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A look into Gassmann's Equation

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Summary

By describing the influence of the pore fluids on seismic properties, we attempt to provide a comprehensive understanding of Gassmann's equation. Throughout the discussion of the rock, fluid, matrix and porous rock frame properties, we try to describe the assumptions and the limitations behind Gassmann's equation, and summarize the steps of performing fluid substitutions by using Gassmann's equation.

Theory and/or Method

The Influence of Pore Fluids on Seismic Properties:

The seismic response of a reservoir is mainly controlled by the compressional and shear velocities, V_p and V_s respectively, along with the density. But V_p and V_s are not the best indicators of any fluid saturation effect because of the coupling between V_p and V_s through the shear modulus and bulk density:

$$V_p = \sqrt{\frac{K + 4/3 \times \mu}{\rho}} = \sqrt{\frac{M}{\rho}} \tag{1}$$

$$V_s = \sqrt{\frac{\mu}{\rho}} \tag{2}$$

Where *K* is bulk modulus, *μ* is shear modulus, *M* is the compressional modulus, and *ρ* is the bulk density.

The bulk modulus is more sensitive to water saturation. The seismic wave causes an increase in pressure in the water filling the pores causing an increase in the bulk modulus. The shear modulus is not affected as the shear deformation does not produce a pore-volume change. Therefore any fluid saturation effect should correlate mainly to a change in the bulk modulus.

Gassmanns's Equation:

Gassmanns's equations relate the saturated bulk modulus of the rock to its porosity, the bulk modulus of the porous rock frame, the bulk modulus of the mineral matrix, and the bulk modulus of the pore-filling fluids:

$$K_{sat} = K^* + \frac{\left(1 - \frac{K^*}{K_o}\right)^2}{\frac{\phi}{K_{fl}} + \frac{(1-\phi)}{K_o} - \frac{K^*}{K_o^2}}, \tag{3}$$

Where K_{sat} is the bulk modulus saturated with pore fluids, K_o is the bulk modulus of the mineral matrix, K_{fl} is the bulk modulus of the pore fluid, K* is the bulk modulus of the porous rock frame, (drained of any pore-filling fluid), and φ is the porosity.

To apply the equation above, we first need to determine the bulk modulus of the porous rock frame (The bulk modulus of the rock drained of any pore-filling fluid), K*, then we calculate the bulk modulus of the rock saturated with any desired fluid.

The dry frame bulk modulus, K*, truly represents the property of the rock with any amount of moisture present (Clark, et al., 1980). So the proper way to refer to K* is by the porous rock frame modulus.

We now need to briefly define and discuss the bulk and shear moduli of the rock, and the bulk and shear moduli of the pore-filling fluids.

Rock Properties:

The bulk modulus of an isotropic rock is defined as the ratio of the hydrostatic stress to the volumetric strain. Bulk modulus values can be obtained either by laboratory measurements or analysis of wireline logs (e.g. sonic logs).
The relationship below relates the bulk modulus of the rock to its compressional velocity, shear velocity and bulk density:

$$K = \rho_B\left(V_p^2 - \frac{4}{3}V_s^2\right) \tag{4}$$

Where ρ_B is the bulk density of the rock, V_p is the compressional velocity, and V_s is its shear velocity

If we use ρ_B, V_p and V_s from the log analysis, the calculated bulk modulus will represent K_{sat}, the bulk modulus of the rock with the in-situ pore-filling fluid, but if we use ρ_B, V_p and V_s from lab measurements of a dried core sample, and the calculated bulk modulus will represent K*.

The shear modulus, G, is defined as the ratio of shear stress to shear strain and is giving by the following equation:

$$G = \rho_B V_s^2 \tag{5}$$

Where ρ_B is the bulk density of the rock and V_s is the shear velocity.

ρ_B and V_s can also be determined by log analysis or lab measurements, but the shear modulus is not sensitive to the fluid filling the pores, meaning means that G_{sat} = G (Biot, 1956).
The bulk modulus is sensitive to pore-fluid composition, while the shear modulus is not, therefore, the shear modulus will not change during the fluid substitution. This concept is one of the fundamental concepts to the application of Gassmann's equation.
For the bulk density, ρ_B, the relationship below relates the porosity (φ), fluid density (ρ_{ft}) and the grain (matrix) density (ρ_g) and allows us to calculate ρ_B:

$$\rho_B = \rho_g(1-\phi) + \rho_{fl}\phi \tag{6}$$

Porosity can be calculated from the core analysis or from the obtained from wireline logs. Log derived porosity represents an indirect measurement and a calibration to core porosity is recommended.

