

Full waveform inversion algorithm using time imaging methods

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Summary

We propose a fast Full Waveform Inversion (FWI) method that is based on the Pre-Stack Time Migration (PSTM) algorithm. The method is fast compared to conventional FWI techniques in forward and inverse iterations; however, since we are doing time migration, we are limited to models with moderate complexity.

Introduction

Recently, seismic Full Waveform Inversion (FWI) (Tarantola, 1984) has become an increasingly practical tool for estimating subsurface parameters. The main obstacles that have prevented its common application in exploration seismology are its computational costs and the need for a starting model (Tarantola, 1984).

FWI requires a huge amount of seismic data prediction and gradient calculation. Various efforts from different perspectives have been expended to reduce the computational costs associated with gradient calculation and data prediction. For example Sirgue and Pratt (2004) and Operto et. al., (2007) limit the number of frequencies in updating model. Vigh and Starr (2006) used plane-wave method for 3D problems and Margrave et. al., (2010) used Phase Shift Plus Interpolation (PSPI) one way wave equation for gradient calculations.

In this work we start with the forward Kirchhoff operator for prediction of shot records from the reflectivity function (Schneider, 1978). Then we derive the wavefield perturbation as a result of the velocity perturbation. This results in a solution of the inverse problem using Prestack Kirchhoff Time Migration (PSTM) for gradient calculation and a de-migration for updating process.

Theory

In the general case where the velocity inside the Earth is varying smoothly, the Double Square Root (DSR) equation serves as a starting point for total traveltimes approximation for PSTM that includes the time from the source to the scatter point plus the time from the scatter point to the receiver

$$t = \sqrt{\frac{\tau^2}{4} + \frac{(x+h)^2}{v^2}} + \sqrt{\frac{\tau^2}{4} + \frac{(x-h)^2}{v^2}}, \quad (1)$$

where the parameters τ is zero offset two-way travel time, h is the half source/receiver offset, x is the distance from source/receiver midpoint to lateral coordination of scatter point and v is the migration velocity. For simplicity equation (1) can be equated to a single square root by

$$t = \sqrt{\tau^2 + \frac{4h_e^2}{v^2}}, \quad (2)$$

where $h_e^2 = x^2 + h^2 - \left(\frac{2xh}{vt}\right)^2$ is equivalent offset that collocate the sources and receivers in PSTM (Bancroft et al, 1998).

The Kirchhoff integral solution to the wave equation was initially presented by Schneider, 1978. In this approximation, assuming migration velocity to be close to RMS velocity the shot record prestack volume $u(x, h, t)$ can be obtained using Common Scatter Point (CSP) gathers $u(x, h_e, \tau)$

$$u(x, h, t) = \int K u(x', h_e(x', h), \tau) = \sqrt{t^2 - \frac{4h_e^2}{v^2}} d\tau dh_e dx', \quad (3)$$

and the adjoint operator for migration is

$$u(x, h_e, \tau) = \int K^* \frac{\partial}{\partial \tau} u(x', h, t) = \sqrt{\tau^2 + \frac{4h_e^2}{v^2}} dt dh dx'. \quad (4)$$

The parameter K is the true amplitude term and K^* is its adjoint operator. Equations (3) distributes the energy of CSP gathers along with DSR equations to model the shot records. The migration operator is the diffractions stack integral in equation (4) that sums the distributed energy and locate it on stacked section $u(x, h_e = 0, \tau)$.

If the lateral velocity variation is negligible, we can approximate the $u(x, \tau) = u(x, h_e = 0, \tau)$ by convolution of reflectivity function with source wavelet $s(\tau)$ using

$$u(x, \tau) = -\frac{1}{4} \frac{\partial}{\partial \tau} \text{Ln} \left(\frac{1}{v(x, \tau)} \right)^2 * s(\tau), \quad (5)$$

where the density is assumed to be constant. The scattered field due to a perturbation to velocity field (i.e., $\delta u(u, t)$ due to $v + \delta v$) can be estimated by linearization of the velocity perturbation and the wavefield perturbation. Using the Taylor expansion we have

$$\frac{1}{(v + \delta v)^2} = \frac{1}{v^2(x, \tau)} - \frac{2\delta v}{v^3(x, \tau)} + O(\delta v)^2, \quad (6)$$

where $O(\delta v)^2$ is higher order of approximation in the Taylor expansion. We obtain

$$u(x, \tau) + \delta u(x, \tau) = -\frac{1}{4} \frac{\partial}{\partial \tau} \left(\frac{1}{v^2(x, \tau)} - \frac{2\delta v(x, \tau)}{v^3(x, \tau)} - 1 \right) * s(\tau), \quad (7)$$

where the logarithmic function has been linearized by Taylor series expansion if $0 < \frac{1}{v^2} - \frac{2\delta v}{v^3} < 2$.

