Detecting Stratigraphic Discontinuities using Wavelet and S-Transform Analysis of Well Log Data

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Summary

Detecting stratigraphic discontinuities using well log data is a fundamental task in petroleum geosciences projects. Wavelet transforms (WT) and S-transform (ST) of well log data can provide objective identification of lithological interfaces to aid in this effort. The continuous wavelet transform (CWT) of a time series gives the wavelet correlation coefficients (i.e. similarity) between the given time series and the wavelet at different spatial scales. In comparison, S-transform (ST) is a hybrid form of short-time Fourier transform and wavelet transform that provides a measure of the local Fourier frequency spectrum of the time series (in this case well log data). We applied Haar continuous wavelet (HCWT) and S-transform on five types of well log data (Gamma Ray (GR), Spontaneous Potential (SP), Deep Resistivity (RD), Bulk Density (RHOB) and Neutron Porosity (NPHI) logs) from an oil sands evaluation well in the Cold Lake area of Alberta to evaluate autonomous detection of lithological interfaces. The wavelet analysis of well log data delivered two-dimensional output (i.e. depth vs. scale) of the given one-dimensional (depth only) well log record. The results of HCWT are presented in terms of wavelet coefficients at different scales whereas the results of ST are shown at different frequencies. The Haar wavelet with its rectangular shape is able to identify the abrupt change in the well log signal encountered at bed boundaries, and therefore it is effective in extracting lithological discontinuities from the well log data analysis. S-transform was applied to median filtered data in order to suppress signal noise. It was observed that in the HCWT of noisy data, there was no appearance of high values of wavelet coefficients (i.e. the HCWT fails to identify change in the noisy log data even after filtering), whereas ST is able to identify the changes in the data. Both, CWT and ST show good depth-frequency resolution in detecting interfaces, however they produce different outputs. These transformation methods are additional and efficient approaches to identify stratigraphic interface boundaries using well log data and useful in the quantitative division of geological layers in the petroleum reservoir characterization.

Key words: Geophysical Well logging, Continuous wavelets transform, S-transform.

Introduction

Petroleum reservoir characterization is a demanding field for geoscientists that includes the quantitative and qualitative analysis of different depth-dependant geological and geophysical properties expressed in geophysical well log data, seismic data, hydrocarbon saturation data, production data, core data, geological and structural information, and pressure data. The geophysical well log data provide a continuous record of the variation in diverse physical properties of a succession of lithological units with excellent depth resolution. In this context, the use of well log data for identification of stratal boundaries expressed as lithological interfaces is arguably the most fundamental process in reservoir characterization problems.

In the last few years, several researchers in the field of earth sciences have documented different applications of time-frequency methods (or spectral analysis techniques) in the interpretation of geological and geophysical data (e.g.: Alvarez et al., 2003; Coconi-Morales et al., 2010; Pan et al., 2008; Sinha et al., 2009; Deng et al., 2007; Pinnegar et al., 2009; Saadatinejad et al., 2011). Wavelet

transform is an advanced approach to analyze non-steady signals and it overcomes the weakness of the short time Fourier transform (STFT) (Andreas et al., 1999; Saadatinejad et al., 2011).

