AAPG Datapages/Search and Discovery Article #90174 CSPG©2014 CSPG/CSEG/CWLS GeoConvention 2012, (Vision) May 14-18, 2012, Calgary, AB, Canada

Monte Carlo Markov Chain methods in seismic deconvolution

Andres Medina*, and Mirko van der Baan**, University of Alberta, Edmonton, Alberta, Canada
*aamedina@aulberta.ca, **mirko.vanderbaan@ualberta.ca.

Summary

One prevailing assumption in reflection seismology is that the observed trace can be described as a convolution of a source wavelet with the Earth's reflectivity plus some noise. In a conventional deconvolution approach one thus estimates a linear deconvolution filter to retrieve the reflectivity series from the observed data. This amounts to taking linear combinations of noisy observations and there is thus always a trade-off between recovery of the underlying reflectivity series and noise amplification.

Here, we investigate the possibilities of sampling methods to recover the desired reflectivity series. This is achieved by specifying an appropriate probability density function sampled by means of Markov Chain Monte Carlo (MCMC), thus producing a sparse, nonlinear deconvolution method that is potentially less prone to noise contamination. Performance tests on realistic synthetic data show that the method is both versatile and reliable.

Introduction

Deconvolution is frequently achieved by means of a linear filter. Wiener filtering is one of the most commonly applied techniques for wavelet removal as it can be shown to be optimal in a least squares sense in terms of Earth's reflectivity recovery versus noise amplification, yet it amounts to taking linear combinations of noisy observations. An alternate approach considers that the reflectivity is sparse, with a probability density function that can be approximated. This means including statistical assumptions on the nature of the reflectivity sequence, an idea that is used by drawing samples of the conditional probability function of the reflectivity given the observed data to retrieve the most likely reflectivity sequence. Such a reflectivity estimation process is non-linear as it cannot be expressed as a linear combination of both the source wavelet and the data, and sparse in the sense of having a considerable amount of its elements equals to zero.

The proposed method is a continuation of the work done by Selvage (2008) and Selvage *et al.* (2009) and is an application of a Monte Carlo Markov chain scheme; it yields a random sequence (Monte Carlo) of numbers (reflectivity sequence), using a sequential scheme (chain) with a memory of one element (Markov property).

Theory and/or Method

The Earth's impulse response to the wavelet acts as a filter and is mathematically expressed as, (Yilmaz, 2001):

$$\boldsymbol{s} = \boldsymbol{w} * \boldsymbol{r} + \boldsymbol{n}, \qquad (1)$$

where $\boldsymbol{s} = (s_1, s_2, \cdots, s_{n_s})$ is the recorded trace, $\boldsymbol{w} = (w_1, w_2, \cdots, w_{n_w})$ the source wave, $\boldsymbol{r} = (r_1, r_2, \cdots, r_{n_r})$ the reflectivity sequence, $\boldsymbol{n}$ the added noise and '$*$' is the convolution operation.

In the seismic trace $\boldsymbol{s}$, presence and absence of reflectors can be reckoned with probability *p* and *(1-p)* respectively. The magnitude of each reflector follows a zero-mean normal distribution with variance σ_r^2, i.e. $N(0,\sigma_r^2)$, therefore, it is said that the reflectivity $\boldsymbol{r}$ follows a *Bernoulli-Gaussian* distribution with parameters *p, 0* and σ_r^2, (Debeye and van Riel, 1990). This is summarized by:

$$P[\,r_t] := BG(p,0,\sigma_r^2) = (1-p)\delta(r_t=0) + pN(0,\sigma_r^2), \qquad (2)$$

for every $t = 1,\cdots,n_r$ and with δ, the Kronecker delta function. The added noise in equation (1) is a zero-mean normally distributed process with variance σ_n^2, i.e. $\boldsymbol{n} \overset{dist}{\longrightarrow} N(0,\sigma_n^2)$. The parameters of the Bernoulli-Gaussian distribution are unknown as well as the noise variance; nevertheless, it is assumed that the noise level is known and $\sigma_n = 1/p$ so that the reflectivity has a unit variance.