Fluid Properties:

The pore space of a rock is typically occupied by two or more fluid phases, and we must calculate the bulk modulus and the density of each individual fluid and then the mix of the fluids keeping in mind one of the main assumptions of Gassmann's equation, which is that the pores are connected and the pressure is at equilibrium. That assumes a uniform distribution of a homogenous fluid throughout the pore space. With this assumption in mind we can calculate the bulk modulus as:

$$K_{fl} = \left[\sum_{i=1}^{n} \frac{S_i}{K_i} \right]^{-1} \quad (7)$$

Where K_{fl} is the bulk modulus of the fluid mixture, K_i is the bulk modulus of the individual phases, and S_i is their saturation.
For a simple hydrocarbon-water system, the equation above becomes:

$$K_{fl} = \left[\frac{S_w}{K_w} + \frac{(1 - S_w)}{K_{hc}} \right]^{-1} \quad (8)$$

Where S_w is the water saturation, K_w is the bulk modulus of the water, and K_{hc} is the bulk modulus of the hydrocarbon

The density of a fluid mixture can be calculated using:

$$\rho_{fl} = \sum_{i=1}^{n} S_i \rho_i \quad (9)$$

Where S_i is the saturation of the individual components, and ρ_i is the density of the individual components.
For a simple hydrocarbon-water system, the above equation becomes:

$$\rho_{fl} = Sw\rho_w + (1 - Sw)\rho_{hc} \quad (10)$$

Where ρ_w is the density of the water, and ρ_{hc} is the density of the hydrocarbon.

Matrix Properties:

Before calculating the bulk modulus of the matrix, K_o, we should understand the mineral composition of the rock. Core analysis is used to obtain the information about the mineral composition. Logs can also be used for this purpose in case of simple mineralogy (Sand and Shale) to calculate V_{shale}.

The Voigt-Reuss-Hill (VRH) average provides a simple way to calculate K_o, which is basically an average of the Reuss average and the Voigt average. For a simple mineralogy of sand and shale, the equations become:

$$V_{Reuss} = \frac{1}{\frac{V_{clay}}{K_{clay}} + \frac{V_{qtz}}{K_{qtz}}} \tag{11}$$

$$V_{Voigt} = V_{clay}K_{clay} + V_{qtz}K_{qtz} \tag{12}$$

$$K_o = K_{VRH} = \frac{1}{2}\left(\left[V_{clay}K_{clay} + V_{qtz}K_{qtz}\right] + \left[\frac{1}{\frac{V_{clay}}{K_{clay}} + \frac{V_{qtz}}{K_{qtz}}}\right]\right) \tag{13}$$

Where V_{clay} is the volume of the clay, $V_{qtz} = 1 - V_{clay}$, K_{clay} and K_{qtz} are the bulk moduli of the clay and quartz respectively.

Porous Rock Frame Properties:

The bulk modulus of the porous rock frame, K*, is determined for the rock drained of any pore-filling fluid. K* that can be determined by the analysis on the controlled humidity-dried core, or by using wireline logs and re-arranging Gassmann's equation to calculate K* as follows (Zhu and McMechan, 1990):

$$K^* = \frac{K_{sat}\left(\frac{\phi K_o}{K_{fl}} + 1 - \phi\right) - K_o}{\frac{\phi K_o}{K_{fl}} + \frac{K_{sat}}{K_o} - 1 - \phi} \tag{14}$$

Once determined, K* is held constant during the fluid substitution.
The shear modulus, G, is also held constant during the fluid substitution because it is insensitive to the pore-fluid composition as we mentioned earlier.

Gassmann's Model Assumptions:

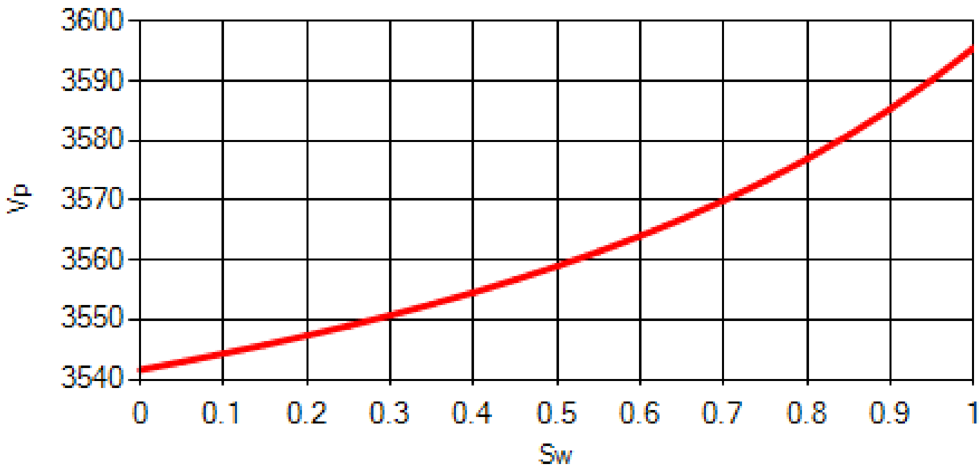
The application of Gasmann's equation is based on several assumptions, some of which we might have already mentioned while we were going over the properties of the rock, fluids, matrix and porous rock frame properties, and it's very important for us to keep them in mind as we perform fluid substitutions:

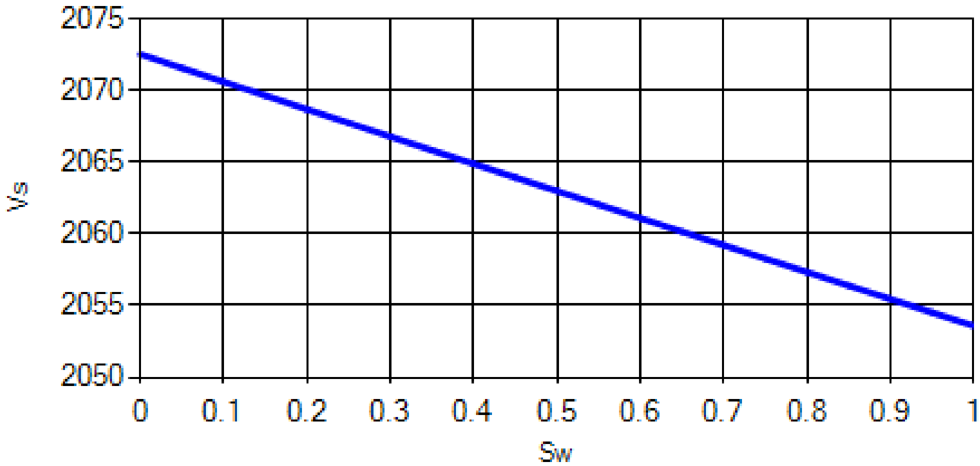
1. The porous material is isotropic, elastic, homogeneous and composed of one type of minerals. This assumption is violated if the rock is composed of multiple minerals with a large contrast is the elasticity of these minerals (Berge, 1998).
2. The porosity remains constant, meaning that the porosity doesn't change with different saturating fluids; In other words, no cementation or dissolution with changing the geochemical conditions in the pores.
3. The pore space is well connected and in pressure equilibrium. This assumption is violated in low-porosity or shaley sands; carbonate rocks can be an example of a diverse pore types and low connectivity between the pores and applying Gassmann's equation would yield unreliable results.
4. The medium is a closed system with no pore-fluid movement across the boundaries.
5. There is no chemical interactions between the fluids and the rock frame meaning that the shear modulus remains constant

6. Gassmann's equation is valid only at a low frequency. Generally, at seismic frequencies (10-100Hz), is the error in using Gassmann's equation and may be negligible. However, higher frequencies will violate this assumption and another formulation by Biot (1956, 1962) must be used.

Example

The following graphs show how Vp and Vs change as the water saturation changes for a given reservoir properties:





The example above was generated using Gassmann's equation and the following reservoir properties:

In-Situ Vp (m/sec)	3500	ρ water (g/cm^3)	1.1
In-Situ Vs (m/sec)	2000	ρ Oil(g/cm^3)	0.91
ρB (g/cm^3)	2.2	Volume of clay (%)	25
Porosity (%)	22	ρ clay (kg/m3)	2.58
Initial Sw (%)	50	ρ quartz (kg/m3)	2.65
Bulk modulus Water (Gpa)	2.2	Bulk modulus clay (Gpa)	14.9
Bulk modulus Oil (Gpa)	1	Bulk modulus quartz (Gpa)	37

Conclusions

Seismic became an important tool in reservoir monitoring and rock physics is the essential link connecting seismic data to reservoir characterization and the pore-filling in-situ fluids. Using Gassmann's equation we can easily calculate the velocity changes resulting from different fluids filling the reservoir, and evaluate the response that should be obtained for a specific fluid or mix of fluids in the pores. However, the validity of the field environment is dependent on the underlying assumptions of Gassmann's equation. These assumptions should be examined carefully, and failure to understand these assumptions could lead to inaccurate models.

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