Well log data are recorded by logging instruments at equal depth intervals (i.e.: 0.1524 m), and can be considered as a time series for time-frequency analysis of the logging signals. Continuous wavelet and S-transform can be applied to the well log data to extract information in two dimensions, i.e. depth-scale and depth-frequency respectively. The objective of the present study is the evaluation of time-frequency analysis of well log data to provide autonomous high-resolution identification of abrupt changes in the data, yielding lithology thickness and sedimentary bounding surface outputs. The flowchart shown in Figure 1 illustrates the decision process for algorithm selection.
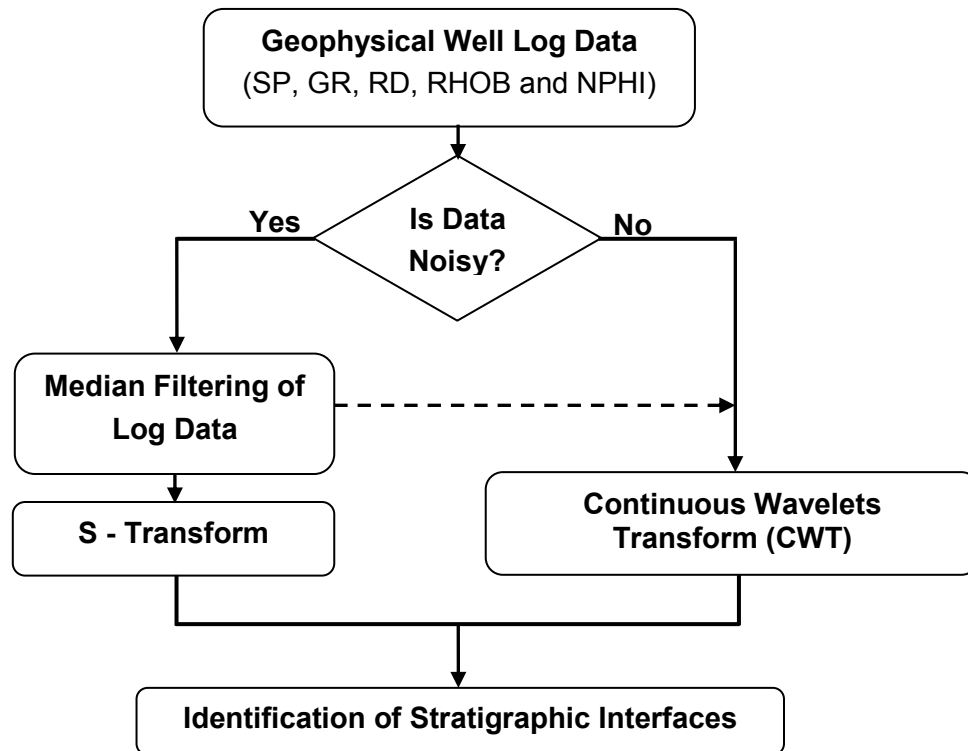

Figure 1: Flowchart of adopted steps (CWT, Median Filtering and ST).

Methodology

The adopted methodologies include continuous wavelet transform and S-transform commonly used for time-frequency analysis of any given time series. We applied these two methods on series of well log data to detect discontinuities (abrupt change) at different spatial scale and frequency resolutions. In the time-frequency analysis, the basic Fourier transforms are limited to sinusoidal signals, whereas the continuous wavelet transform can be performed based on many wavelet types, which may produce different results when analyzing the same signal. The standard definition of continuous wavelet transform and S-transform are discussed below.

Continuous wavelet transform (CWT): Suppose h (t) is a given time series and Ψ (t) $\in L^2(\mathbb{C})$ is a wavelet (or Mother wavelet which is a real or complex-value continuous-time function) which is square integrable, then the continuous wavelet transform is defined as (Grossman and Morlet 1984; Daubechies 1990, 1992; Misiti et al., 2000):

$$W(a,b) \equiv \frac{1}{\sqrt{|a|}} \int h(t) \psi^* \left(\frac{t-b}{a} \right) dt \qquad (1)$$

Or,

$$W(a,b) = \int h(t) \psi^*_{a,b}(t) dt = \langle h(t), \psi_{a,b}(t) \rangle \qquad (2)$$

where 'a' (scale parameter, a>0) and 'b' (time parameter) are real numbers. The symbols "*" and "$\langle . \rangle$" represent the complex conjugate and inner product respectively and

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi \left(\frac{t-b}{a} \right) \qquad (3)$$

The expression for continuous wavelet transform in frequency domain is

$$W(a,b) = \frac{\sqrt{a}}{2\pi} \int H(\omega) \psi^*(a\omega) e^{j\omega b} d\omega \qquad (4)$$

where $H(\omega) = \int h(t) e^{-j\omega t} dt$. From the above expressions, it is clear that CWT provides the adoptive resolution for time-frequency decomposition of a signal at different scales. The Haar wavelet with its rectangular shape (like step function) is discontinuous in amplitude and is well-suited for the detection of abrupt change in the data.