The probability function in equation (2) is called the *prior* and does not depend on the data; hence, it is natural to demand for a function that includes the recorded data. Conditioning the prior to the recorded data renders a distribution $P[\boldsymbol{r}|\boldsymbol{s}]$ called *posterior*; it is an adjustment of the prior to the data and samples will be drawn from it in order to estimate the Earth's reflectivity.

Assuming that the wavelet and variances of the noise and the reflector magnitudes are known, the posterior and its parameters are completely determined:

$$P\left[r_j\middle|\boldsymbol{r}_{-j},\boldsymbol{s}\right] = p_j\delta\left(r_j=0\right) + \left(1-p_j\right)N\left(\widetilde{r_j}\,,\frac{1}{p_j}\right), \qquad (3)$$

where $\boldsymbol{r}_{-j} = (r_1,\cdots,r_{j-1},r_{j+1},\cdots,r_{n_r})$, and:

$$\widetilde{r_j} = \frac{\sum_{i=0}^{n_w}(s_{j+i}w_i - \sum_{k=0,k\neq i}^{n_w} w_k\, r_j\, w_i)}{p\sigma_n\sum_{i=0}^{n_w} w_i^2}\,,$$

$$p_j = \frac{p}{p+(1-p)\frac{\exp\left(-\widetilde{r_j}^2\right)/2\sigma_j^2}{\sqrt{p\sigma_j^2}}}\,, \text{ and}$$

$$\sigma_j = \frac{\sigma_n}{p\sigma_n + \sum_{i=0}^{n_w} w_i^2}\,.$$

Continuous sampling from the posterior yields several reflectivity sequences that are then averaged, this will be the estimated $\hat{\boldsymbol{r}} = E[\boldsymbol{r}|\boldsymbol{s}]$.

Schematically, the sampling process starts with an initial reflectivity sequence $\boldsymbol{r}^{(0)} = (r_1^{(0)}, r_2^{(0)},\cdots,r_{n_r}^{(0)})$ from which the first-iteration reflectivity sequence $\boldsymbol{r}^{(1)}$ is built in an element-wise fashion. The first element of the sequence $r_1^{(1)}$ is sampled out from $P[r_1^{(1)}|\, r_2^{(0)},\cdots,r_{n_r}^{(0)},\boldsymbol{s}]$, to then draw the second element of the sequence $r_2^{(1)}$ from the updated conditional probability $P[r_2^{(1)}|\, r_1^{(1)}, r_3^{(0)},\cdots,r_{n_r}^{(0)},\boldsymbol{s}]$. This process continues until all the n_r indexes had been visited. Then, it is said that a *cycle* has been completed. Repeating this process indefinitely yields a sequence of the form: $(\boldsymbol{r}^{(1)},\boldsymbol{r}^{(2)},\cdots)$. The elements of this sequence are averaged to obtain the wanted reflectivity estimate $\hat{\boldsymbol{r}}$. Such an averaged reflectivity minimizes the minimum square error function $MSE = E[(\boldsymbol{r}-\hat{\boldsymbol{r}})^2]$, (Winkler, 1995).

The sequences obtained using this sampling routine, also known as Gibbs sampling, are random in nature, thence the name *Monte Carlo*, and comply with the property:

$$P\left[r_j^{(i+1)}\middle|(r_1^{(i+1)},\cdots r_{(j-1)}^{(i+1)}, r_{(j+1)}^{(i)},\cdots,r_{n_r}^{(i+1)}),\boldsymbol{r}^{(i-1)},\boldsymbol{r}^{(i-2)},\cdots,\boldsymbol{r}^{(0)},\boldsymbol{s}\right] =$$
$$P\left[r_j^{(i+1)}\middle|r_1^{(i+1)},\cdots r_{(j-1)}^{(i+1)}, r_{(j+1)}^{(i)},\cdots,r_{n_r}^{(i+1)},\boldsymbol{s}\right]. \qquad (4)$$

This transition probability depends only on the immediate state of the *chain* to generate the next element of the chain and not on the whole history of the chain, that is, it has a memory of one iteration.