Using similar Taylor expansion of equation (5) for $u(x, \tau)$ from (7) we readily obtain

$$\delta u(x, \tau) = \frac{\delta v(x, \tau)}{2v^3(x, \tau)} * \dot{s}(\tau) + O(\delta v, \delta u)^2. \quad (8)$$

Here we used the linear property of derivative of convolution operator. Finally from (3) and (8) we have

$$\delta u(x, h, t) = \int K^* \left(\frac{\delta v(x', \tau = \sqrt{t^2 - \frac{4h_e^2}{v^2}})}{2v^3(x', \tau)} * \dot{s}(\tau) \right) d\tau dh_e dx'. \quad (9)$$

Equation (9) describes the change of recorded shot record $\delta v(x, \tau)$ if the velocity field is perturbed by $\delta v(x, \tau)$. Let us point out that (9) is similar to (10) of Tarantola (1984) and (6) of Cohen and Bleistein (1979). The main difference is that here we have $\delta v(x, \tau)$ which is function of time and not on depth (i.e., $\delta v(x, z)$). This follows that without the loss of generality, to estimate $\delta v(x, \tau)$, we can impose similar FWI inversion algorithms that are used for inversion of $\delta v(x, z)$.

Marmousi numerical example

To perform a successful velocity update, we select the left part of Marmousi velocity model (Figure 1a). The small lateral variation of the velocity within the model ensures that the Kirchhoff forward operator for shot records has fair match with that from finite difference wave propagations techniques. Using depth to time conversion algorithm, the $v(x, z)$ converted to $v(x, \tau)$ in Figure 1b and then 21 shots are recorded each with 201 receivers in split spread configuration. The first shot is at surface of 1250m lateral coordination and the last shot is at 2250m. The wavelet is minimum phase and the dominant frequency changes from 5 Hz to 12 Hz depending on iterations. The starting velocity was obtained from the smoothed true model using 700m Gaussian smoother (Figure 1c). The iteration starts with 5 Hz shots record to 12 Hz to update the velocity. Figures 1d and Figure 2 illustrate the success of the method. Figure 1d shows the updated velocity after 45 iterations which indicate that we inverted the velocities of different layers. As an example shown in Figure 2 the updated velocity at 1800m of the model gives overall estimation of true model (see e.g., high velocity layer at 1.75s).

Conclusions

The main conclusion of this work is that the linearized solution of seismic reflection inverse problem can be obtained using the fast PSTM and corresponding forward modeling. It requires updating the velocity in time and it incorporates accurate diffraction stack weighting of the PSTM data. The result of the method will be an updated velocity in time that can be used in time to depth conversion. The accuracy of this approach is higher in the mediums with smaller lateral velocity variations.

We used Equivalent Offset Migration (EOM) that is based on Kirchhoff PSTM. The advantage of using EOM is the starting velocity in update procedures that are obtained from the Common Scatter Point gathers.

We showed a P-P inversion example in this study and intend to use this algorithm to simultaneously invert both P-P and P-S data.