S - Transform (ST): S-transform is an extended version of short time Fourier transforms (STFT) and wavelet transforms (WT); and it provides a measure of the local Fourier frequency spectrum of the time series (in this case well log data). The ST for a time series *h* (t) is defined as (Stockwell et al., 1996):

$$S(\tau, f) \equiv \int h(t) \frac{|f|}{\sqrt{2\pi}} e^{\frac{-(\tau-t)^2 f^2}{2}} e^{-i2\pi ft} dt \qquad (5)$$

where 'f' represents the frequency.

Further detail about S-transform and its application can be found in Stockwell et al., 1996; Mansinha et al., 1997; Pinnegar et al., 2009; Stockwell, 2007. For noisy data like the RHOB and NPHI in this example, median filtering (Pratt, 1978) is used to smooth the data prior to applying S-transform to the filtered data. All the computations are done using MatLab R2009b.

Results and Discussion

In this study, five well log data series are analyzed using wavelet and S-transforms. The Haar continuous wavelet transformation (HCWT) is applied on three types of well log data (Gamma Ray (GR), Spontaneous Potential (SP) and Deep Resistivity (RD)) to identify stratigraphic interfaces (abrupt change in data), whereas, S-transform, a time-frequency method, is used to analyse bulk density (RHOB) and neutron porosity (NPHI) data. These two data were noisy, so we applied median filtering before applying S-transform. The purpose of time-frequency (or time-scale) analysis of GR, SP, RD, RHOB and NPHI logs are to identify abrupt changes in the data that correspond to the change in lithology. The data used in these analyses are from 1AA/12-21-064-03W4 in the Cold Lake area, in the depth interval from 220 – 490 metres TVD below Kelly Bushing.

The continuous wavelet transform of well log signal provides the correlation between the mother wavelet and the log data and the results are presented in terms of correlation coefficients (or wavelet coefficients) at different spatial scales. Scales are related to the stretching of the mother wavelet and can be related to the frequency. The HCWT of GR, SP and RD logs are shown in Figures 2, 3 and 4 respectively. Whereas, the results of S-transform on RHOB and NPHI are presented is Figures 5 and 6 respectively.
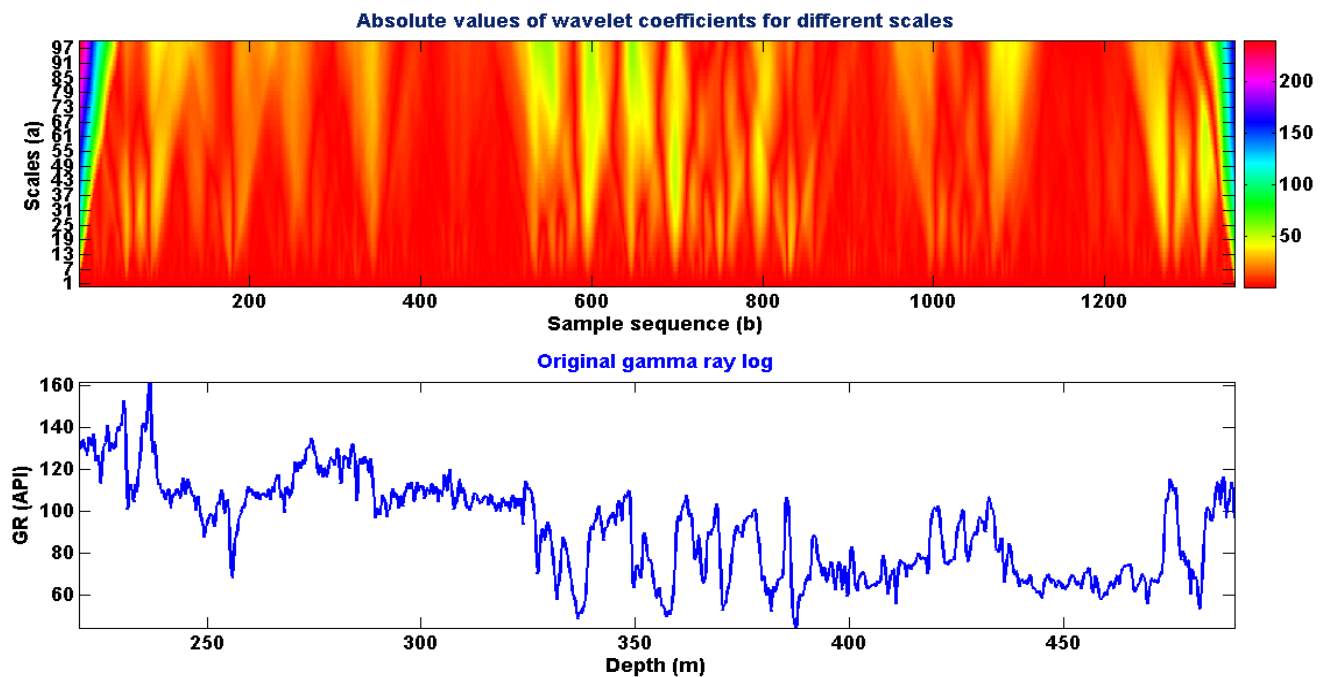

