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Short-time wavelet estimation in the homomorphic domain

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Summary

Wavelet estimation plays an important role in many seismic processes like impedance inversion, amplitude versus offset (AVO) and full waveform inversion (FWI). Statistical methods of wavelet estimation away from well control are a desirable tool to support seismic signal processing. One of these methods based on Homomorphic analysis has long intrigued as a potentially elegant solution to the wavelet estimation problem. Yet a successful implementation has proven difficult. We propose here a method based short-time homomorphic analysis which includes elements of the classical cepstrum analysis and log spectral averaging. Our proposal increases the number of segments, thus reducing estimation variances. Results show good performance on realistic synthetic examples.

Introduction

Wavelet estimation in seismic applications by homomorphic analysis was first proposed by Ulrych (1971). The implementation was conceptually simple and elegant requiring only a forward Fourier transform, then logarithm, followed by an inverse Fourier transform. In addition, no assumption regarding the phase of the wavelet is made, nor is the reflectivity assumed white.

Despite its promise, the method has often produced mixed results. This is partially due to an often overlooked condition, namely that the reflectivity series must be sufficiently sparse (Ulrych, 1971). Also the method forces the resulting wavelet to replicate the strongest arrival in the data, often the first arrival in non-AGC'd data (Tribolet, 1978).

In our approach we combine elements of Ulrych's (1971) classic application of homomorphic analysis, Tribolet's (1978) short-time cepstral deconvolution with homomorphic wavelet estimation in the log spectral domain (Otis and Smith, 1977). This relaxes some of the original assumptions; for instance, the assumption of a sufficiently sparse reflectivity is replaced by a white, random one. The minimum reflectivity assumption is replaced by short mixed phase segments by windowing. This produces better wavelet estimates while extending the ability to handle nonminimum phases in both the wavelet and the reflectivity.

Theory

The seismic signal is described by the convolutional model (Ulrych, 1971):

$$s(t) = w(t) \star r(t), \tag{1}$$

where $r(t)$ is the reflectivity, $w(t)$ is stationary the wavelet, $\star$ stands for the convolution operator and $s(t)$ is the seismic trace. In the cepstral domain convolution is mapped into an addition:

$$\hat{s}(t) = \hat{w}(t) + \hat{r}(t), \tag{2}$$

where $\hat{s}(t) = FT^{-1}\{\ln[FT\{s(t)\}]\}$ is the complex cepstrum of $s(t)$ and $\hat{w}(t)$ and $\hat{r}(t)$ are the complex cepstra of the wavelet and the reflectivity respectively.

Ulrych (1971) noticed that if the wavelet is time-invariant and has a smooth spectrum its contribution to the complex cepstrum will be located near the origin. While a rapidly varying reflectivity will have contributions at higher quefrencies. In addition if the reflectivity is minimum phase its complex cepstrum will be right-sided, making the separation of the additive terms easier. The third condition he observed is related to the sparsity: if the interval time is larger than the wavelet cepstrum the two signals can be separated in the cepstral domain. In this later condition the logarithm of the spectrum plays the role of whitening the wavelet spectrum to shorten its cepstral representation.

After windowing, the resulting signal is transformed to the quefrency domain, that is:

$$\tilde{s}_i(t) = FT^{-1}\{\exp(FT\{\hat{s}(t)\hat{l}(t)\})\}, \tag{3}$$

with $\hat{l}$ the windowing filter and $\tilde{s}_i(t)$ is the estimated signal after the liftering process. Theoretically a low-cut time lifter will produce $\tilde{s}_i(t) = \tilde{w}(t)$ and a high-cut time lifter will give $\tilde{s}_i(t) = \tilde{r}_i(t)$.

Wavelet estimation by cepstral gating is largely dependent on the lifter applied to the complex cepstrum. To avoid this problem Otis and Smith (1977) proposed to average several traces assuming the quefrency responses of the reflectivities is random while the wavelet is stationary. By averaging the complex cepstra of a seismic profile, the constant wavelet can be estimated since the variable reflectivity with zero mean will tend to zero:

$$\hat{s}(t) = \frac{1}{N}\sum_{i=1}^{N}\hat{s}_i(t) = w(t) + \frac{1}{N}\sum_{i=1}^{N}\hat{r}_i(t), \tag{4}$$

where i represents the i_{th} trace in a seismic profile of N traces.