Acknowledgements

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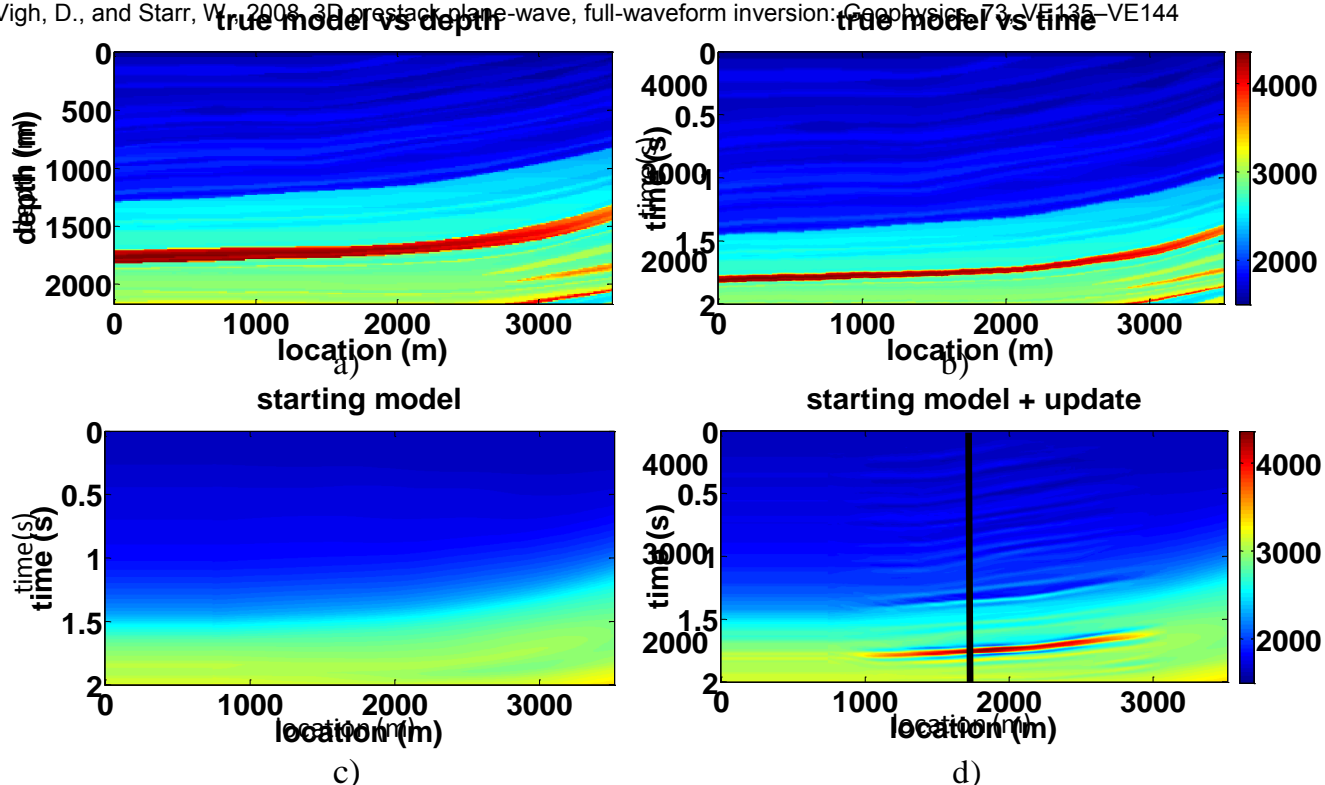


Figure 1: Numerical example FWI using PSTM imaging theory a) True velocity vs depth b) True velocity vs time c) starting velocity d) The inverted velocity after 45 iterations. Color scale is the velocity and the vertical black line indicates the well location at 1800m.

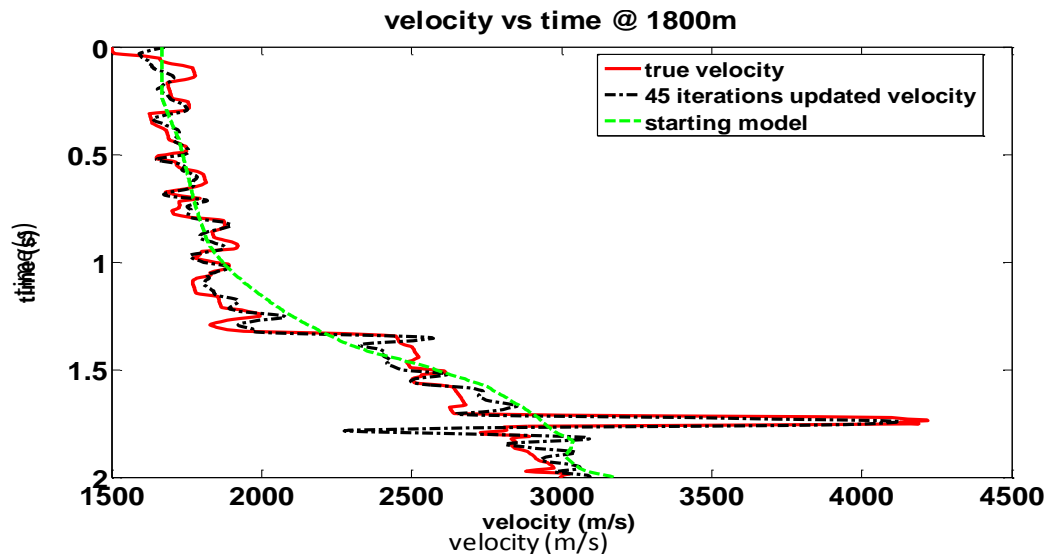


Figure 2: Comparison between the true velocity model (solid red) the starting model (green dashed) and the FWI model (dashed dot black) using the PSTM strategy at 1800m lateral position.