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Seismic Denoising by Time-Frequency Reassignment

Li Han*, Jilin University, Changchun, China
Department of Physics, University of Alberta, Edmonton, Canada
hanli0623@gmail.com
David Bonar and Mauricio Sacchi
Department of Physics, University of Alberta, Edmonton, Canada

Summary

A new random noise attenuation method that is based upon time-frequency reassignment is presented for seismic data. Time-frequency reassignment aims at increasing the concentration of energy in time-frequency maps. Through thresholding in this reassigned domain, random noise is more easily attenuated since seismic events are more compactly represented with a relatively larger energy than the noise. In order to utilize reassignment methods for data processing, an inverse reassignment process that permits to synthesize seismic data from the reassigned time-frequency maps must also be developed. To help reduce the undesirable concentration of pure random noise in the reassigned domain, mutlitapering was introduced to the reassignment algorithm. The effectiveness of this method is demonstrated with 1D and 2D synthetic examples.

Introduction

The idea of reassignment was first proposed by Kodera et al. (1976) for the analysis of non-stationary signals. Auger and Flandrin (1995) reformulated the method into more practical terms by describing reassignment as the process of de-smoothing a time-frequency distribution. A reassigned time-frequency spectrum can better localize signals because the process concentrates the time-frequency energy of each point in the original spectrum to a time-frequency location that matches the signal's instantaneous frequency and local group delay. Previous applications of this property were mainly used for signal analysis, and also a few geophysical applications (Pedersen et al., 2003; Hyslop and Diallo, 2010).

Reassignment generates concentrated time-frequency maps for signals. However, it also generates concentrated energy for random noise. This problem can be minimized with multi-tapering, since reassigned spectrums from different windows enhance the localization of chirp-like signals while reduce energy concentration of random noise (Chassande-Mottin et al.,1996; Xiao and Flandrin, 2007).

In this paper, we introduce a method of inverse reassignment through constructing a weighting factor during the forward process. To accomplish denoising, thresholding of the reassigned domain is applied before the signal is reconstructed in the time domain. Synthetic data examples in both 1D and 2D are utilized to test the proposed method.

Theory

Forward Reassignment

The well-known Short Time Fourier Transform (STFT) of the signal $x(t)$ can be defined as,

$$G(t,\omega) = \int_{-\infty}^{+\infty} x(\tau) h^{*}(\tau - t) e^{-i\omega\tau} d\tau \ , \tag{1}$$

where h represents the analysis window. If one adopts a Gaussian function window for h, the STFT is known as the Gabor transform. Inherent within the time-frequency map, $G(t,\omega)$, is a tradeoff between time and frequency resolution. We assume that there is a desired spectrum, $G_R(\hat{t},\hat{\omega})$, of the signal with variables of instantaneous frequency, $\hat{\omega}$, and local group delay, $\hat{t}$,

$$\hat{t}(t,\omega) = -\partial\phi(t,\omega)/\partial\omega \,, \tag{2a}$$

$$\hat{\omega}(t,\omega) = \partial\phi(t,\omega)/\partial t \,, \tag{2b}$$

where $\phi(t,\omega)$ represents the phase of $G(t,\omega)$. Essentially, the energy of each point in Gabor spectrum, $G(t,\omega)$, comes from a sum of energies in an area near coordinate (t,ω) of $G_R(\hat{t},\hat{\omega})$. In this sense, $G(t,\omega)$ can be considered as a smoothed version of $RG(\hat{t},\hat{\omega})$. The de-smoothing process to obtain the spectrum $G_R(\hat{t},\hat{\omega})$ requires an efficient coordinate transform to reassign the energy in Gabor spectrum from (t,ω) to $(\hat{t},\hat{\omega})$. Previous studies (Auger and Flandrin, 1995; Flandrin et al., 2003) have derived an equivalent coordinate transform to avoid 2D phase unwrapping as shown in Equation (3),

$$\hat{t}(t,\omega) = t + \mathrm{Re}\{G_{Th}(t,\omega)/G(t,\omega)\} \,, \tag{3a}$$

$$\hat{\omega}(t,\omega) = \omega - \mathrm{Im}\{G_{Dh}(t,\omega)/G(t,\omega)\} \,, \tag{3b}$$

where $G_{Th}(t,\omega)$ and $G_{Dh}(t,\omega)$ are the spectrum of the STFT with analysis windows $Th(t) = t \times h(t)$ and $Dh(t) = \partial h(t)/dt$, respectively. The reassigned spectrum, $G_R(\hat{t},\hat{\omega})$, can then be generated by,

$$G_R(\hat{t},\hat{\omega}) = \int\int_{-\infty}^{+\infty} G(\tau,v)\delta(t-\hat{t}(\tau,v),\omega-\hat{\omega}(\tau,v))d\tau dv \,. \tag{4}$$