Example

The method is applied to a realistic synthetic example resembling a 2D-stacked section in a trace-wise fashion. Gaussian noise is added to the original data set such that the signal-to-noise ratio is 5. A global sparsity parameter $p = 0.23$ is used. The wavelet is estimated using a kurtosis maximization-based algorithm (van der Baan, 2008). Here, 500 *warm-up*, or training cycles are used for the algorithm to achieve a Bernoulli-Gaussian distribution, then 250 cycles are deployed in the averaging of the reflectivity sequences.

Performance of the method is evaluated by calculation of correlations and the l_2-norm between original data traces and the reconstruction obtained by convolving MCMC-sequences and the estimated wavelet. Typical values for such reconstruction evaluation in a particular trace of the 2D-section are displayed in Figure 3. For most of the traces, l_2-norm values are around 0.4 and correlations around 0.92.

Conclusions

MCMC has the advantage that it uses a sampling approach instead of linear filtering and is therefore potentially less prone to noise contamination. Here, MCMC is used to produce sparse reflectivity series; yet it has many applications to solve a large variety of general inverse problems. It also improves linear deconvolution results reducing noise components as they can be used as initialization of the proposed MCMC method.

Acknowledgements

We thank Chevron and Statoil for financial support of the project Blind Identification of Seismic Signals, Chevron for permission to use the synthetic 2D dataset, James Selvage and Dinh Tuan Pham for invaluable discussions and contribution.

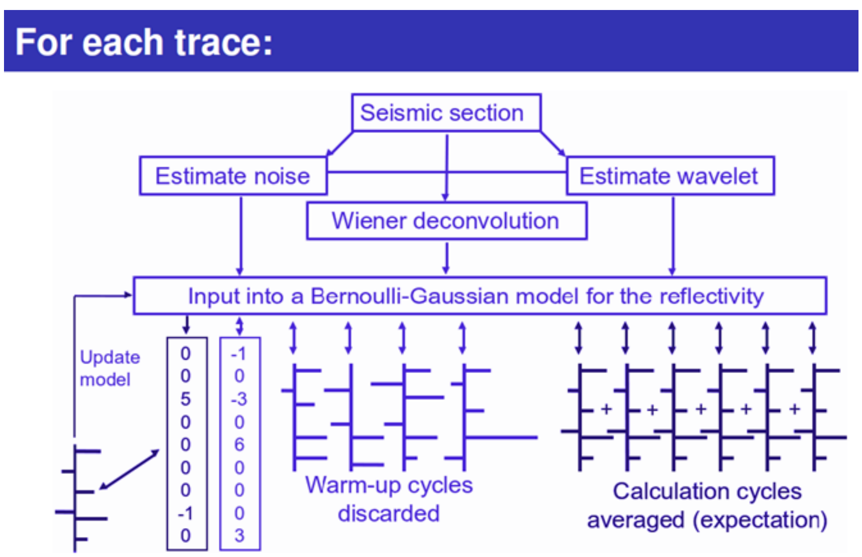

Figure 1: Scheme deployed in the non-linear estimation of the reflectivity, after Selvage (2008).

