

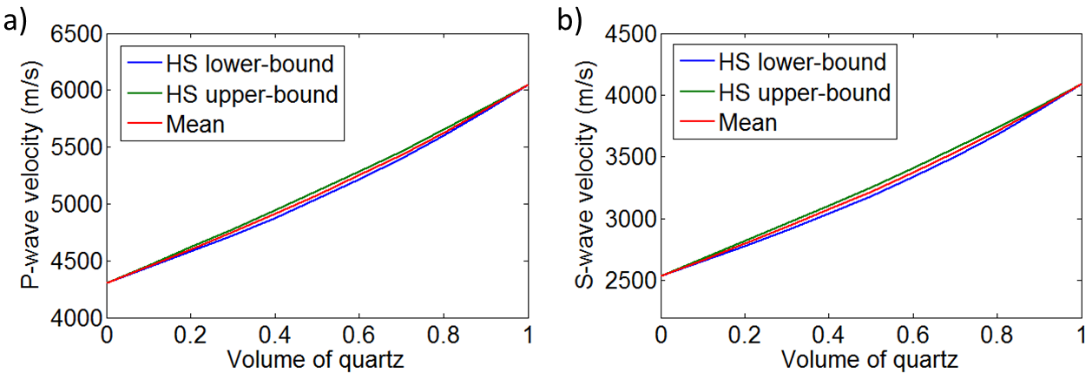

Figure 1: Hashin-Shtrikman upper and lower bounds and mean for the a) P- and b) S-wave velocities for a two-phase material consisting of quartz and clay.

Table 1: Density (ρ) and P- (α) and S-wave (β) velocities used for mineral end members (From Greenberg and Castagna, 1992).

Mineral	ρ (g/cc)	α (km/s)	β (km/s)
Quartz	2.65	6.05	4.09
Clay	2.66	4.32	2.54

Dynamical Modeling

As the dynamics of the hydraulic fracturing problem are not easily amendable to analytical treatment, we solve the system numerically and analysis the corresponding results. The simulations were performed in 2D for each set of mineral fractions ranging from pure clay to pure quartz. As mentioned above, we attribute a successful hydraulic fracture treatment with a cascade event triggered by a certain number of initially active nodes. The properties of interest are then the number of seed nodes required to trigger a cascade, the areal extent of the cascade and the energy output for each set of mineral fractions.

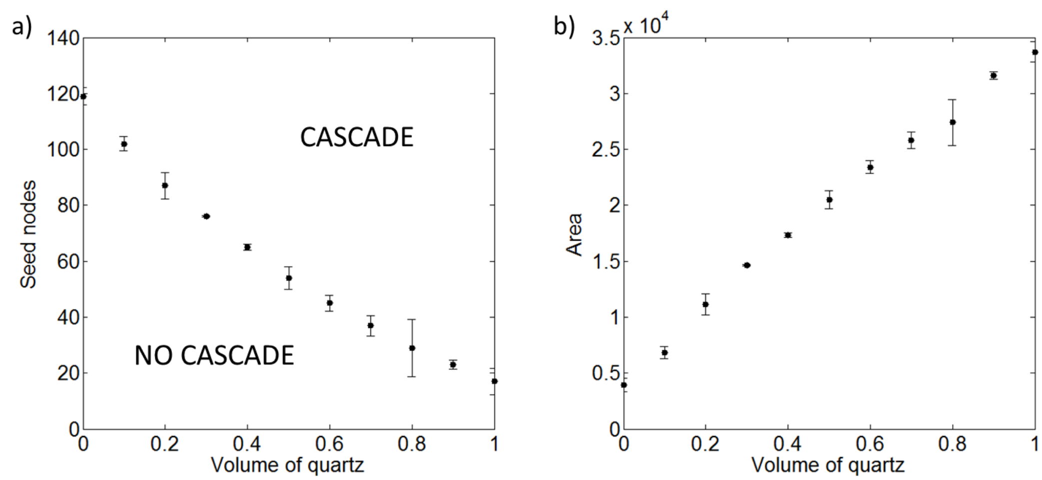

Figure 2: a) Cascade boundary and b) simulated area as a function of the volume of quartz.

Figure 2 shows the phase diagram illustrating the cascade boundary (a) and the stimulated area (b) as a function of the volume of quartz. The simulations demonstrate that less effort is required to achieve a cascade and a larger area is stimulated for a more brittle rock, which is consistent with empirical observations.

