Inversion approach to calculate instantaneous frequency

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Summary

This paper revisits complex trace analysis and proposes an inversion approach to calculate instantaneous frequency. Computed frequencies tend to be smooth and meaningful with the help of shaping operator. Synthetic examples demonstrate the suitability of the inversion method to calculate instantaneous frequency, and combined with empirical mode decomposition (EMD) or ensemble empirical mode decomposition (EEMD), it shows potential as a time-frequency analysis tool with a high resolution.

Introduction

Taner et al. (1979) systematically examined the complex trace analysis theory. Using the analytic signal and its amplitude information, they developed several poststack seismic attributes. The resulting instantaneous frequency has been proven useful to detect meandering channels and determine their thickness. However, the notion of instantaneous frequency has always been controversial. Saha (1987) discussed the relationship between instantaneous frequency and Fourier frequency, and pointed out instantaneous frequency is identical to Fourier mean frequency. Recently, Huang et al. (2009) summarized the applicable conditions of instantaneous frequency: the analysis time series needs to be not only mono-component, but also narrow band. Analysis of instantaneous frequencies has been gradually replaced by spectral decomposition techniques in early 1990s due to their increased flexibility. The empirical mode decomposition (EMD) developed by Huang et al. (1998) is an alternative method for time-frequency analysis. EMD decomposes a data series into a finite set of signals, called intrinsic mode functions (IMFs). The IMFs represent the different oscillations embedded in the data. They are functions that satisfy two conditions: (1) in the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one; and (2) at any point, the mean value of the envelope defined by the local maxima and the one defined by the local minima is zero. These conditions are necessary to ensure that each IMF has a localised frequency content by preventing frequency spreading due to asymmetric waveforms. Based on the filter bank structure of EMD (Flandrin. et al. 2004), Wu and Huang (2009) proposed the ensemble empirical mode decomposition (EEMD) to solve the mode mixing problem of EMD, which has proved to be of practical importance. Instantaneous frequency combined with EMD/EEMD has potential contributions in seismic processing (Battista et al. 2007; Magrin-Chagnolleau and Baraniuk 1999) and seismic attribute analysis (Huang and Milkereit, 2009; Han and van der Baan, 2011).

Inversion approach to calculate instantaneous frequency

First, we briefly review complex trace analysis. Both instantaneous amplitude and instantaneous phase are derived from the conventional seismic trace. If the seismic trace is $x(t)$ and its Hilbert transform $y(t)$, then the complex trace is formed by

$$z(t) = x(t) + iy(t) = R(t)\exp[i\theta(t)]. \tag{1}$$

where $R(t)$ and $\theta(t)$ denote the instantaneous amplitude and instantaneous phase, respectively. Instantaneous frequency $f(t)$ is defined as the derivative of instantaneous phase. Thus,

$$f(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \ . \tag{2}$$

Although instantaneous frequency comprises a powerful tool for detecting hydrocarbon, fracture zone and bed thickness (Taner 2001), its disadvantages is obvious: it fluctuates rapidly with the spatial and temporal location, and it can become negative which has no physical meaning (Barnes 2007).

Revisited the complex trace analysis theory, we design a new way to calculate instantaneous frequency. Our approach is to build up an inversion formulation, by setting regularization constraints, instantaneous frequency tends to become smooth and continuous.

From equation (2), instantaneous frequency is the derivative of instantaneous phase; in other words, instantaneous phase is the time integral of instantaneous frequency

$$\theta(t_0) = 2\pi\int_0^{t_0} f(t)dt \ . \tag{3}$$

where t_0 is the time sample. The right side of equation (3) is a definite integral. We can use the Trapezoidal rule, or more accurately, the Simpson integral rule for discretization. Then, equation (3) transforms to

$$\theta = H * f \ . \tag{4}$$

where θ and f are the same definition as before, H is the matrix of discretization coefficients. This is an ill-posed matrix.

To solve equation (4), the standard regularized least squares approach is to minimize $\|Hf - \theta\|_2^2 + \varepsilon^2\|Df\|_2^2$, where D is the classic Tikhonov regularization operator and ε is a scaling parameter. The formal solution is

$$\hat{f} = (H^T H + \varepsilon^2 D^T D)^{-1} H^T \theta \ . \tag{5}$$

In our approach, we utilize the shaping operator S to get a smooth output (Fomel 2007). The solution of equation (4) then becomes

$$\hat{f} = [\lambda^2 I + S(H^T H - \lambda^2 I)]^{-1} SH^T \theta \ . \tag{6}$$

The shaping operator S can be defined as a triangle or Gaussian smoothing operator.