Inverse Reassignment

A weighting factor $w(t,\omega)$ is created during the forward reassignment process to save the information of how much energy is transferred from a point (t,ω) in $G(t,\omega)$ to the point $(\hat{t},\hat{\omega})$ in $G_R(\hat{t},\hat{\omega})$ according to,

$$w(t,\omega) = G(t,\omega) \,/\, G_R(\hat{t},\hat{\omega}) \tag{5}$$

After thresholding or masking $G_R(\hat{t},\hat{\omega})$ to attenuate random noise, the Gabor spectrum can be reconstructed with this pre-computed weighting factor and coordinate transform by,

$$\tilde{G}(t,\omega) = w(t,\omega)\tilde{G}_R(\hat{t},\hat{\omega}) \,, \tag{6}$$

where $\tilde{G}(t,\omega)$ and $\tilde{G}_R(\hat{t},\hat{\omega})$ represent the Gabor and reassigned Gabor spectrum after thresholding. The denoised seismic signal is then obtained through applying the inverse STFT to $\tilde{G}(t,\omega)$.

Multitaper Reassignment

Reassignment can generate a concentrated time-frequency map for a signal regardless of the choice of analysis window (Xiao and Flandrin, 2007). However, when there is noise present within the signal, reassignment also generates concentrated energy for random noise. To overcome the problem of random noise concentration, multiple analysis windows are utilized for the Gabor transform and reassignment through a process called multitapering (Xiao and Flandrin, 2007). A set of tapered windows are utilized based upon Hermite functions,

$$h_n(t) = h(t)H_n(t)\big/\sqrt{2^n!} \,,\ n = 0,1,\ldots,N \,, \tag{7}$$

where N represents the number of windows and $H_n(t)$ represents the Hermite polynomial that satisfies the recursion,

$$H_n(t) = 2tH_{n-1}(t) - 2(t-2)H_{n-2}(t), \quad n \ge 2 \tag{8}$$

with the initialization of $H_0(t) = 1$ and $H_1(t) = 2t$. When $n = 0$, the tapered window is the original Gaussian window. Utilizing this set of tapered windows, N different reassigned time-frequency maps are generated and subsequently averaged. This averaged reassigned time-frequency map is then employed to create a masking function, which maintains all values above a certain threshold (i.e. estimated noise level) of the time-frequency map and eliminates all others. The masking function is applied to the time-frequency map corresponding to $n = 0$ to generate a denoised reassigned time-frequency map.

Examples

A synthetic seismic example of forward and inverse reassignment presented in Figure 1 shows that the reassigned Gabor spectrum has a better energy concentration than the traditional Gabor spectrum. Also, the obtained signal from the reassigned Gabor spectrum provides an exact reconstruction of the original signal. Figure 2 presents the denoising of the noise contaminated (SNR=2) synthetic trace from Figure 1. The single taper reassigned spectrum has a better energy concentration than the Gabor spectrum, but is also contaminated by concentrated random noise energy. This contaminated random noise energy is reduced with in multitaper reassigned spectrum. Clearly, denoising by thresholding in the reassigned domain provides a much cleaner signal than thresholding in the Gabor domain. Slight improvements for denoising are seen when utilizing the multitaper reassigned spectrum with $N = 5$ in comparison to the single taper (analysis window), reassigned spectrum.

