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AVAZ inversion for anisotropy parameters of a fractured medium

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Summary

We present a prestack amplitude inversion of PP data, collected through physical modelling, for the Thomsen anisotropy parameters (ε, δ, and γ) of a simulated fractured medium. 3D physically-modeled PP data have been acquired along several azimuths over a phenolic LE™ layer using the physical modeling facility at the University of Calgary. The PP amplitudes picked from the reflection off the top of the fractured layer for several azimuths were used as the input data for the inversion. A linearized PP reflection coefficient approximation for a HTI (horizontal transverse isotropy) medium was used to facilitate the inversion. Some constraints on the vertical velocities and density were also incorporated in the inversion process. The results for all three anisotropy parameters from AVAZ inversion compared very favourably to those obtained previously by a traveltime inversion. This result makes it possible to compute the shear-wave splitting parameter, γ, (historically determined from shear-wave data) directly related to fracture density from a quantitative analysis of the PP data.

Introduction

Geophysical research on naturally fractured reservoirs is mainly focused on vertical fractures of finely layering background. Vertical fractures are the result of a stress regime in which the vertical axis is the maximum or intermediate direction. Depending on the stress regime that causes fractures, the fracture orientation (however random) has a dominant direction (Nelson, 2001). In this work, the fracture orientation is considered to be the direction of fracture planes. Vertical fractures with typical size of millimetres are impossible to image using seismic waves with wavelengths of tens of meters; however a 3D seismic image can provide indirect information about a fractured medium. The dominant orientation of fracture networks makes the fractured medium azimuthally anisotropic in seismic wave propagation; seismic waves travel fastest in the direction of the fracture planes. Thus, fractures can be detected due to their seismic velocity anisotropy.

HT is a first-order approximate symmetry model to describe vertical fractures; in this model, the fracture plane is considered to be isotropic and the direction normal to it is referred as the symmetry axis of the system. Three Thomsen-style anisotropy parameters (ε, δ, γ) in addition to the vertical P-velocity, V_{Pz}, and S-velocity, V_{Sz}, (the Sv-wave propagating in (x_2,x_3) plane) can be considered to be the five parameters required to describe a HTI model. In a reference system where the x_1-axis is along the symmetry axis of the fractured system (see Figure 1), the HTI anisotropy parameters are defined as in Table 1. The anisotropy parameters of a fractured medium are needed for proper seismic imaging of the fractured and underlying layers both for the velocity analysis and depth migration.

Table 1: HTI anisotropy parameters for the reference system in Figure 1, A_{13} is density normalized C_{13}.

ε	γ	δ
$\dfrac{V_{Px}-V_{Pz}}{V_{Pz}}$	$\dfrac{V_{Sx}-V_{Sz}}{V_{Sz}}$	$\dfrac{(A_{13}+V_{Sz}^2)^2-(V_{Pz}^2-V_{Sz}^2)^2}{2V_{Pz}^2(V_{Pz}^2-V_{Sz}^2)}$

Azimuthal anisotropy due to vertical fractures has a strong effect on all seismic wave propagation aspects; it causes shear wave splitting, azimuthally-dependent NMO velocity, and amplitude variation of P- and S-wave reflections with azimuth. The anisotropy parameter γ has historically been determined from the analysis of time delays of split shear waves and is directly related to fracture density (Crampin, 1981). The ε and δ parameters have been measured from azimuthally dependent NMO velocity analysis (e.g. Tsvankin, 2001), which the parameter δ dominates the near vertical propagation.

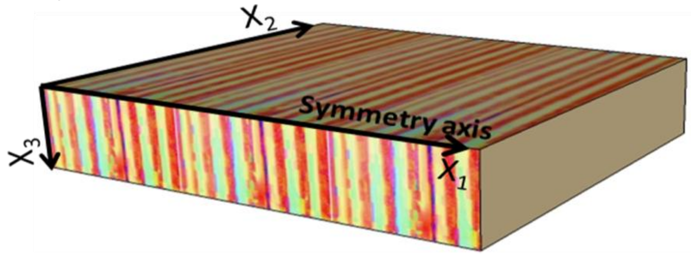

Figure 1: Reference system along fracture system.