Averaging in the cepstral domain is equivalent to averaging in the spectral domain, since the Fourier transform is a linear process. Thus,

$$\hat{S}(f) = \hat{W}(f) + \frac{1}{N}\sum_{i=1}^{N}\hat{R}_i(f), \tag{5}$$

where $\hat{S}(f) = \ln S(f)$ and $S(f)$ is the Fourier transform of $s(t)$, likewise for the wavelet and the reflectivity.

Log spectral averaging will produce successful results when the wavelet is spatially stationary and the reflector series are white. The latter condition implies that the geological structure changes at each shot point (Tribolet, 1979). This can be best realized by combining traces from different parts of the 3D volume. The minimum phase reflectivity assumption is also used in the log spectral averaging method.

Seismic signals are nonstationary, i.e. they follow the time-invariant convolutional model only on a short-time basis. Thus assuming that the convolutional model in (2) is true within a small window where the wavelet could be considered stationary; the complex cepstrum of the trace will be:

$$\hat{s}_{ik}(t) = \hat{w}(t) + \hat{r}_{ik}(t), \quad 0 \le t \le L, \tag{6}$$

where k represents the segment of length L used to compute the complex cepstrum.

If the cepstral structure $\hat{r}_{ik}(t)$ is independent between segments, the estimated wavelet in the log spectrum is:

$$\hat{W}_e(f) = \frac{1}{NM}\sum_{i=1}^{N}\sum_{k=1}^{M}\hat{S}_{ik}(f) = \hat{W}(f) + \frac{1}{NM}\sum_{i=1}^{N}\sum_{k=1}^{M}\hat{R}_{ik}(f), \tag{7}$$

where $\hat{S}_{ik}(f) = \ln S_{ik}(f)$ and $S_{ik}(f)$ is the Fourier transform of $s_{ik}(t)$.

The estimated wavelet $\hat{W}_e(f)$ converges to the true wavelet $\hat{W}(f)$ if the reflectivity series is white and random, and the propagating wavelet is stationary. Averaging zero mean random reflectivities over the ensemble of windows makes the the real-valued part of the last term converges to a constant, and the imaginary part to zero. Hence the resultant process converges to the seismic wavelet. The complex log spectrum of the averaged reflectivity is:

$$\hat{R}(f) = \ln R(f) = \ln|R(f)| + j\phi_R(f), \tag{8}$$

where the $\ln|R(f)|$ converges to $\ln\sigma_r$ which approaches 0 as reflectivity series has assumed variance $\sigma_r = 1$. The reflectivity phase $\phi_R \approx 0$ after deramping and phase unwrapping since we assume that (1) ϕ_R is uniformly distributed between $-\pi$ and π and (2) the reflectivity series is dominated by a few large reflectors.

Expanding the complex logarithm in equation (7) in terms of the amplitude and phase means that we have to deal with the phase unwrapping problem (Herrera and van der Baan, 2011):

$$\hat{W}_e(f) = \frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M}\ln|S_{ik}(f)| + \frac{j}{NM}\sum_{i=1}^{N}\sum_{k=1}^{M}\arg\{S_{ik}(f)\}, \tag{9}$$

where the function $\arg$ refers to the continuous unwrapped phase $arg\{S(f)\} = \phi_S(f) + 2\pi n$ with n integer. We need a continuous function of f to guarantee the uniqueness of the solution. That is why at every short trace we estimate the unwrapped and deramped phase. Removing the phase linear trend suppresses the wavelet timing errors prior to the averaging process.

The estimated wavelet in the time-domain is finally given by:

$$w_e(t) = FT^{-1}\{\exp[\hat{W}_e(f)]\}. \tag{10}$$

Example

The dataset used in the experiments consists of a stacked seismic section of 400 traces of 560 samples each one (Figure 1). This is a realistic synthetic noiseless data. These data have been produced with a mixed-phase and narrow-bandwidth wavelet.