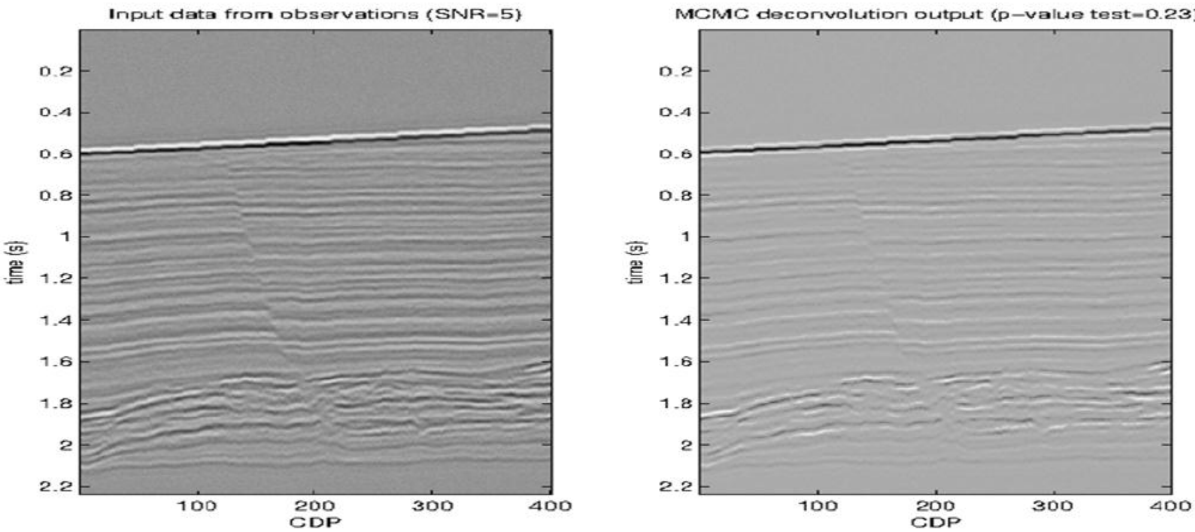

Figure 2: *Left:* A realistic example resembling stacked noisy data. *Right:* MCMC deconvolution output testing a *p*-value of 0.23, 500 warm-up cycles and 250 iteration cycles.

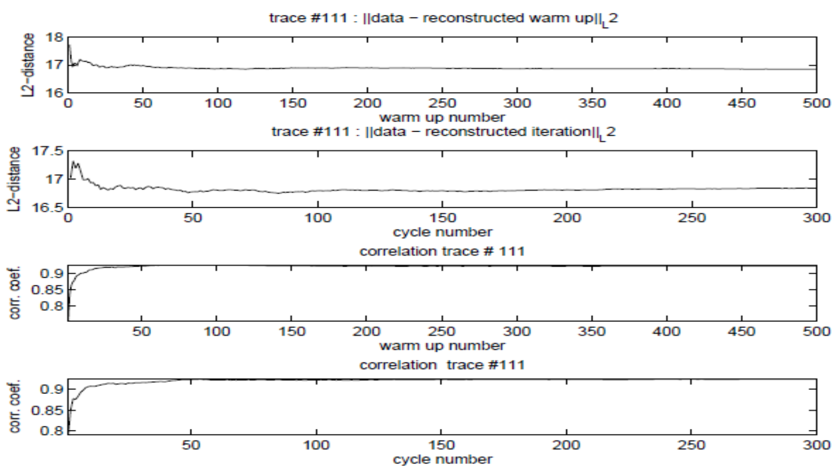

Figure 3: l_2-norm values and correlations between an original trace (#111) and the reconstruction, using the MCMC reflectivity and estimated wavelet, (500 warm-up cycles and 300 iteration cycles.)

References

Debeye, H., and van Riel, P., 1990, Lp-norm deconvolution: Geophysical Prospecting, **38**, n. 4, 381-403.

Selavge J., Pham, D. T., and Van der Baan, M. 2009, Nonlinear deconvolution using Markov Chain Monte Carlo for sparse reflectivity estimation. 79th Ann. Int. Mtg., SEG, Houston, SPNA2.2

Selvage, J., 2008, Acoustic Impedance Inversion Through Nonlinear Deconvolution Using Markov Chain Monte Carlo: Master's thesis, University of Leeds.

Van der Baan M., 2008, Time-varying wavelet estimation and deconvolution using kurtosis maximization. Gephysics, 73(2), V11-V18.

Winkler, G., 1995, Image Analysis, Random Fields and Dynamic Monte Carlo Methods: Volume 27 of Applications of Mathematics. Springer-Verlag.

Yilmaz, O., 2001, Seismic Data Analysis: Volume I of Investigations in Geophysics. Society of Exploration Geophysicists.