AVAZ analysis has been used quite frequently in recent years for detecting fracture orientation and providing qualitative information on fracture density. The difference in seismic AVO responses parallel and perpendicular to fractures makes AVAZ a viable method in analyzing fractures. Here we present an AVAZ inversion based on the linear PP reflection coefficient for HTI medium by Rüger (1997). To evaluate our proposed AVAZ inversion, we used physical model data acquired over a fractured layer; see Figure 2 for the model description. The elastic constants (C_{ij}), as well as the anisotropy parameters of the fractured layer were determined by Mahmoudian et al. (2011) through traveltime inversion. The input data to the AVAZ inversion are the amplitudes reflected from top of the fractured layer (from nine different azimuths between 0º and 90º), and corrected to represent reflectivity. Figure 2 (right) shows amplitudes versus incident angle for three azimuths; the azimuthal variation due to fractured layer (although small) is clear. Note there is also little AVA variation for the incident angle less than 30º.

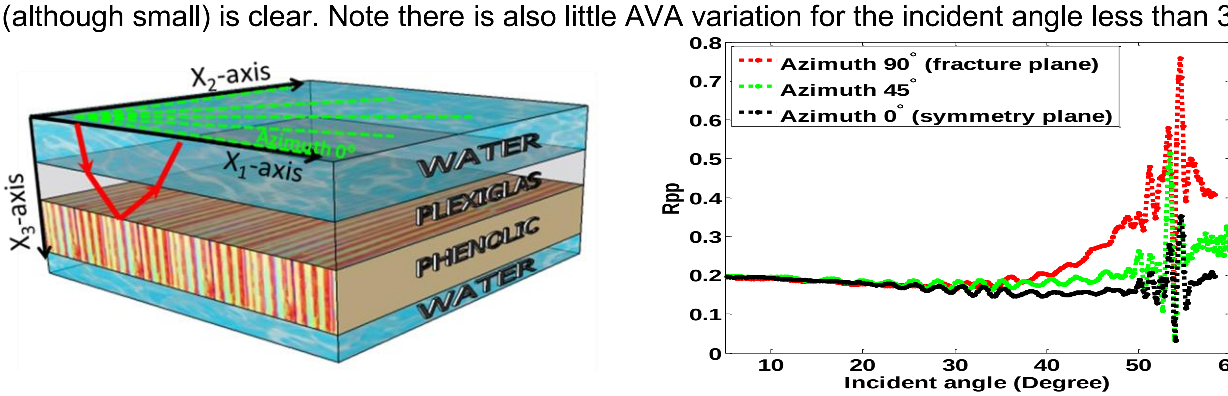

Figure 2: (left) Physical earth model, the azimuthally anisotropic phenolic layer simulates a fractured medium and is overlain by isotropic plexiglas layer. (right) Amplitude data (input to inversion) from azimuths 0º, 45º, and 90º.

Rüger's equation

There are linearized approximations for the reflection coefficient with respect to changes in medium parameter for both isotropic and anisotropic media. The linear Aki and Richards (1980) approximation for PP reflection coefficient (for a boundary between two isotropic layers) is

$$R_{PP}(\theta,\varphi)=\left(\frac{1}{2\cos^2\theta}\right)\frac{\Delta\alpha}{\bar{\alpha}}-\left(\frac{4\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\beta}{\bar{\beta}}+\left(\frac{1}{2}-\frac{2\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\bar{\rho}}, \tag{1}$$

where θ is the incident angle, α and β are the P- and S-wave velocities, ρ is density, $(\bar{\alpha},\bar{\beta},\bar{\rho})$ are the average values, and $(\Delta\alpha,\Delta\beta,\Delta\rho)$ are the difference of the values in the two layers. For HTI anisotropy assuming x_1-axis as the symmetry axis, the PP reflection coefficient approximation given by Rüger (1997) can be written as