Figure 2: Haar continuous wavelet transform of gamma ray (GR) log at different scales.

From Figures 2, 3 and 4, it can be clearly seen that high values of wavelet coefficients correspond to the abrupt change in the well log properties and indicate lithological boundaries. It provides better resolution in 2-D domains (i.e. depth-scale) in the identification of stratigraphic changes and hence it is demonstrated that the Haar continuous wavelet analysis with its rectangular shape (like step function) is able to detect abrupt changes in the lithological properties. On the other hand, we applied median filtering on RHOB and NPHI data to remove noise and evaluated HCWT on filtered and unfiltered data but very low values of wavelet coefficients were seen, that means HCWT is not suitable in the identification of abrupt changes in the noisy data even after filtering. We also tried with other wavelets (e.g. Symlets, Coiflets, Morlet and many others) on noisy data but were unable to get satisfactory results. Consequently, the alternative approach of time-frequency analysis called "S-transform" was evaluated, and it was observed that ST provides better result than CWT for RHOB and NPHI data (Figure 5 and 6).
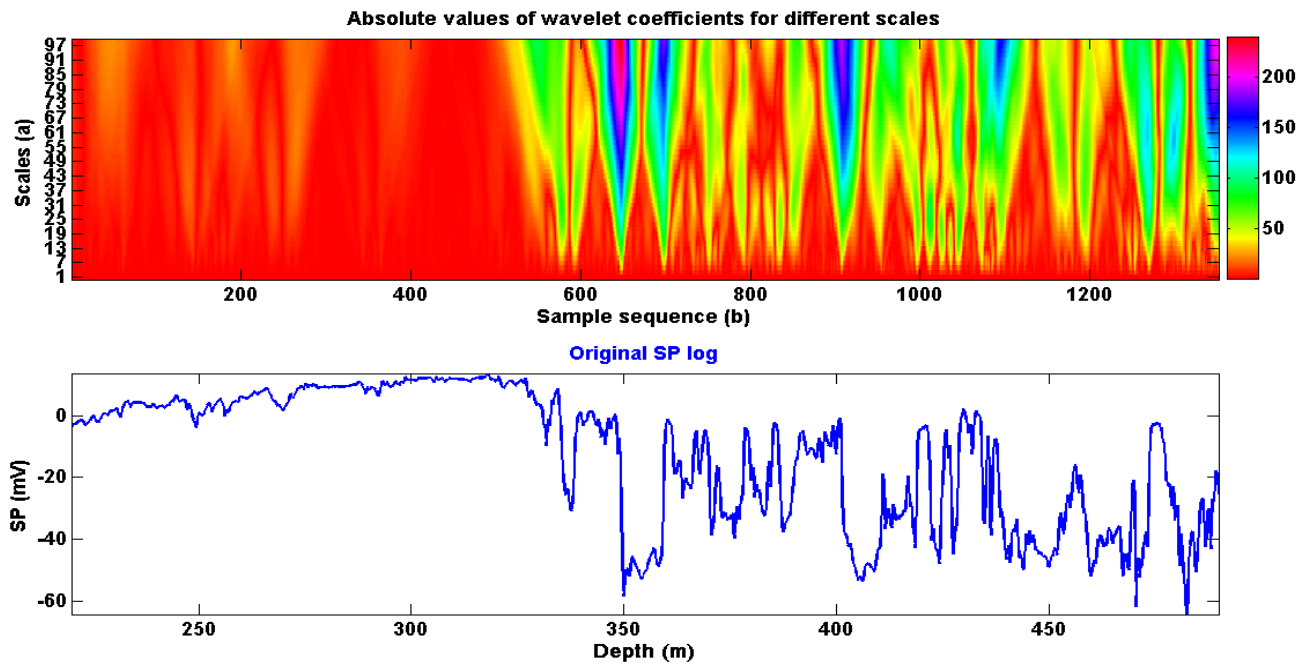
Figure 3: Haar continuous wavelet transform of spontaneous potential (SP) log at different scales.
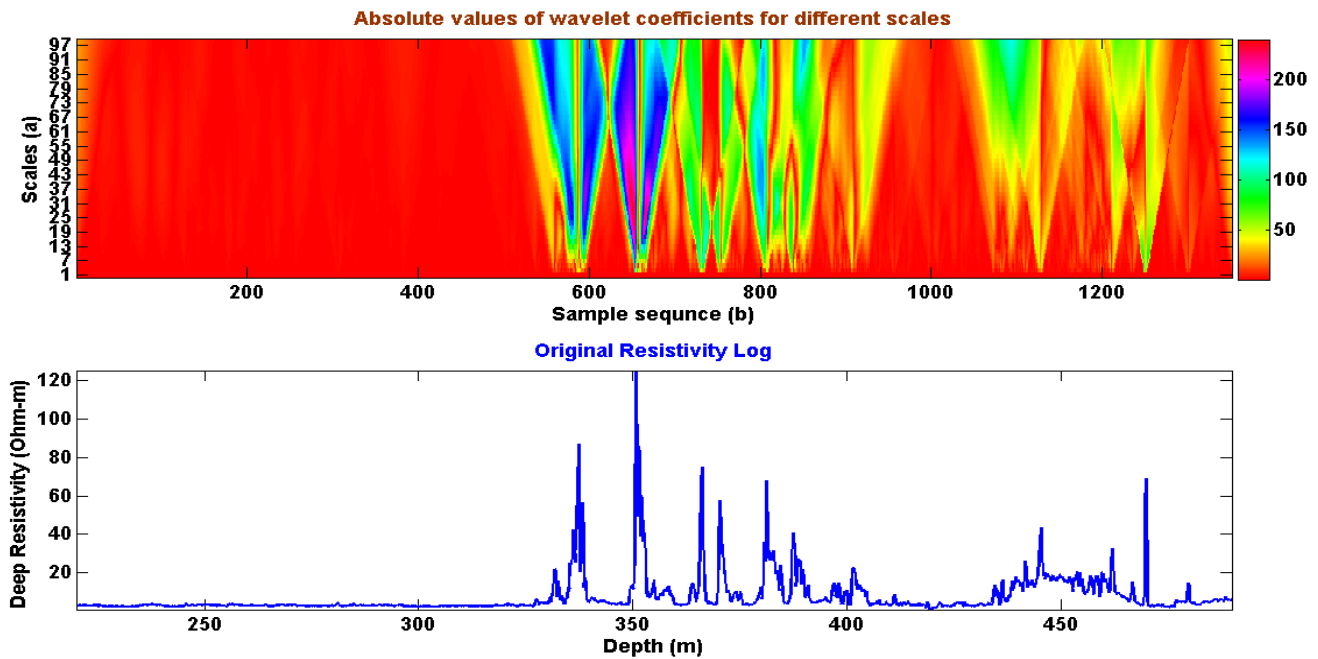