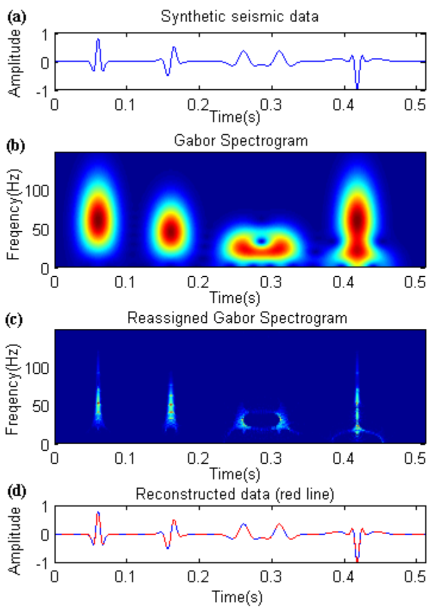

Figure 1: A synthetic seismic example of forward and inverse reassignment.

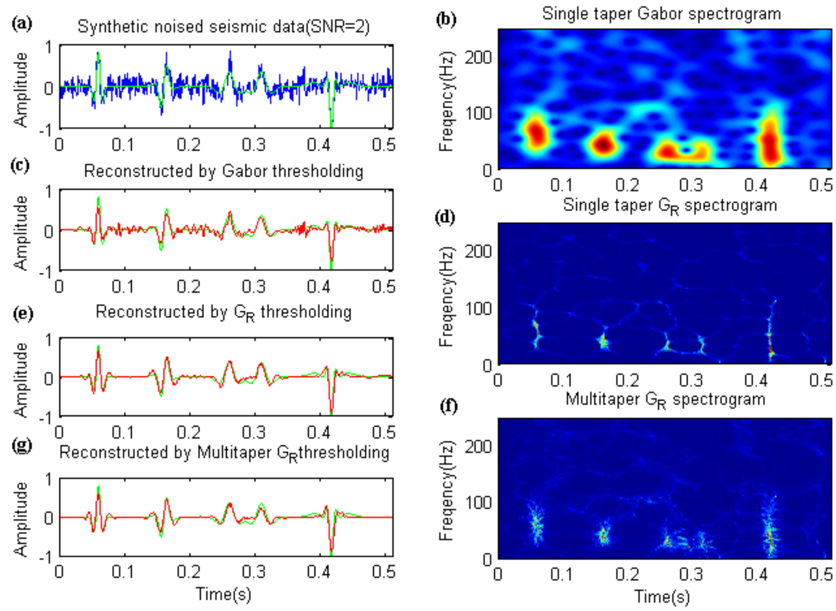

Figure 2: 1D synthetic denoising example. Green lines in (a), (c), (e), and (g) are noise free data for comparison.

Figure 3 provides a 2D synthetic denoising example. The example cross section was contaminated to contain a SNR = 1. For a comparison to common 2D random noise attenuation methods, the noisy cross section was also denoised with f-x deconvolution applied in small spatial windows as shown in Figure 3(b). It is clear that multitaper reassigned denoising generate a better performance than f-x deconvolution denoising especially for hyperbolic events.

Conclusions

We have proposed a new seismic denoising method based on time-frequency reassignment. We introduced an inverse reassignment process that permits to go back from the reassigned domain after thresholding. Multitapering was used to reduce the problem of random noise concentration to improve reassignment denoising. Synthetic examples verified that the proposed method has a good performance on denoising and is better than denoising in Gabor spectrum domain and f-x deconvolution denoising.