$$R_{PP}^{HTI}(\theta,\varphi)=\left(\frac{1}{2\cos^2\theta}\right)\frac{\Delta\alpha}{\bar{\alpha}}-\left(\frac{4\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\beta}{\bar{\beta}}+\left(\frac{1}{2}-\frac{2\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\bar{\rho}}$$
$$+\frac{1}{2}\left(\cos^2\varphi\sin^2\theta+\cos^2\varphi\sin^2\varphi\sin^2\theta\tan^2\theta\right)\Delta\delta+\left(\frac{1}{2}\cos^4\varphi\sin^2\theta\tan^2\theta\right)\Delta\varepsilon \quad (2)$$
$$+\left(\frac{4\beta^2}{\alpha^2}\cos^2\varphi\sin^2\theta\right)\Delta\gamma,$$

where α and β are vertical P-wave and S-wave (Sv-wave propagating in (x_2,x_3) plane) velocities, and φ is the azimuth. The PP reflection coefficient as in equation (2) can be considered as a function of six parameters $(\Delta\alpha/\alpha,\Delta\beta/\beta,\Delta\rho/\rho,\Delta\delta,\Delta\varepsilon,\Delta\gamma)$.Therefore, inverting the prestack amplitudes (assuming that after AVO-friendly pre-processing they represent reflectivity) from different incident angles and azimuths, potentially allows the direct estimation of all three anisotropy parameters from PP data. We used equation (2) as the theoretical basis for our amplitude inversion to estimate the anisotropy parameters.

AVAZ inversion

Equation (2) in a simpler way can be written as

$$R= A\frac{\Delta\alpha}{\bar{\alpha}}+B\frac{\Delta\beta}{\bar{\beta}}+C\frac{\Delta\rho}{\bar{\rho}}+D\Delta\delta+E\Delta\varepsilon+F\Delta\gamma, \quad (3)$$

where *R* is the prestack amplitudes, and the coefficients A, B, C, D, E, and F are expressed in the details of equation (2). Incorporating the amplitudes from different azimuths and incident angles up to 44º as the input data (R_{mn} below), the equation (3) can be used to express a linear system of *n* equations and six unknowns:

$$\begin{bmatrix} A_{1\varphi_1} & B_{1\varphi_1} & C_{1\varphi_1} & D_{1\varphi_1} & E_{1\varphi_1} & F_{1\varphi_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{n\varphi_1} & B_{n\varphi_1} & C_{n\varphi_1} & D_{n\varphi_1} & E_{n\varphi_1} & F_{n\varphi_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{1\varphi_m} & B_{1\varphi_m} & C_{1\varphi_m} & D_{1\varphi_m} & E_{1\varphi_m} & F_{1\varphi_m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{n\varphi_m} & B_{n\varphi_m} & C_{n\varphi_m} & D_{n\varphi_m} & E_{n\varphi_m} & F_{n\varphi_m} \end{bmatrix}_{(nm\times 6)} \begin{bmatrix} \Delta\alpha/\alpha \\ \Delta\beta/\beta \\ \Delta\rho/\rho \\ \Delta\delta \\ \Delta\varepsilon \\ \Delta\gamma \end{bmatrix}_{(6\times 1)} = \begin{bmatrix} R_{11} \\ \vdots \\ R_{n1} \\ \vdots \\ \vdots \\ R_{1m} \\ \vdots \\ R_{nm} \end{bmatrix}_{(nm\times 1)} \quad (4)$$

where *m* = 9 is the number of azimuths, and *n* is the number of incident angles. The coefficients are calculated using a smooth background isotropic velocity. Equation (4) in matrix form can be written as,