To evaluate the performance of our proposal, the Short-Time Homomorphic Wavelet Estimation (STHWE), we will compare its results with the well-established method of Kurtosis Phase Estimation (KPE) (van der Baan, 2008) and with the first arrival of the seismic trace.

The time window is selected to be three times the wavelet length L = 660 ms, and a Hamming window with 50 % overlap. Two cycles of averaging are implicit in our implementation, firstly one wavelet is estimated for each trace and the final estimated wavelet is the result of the lateral averaging of all vertical wavelets. For each trace we have k = 19 segments and the total set is NM = 7600 segments.

The first arrival is shown in Figure 1 (center plot in black) along with the superposition of the KPE-wavelet (red) and the STHWE-wavelet (blue). The three wavelets have similar time-domain waveforms and spectral content. KPE gives a constant phase of $\phi_{KPE} = 65.30$ degrees. STHWE has a frequency variant nature since the phase was estimated in a short-time basis. Fluctuations around the mean value of $\phi_{STHWE} = 68.14$ degrees are observed, this result is close to the value of the first arrival $\phi_{FA} = 58.42$ degrees.

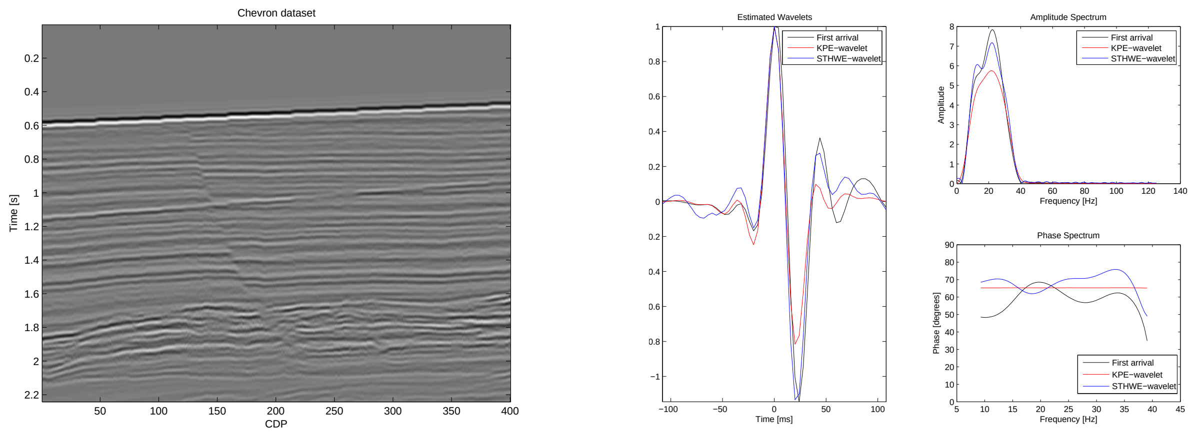

Figure 1 *Dataset used to compute the wavelet (left). Center plot shows the estimated wavelets, in black is the first arrival (FA), the KPE method (red) and the short-time homomorphic method (blue). In the upper right panel the three amplitude spectrums are plotted and in the right bottom their corresponding phase spectra.*

Conclusions

Homomorphic wavelet estimation was first introduced over 40 years ago and has been revisited often with its promise of nonminimum-phase wavelet estimation. The original method of cepstral filtering assumes the wavelet is simple, and that the reflectivity series is minimum phase and sufficiently sparse. However, for the most part, the latter assumption is rarely honored. Log spectral averaging mitigates the need for the sparsity constraint, but requires a large number of independent reflectivity series while maintaining the minimum phase constraint. The method of log spectral averaging using a short-term Fourier transform increases the number of traces, thus reducing estimation variances. Furthermore, no assumptions regarding the phase of the wavelet or the reflectivity are required. A comparison using realistic example shows similar results, with regards to constant-phase wavelet estimation based on kurtosis maximization. All three estimated wavelets are similar but the short-time homomorphic technique allows for a frequency-dependent phase estimation, whereas the kurtosis-based method assumes a constant phase.

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