Acknowledgements

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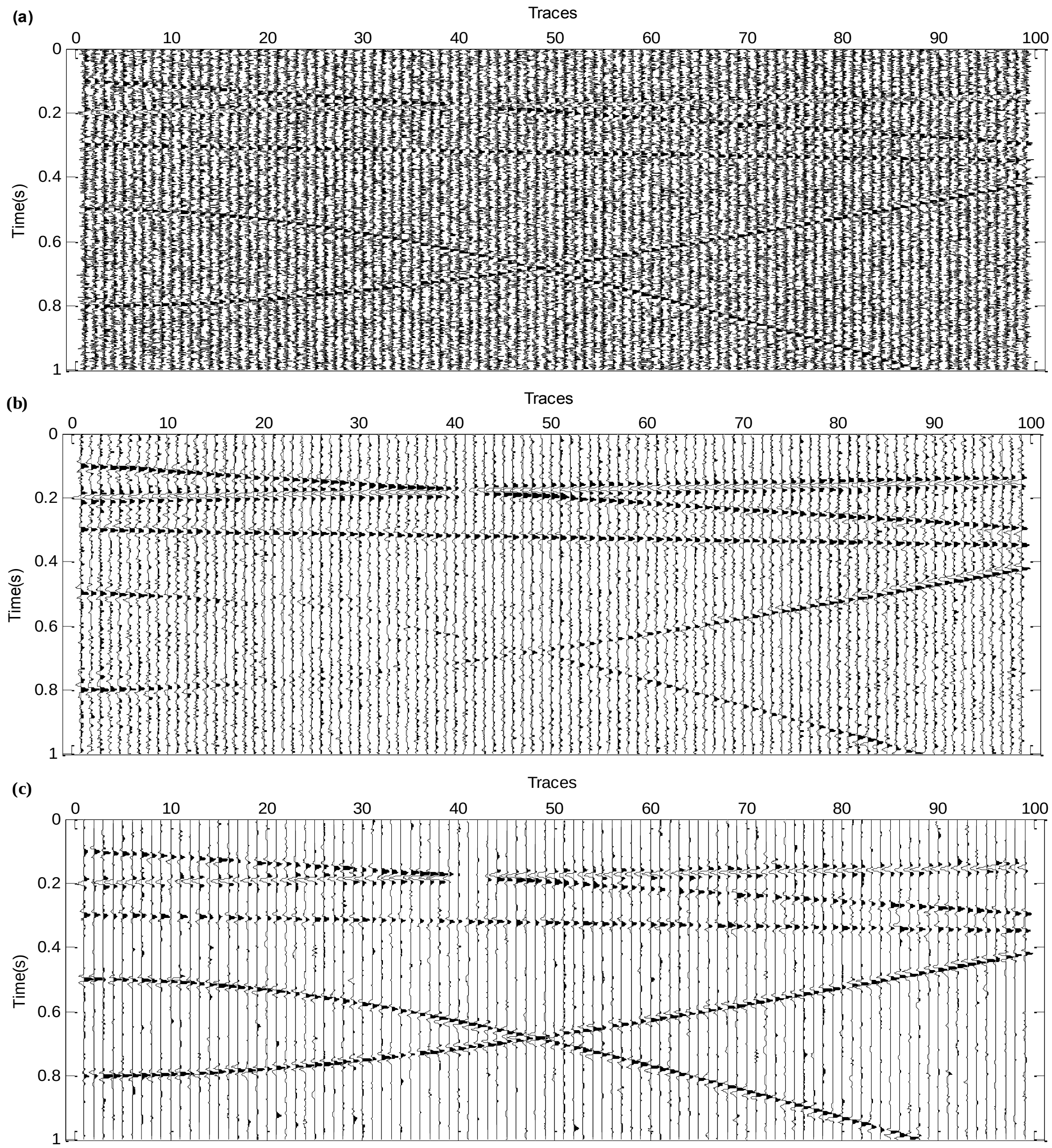

Figure 3: 2D synthetic denoising example. (a) synthetic noisy seismic data (SNR = 1); (b) conventional f-x deconvolution denoising; (c) multitaper reassignment denoising.

References

Auger, F., and P. Flandrin, 1995, Improving the readability of time-frequency and time-scale representations by the reassignment method: Signal Processing, IEEE Transactions on signal processing, **43**, 1068-1089.

Chassande-Mottin, E., F. Auger, and P. Flandrin, 1996, Supervised time-frequency reassignment: Time-Frequency and Time-Scale Analysis, Proc. of the IEEE International Symposium, 517-520.

Flandrin, P., F. Auger, and E. Chassande-Mottin, 2003, Time-frequency reassignment: from principles to algorithms: Applications in Time-Frequency Signal Processing, 179-203.

Hyslop, C. W., and M. S. Diallo, 2010, Energy Reassignment of an Image for Improved Picking of Velocity Dispersion Curves: EAGE, 14-17.

Kodera, K., C. De Villedary, and R. Gendrin, 1976, A new method for the numerical analysis of non-stationary signals: Physics of the Earth and Planetary Interiors, **12**, 142-150.

Pedersen, H. A., J. I. Mars, and P.-O. Amblard, 2003, Improving surface-wave group velocity measurements by energy reassignment: Geophyscs, **68**, 677-684.

Xiao, J., and Flandrin, P. (2007). Multitaper Time-Frequency Reassignment for Nonstationary Spectrum Estimation and Chirp Enhancement. IEEE Transactions on Signal Processing, **55**(6), 2851-2860.