$$G_{nm\times 6}m_{6\times 1}=R_{nm\times 1}. \quad (5)$$

The unknown vector *m* will result from a damped least-squares inversion, as $m=(G^TG+\mu)^{-1}G^TR$ where μ is the damping factor. In the first implementation of AVAZ inversion, we tried to simultaneously estimate all six parameters $(\Delta\alpha/\alpha,\Delta\beta/\beta,\Delta\rho/\rho,\Delta\delta,\Delta\varepsilon,\Delta\gamma)$; Figure 3 (left) shows that this inversion gives reasonable results only for $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$. The six-parameter inversion was not very stable due to the three very small singular values in inverting the coefficient matrix *G*. Data from incident angles less than 44º were used in the inversion, as incorporating data from larger angles would introduce larger errors to the inversion because of our use of a linearized equation (2) which is not valid near the critical angle. Still, the result is not disappointing, since the azimuthal variation and the AVA was small to begin with.

In an effort to stabilize the inversion, we used a second implementation in which we applied some constraints to the first three variables $(\Delta\alpha / \alpha, \Delta\beta / \beta, \Delta\rho / \rho)$. There are many other methods available to estimate the vertical P, S-wave velocity and density (such as AVA inversion of single azimuth data or incorporating well log information). Constraints on the first three variables were obtained by using the azimuth 90º data in an AVA inversion since the azimuth 90º coincides with the isotropic fracture plane. With such constraints, the inversion results for the anisotropy parameters $(\varepsilon, \delta, \gamma)$ are as in Figure 3 (right). As the results of constrains, the inversion results for ε, δ, and γ very favourably agree with those obtained previously by a traveltime inversion.

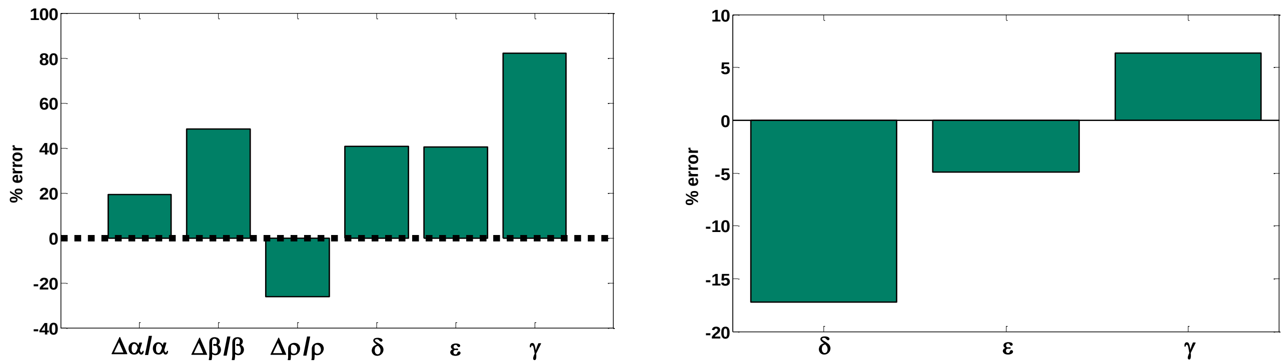

Figure 3: (left) Estimation of six parameters $(\Delta\alpha / \alpha, \Delta\beta / \beta, \Delta\rho / \rho, \Delta\delta, \Delta\varepsilon, \Delta\gamma)$ from AVAZ inversion of nine-azimuth data. (right) Anisotropy parameters from the constrained AVAZ inversion.

Conclusions

We applied an AVAZ inversion for the Thomsen anisotropy parameters using amplitudes picked from the reflection of an isotropic-HTI interface recorded in a 3D physical model experiment. All three anisotropy parameters (ε, δ, γ) were successfully estimated, where the estimation of the shear-wave splitting parameter (γ) (related to fracture density) directly from compressional data was of great importance. This AVAZ inversion can be applied to seismic field data only if there is enough information about the fractured layer overburden. Also, pre-knowledge of the orientation of the fracture plane is essential in this method. Future application of this AVAZ inversion may be more successful if stronger azimuthal or AVA variation is observed. From current work, we think AVAZ inversion is also possible without the pre-knowledge of the fracture orientation with some modification to the theory.

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