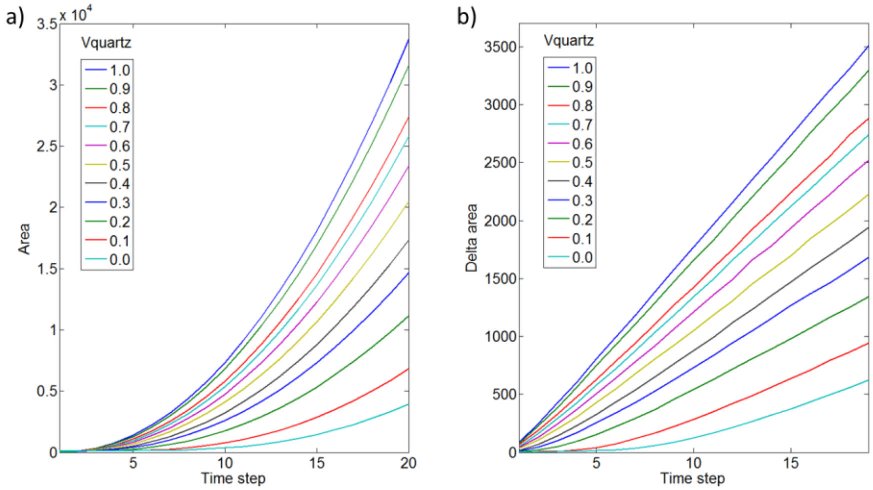

Figure 3: a) Stimulated area and b) change in stimulated area as a function of time.

Figure 3 shows the stimulated area (a) and the change in stimulated area (b) as a function of time. In Figure 3b, note the nonlinear behavior at small time steps for low values of the volume of quartz. This is attributed to the interactions between component parts that result in nonlinearities in defining the properties of the system as a whole.

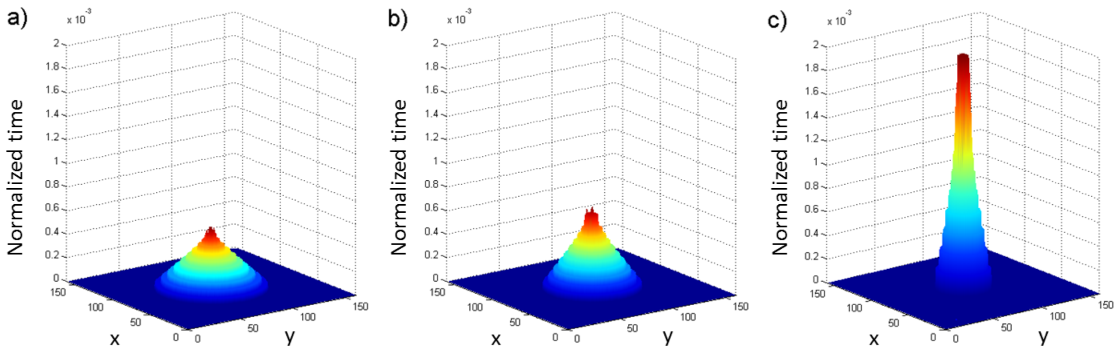

Figure 4: Spatial distribution of total energy output for a) 70%, b) 40% and c) 10% quartz.

Figure 4 shows the normalized spatial distribution of activated nodes for various sets of mineral fractions that provide an indication for the spatial distribution of total energy output. Since we associate the activation of a node as a fracture creation event, the energy output corresponds to the generation of a microseism. Therefore, the distributions can be related to the microseismic moment density in different rock types. As the volume of quartz decreases, the energy becomes more localized, which is consistent with the observation that microseismic moment densities increase in more ductile rock.

Conclusions

The dynamics of the hydraulic fracturing process were evaluated through a spreading model in networked systems for rock types consisting of varying mineral fractions of quartz and clay. This was performed to provide an alternative view of the mechanisms that underlie the empirical observations documented by various authors concerning the fracture propagation response in brittle and ductile rock. The results of the numerical simulations were in qualitative agreement with the observations regarding the relationship between ISIP and Poisson’s ratio and the microseismic response in various rock types. As the hydraulic fracturing process is a dynamical system consisting of numerous interacting rock elements, the interactions between component parts as well as the parts themselves must be considered in understanding the properties of the system. For this reason, nonlinearities are anticipated and are observed in the numerical modeling.

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References

Greenberg, M. L., and Castagna, J. P., 1992, Shear-wave velocity estimation in porous rocks: Theoretical formulation, preliminary verification and applications: Geophysical Prospecting, 40, 195-209.

Hashin, Z., and Shtrikman, S., 1963, A variational approach to the elastic behavior of multiphase minerals: J. Mech. Phys. Solids, 11, 127-140.

Lutz, E. E., 1991, Numerical methods for hypersingular and near-singular boundary integrals in fracture mechanics, Ph.D. Thesis, Cornell University.

Maxwell, S. C., Cho, D., Pope, T., Jones, M., Cipolla, C., Mack, M., Henery, F., Norton, M., and Leonard, J., 2011, Enhanced reservoir characterization using hydraulic fracture microseismicity: SPE Hydraulic Fracturing Technology Conference, SPE 140449.

Motter, A. E., and Albert, R., 2012, Networks in motion: Physics Today, vol. 65, no. 4, page 43.

Watts, D. J., 2002, A simple model of global cascades on random networks: Proc Natl Acad Sci USA, Applied Mathematics, 99(9), 5766-5771.