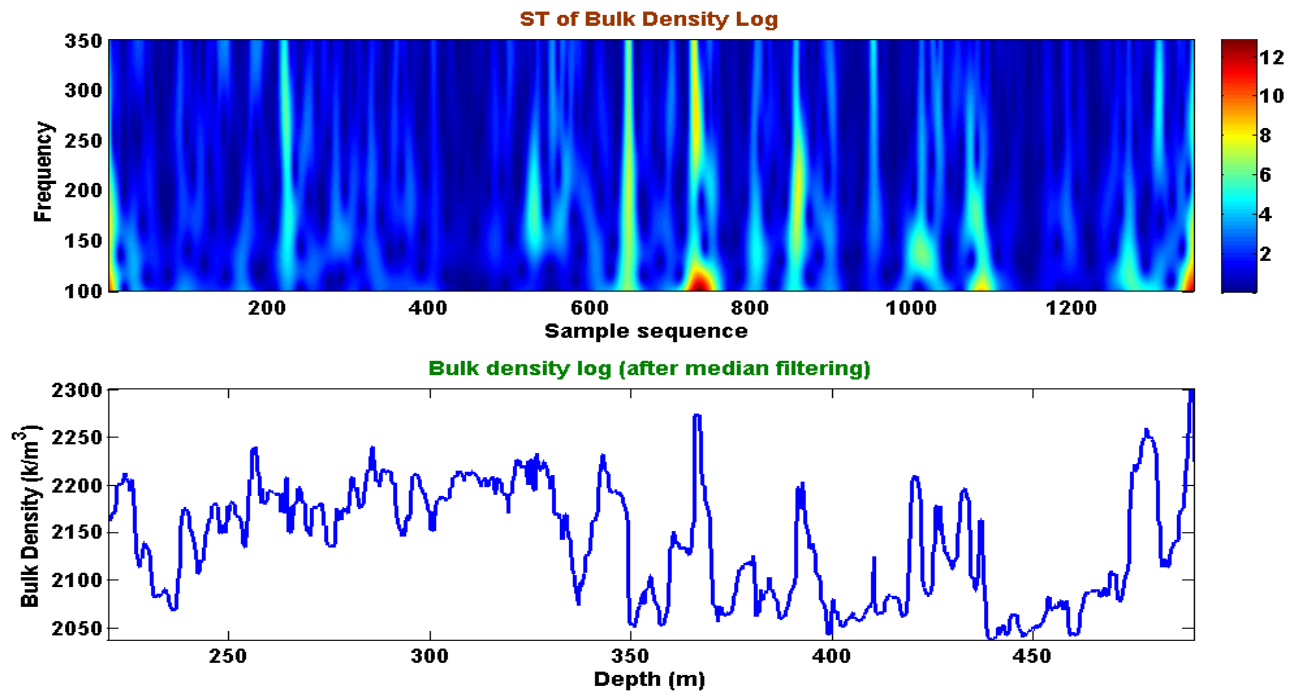Figure 4: Haar continuous wavelet transform of deep resistivity (RD) log at different scales.

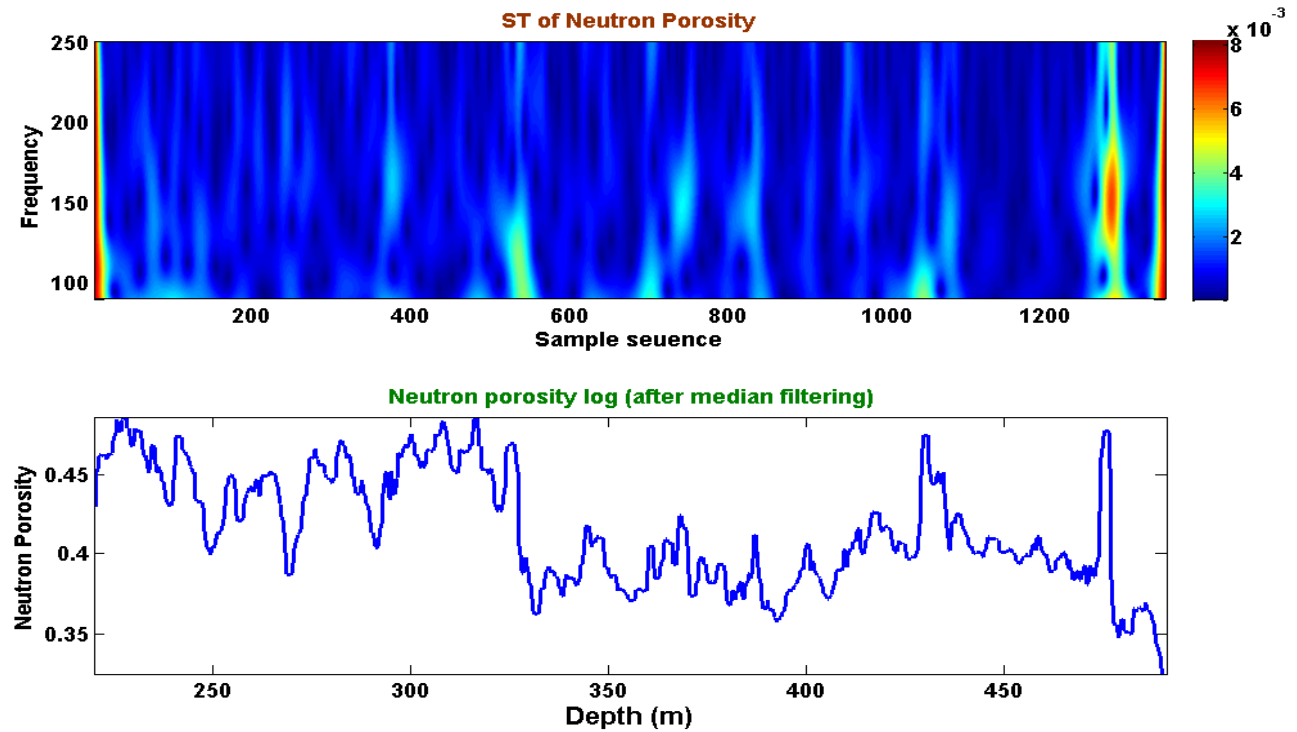Figure 5: S- transform of bulk density (RHOB after median filtering) log.

Figure 6: S- transform of neutron porosity (NPHI after median filtering) log.

Conclusions

The time-frequency (equivalent to depth-frequency) analysis of geophysical well log data provides high-resolution information that is useful in the interpretation of different lithological units. In this study, we applied continuous wavelet transform and S-transform to identify stratigraphic interfaces from five types of well log data (GR, SP, RD, RHOB and NPHI). The rectangular shape (like step function) of the Haar wavelet is demonstrated to be useful in the identification of abrupt change (i.e. interfaces) in the well log data. At the same time, for noisy data (in this case, RHOB and NPHI), the HCWT (CWT) does not show high values of wavelet coefficients and fails to identify the abrupt changes in the data. Application of the S-transform on median-filtered data is satisfactory in identification of interface-related changes in the RHOB and NPHI data sets. The high values of wavelet coefficients in the HCWT and amplitude in ST correspond to abrupt shifts in the well log data. Both CWT and ST show good depth-frequency resolution in detecting interfaces, however they are different in their output characteristics. These transformation methods are computationally efficient approaches to identify stratigraphic interface boundaries in digital well log data and useful in the quantitative division of geological layers for petroleum reservoir characterization.

Acknowledgements

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