Examples

Due to the rigorous applicability conditions of instantaneous frequency, direct calculation often leads to fluctuating and unreasonable values (Han and van der Baan 2011). EMD can extract the intrinsic components of the original signal efficiently, thereby playing a precursory role for instantaneous spectral analysis. In our applications, we decompose the data using EMD or EEMD before calculating the instantaneous spectrum.

The signal in Figure 1 is the sum of two chirp signals with different frequencies and amplitudes. After EMD, two chirp signals can be extracted perfectly. Calculating the instantaneous frequencies of two IMFs using the inversion approach with smoothing radius of 3 points (Figure 2), both frequency and amplitude information are correctly recovered. A Gaussian weighted filter was applied to widen the instantaneous spectrum for display purposes.
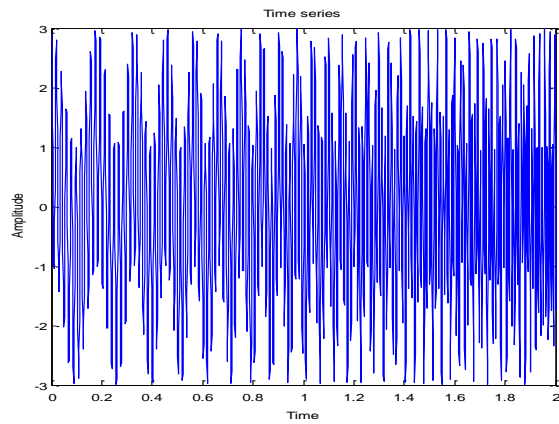
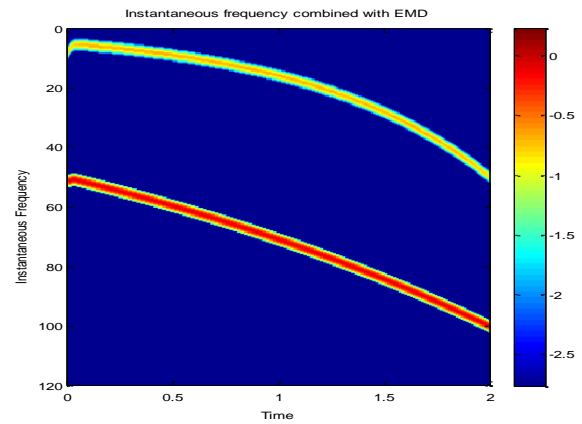Figure 1: Sum of two chirp signals with different frequency distributions.

Figure 2: Instantaneous spectrum obtained using the inversion approach after EMD. Both frequency and amplitude information are correctly recovered.

After testing the validity of the inversion approach, we apply it on a more intricate synthetic example. The signal in Figure 3 is comprised of an initial 20 Hz cosine wave, superposed 100Hz Morlet wavelet at 0.3s, two 30Hz Ricker wavelets at 1.1s, and other three different frequency components between 1.3s and 1.7s.

We compare the instantaneous spectrum analysis with the Short-time Fourier and wavelet transforms. All three methods can discriminate the frequency components between 1.2s and 2s with acceptable temporal and spectral resolution. The short-time Fourier transform with a 170ms time window (Figure 4) does not distinguish the two Ricker wavelets clearly at 1.1s due to its fixed time-frequency resolution. Wavelet analysis (Figure 5) shows better result; however, the spectral resolution for the 100Hz Morlet wavelet at 0.3s is poor. Figure 6 displays the instantaneous spectrum after EEMD of the input signal. The100Hz Morlet wavelet, both 30Hz Ricker wavelets and all other frequency components are recovered with the highest time-frequency resolution.
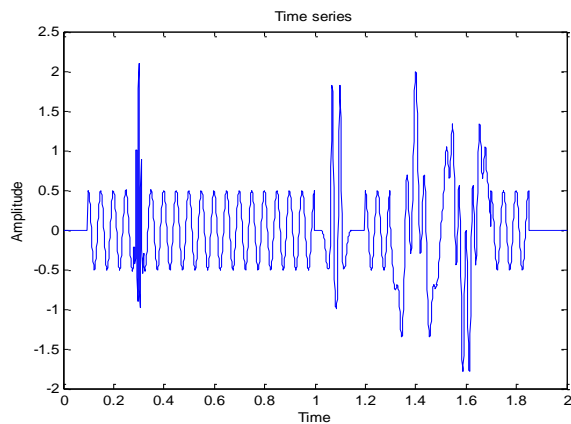

Figure 3: Synthetic example: 20Hz cosine wave from 0.1s to 1.85s, 100Hz Morlet wavelet at 0.3s, two 30 Hz Ricker wavelet at 1.1s, and other three frequency components between 1.3s and 1.7s.
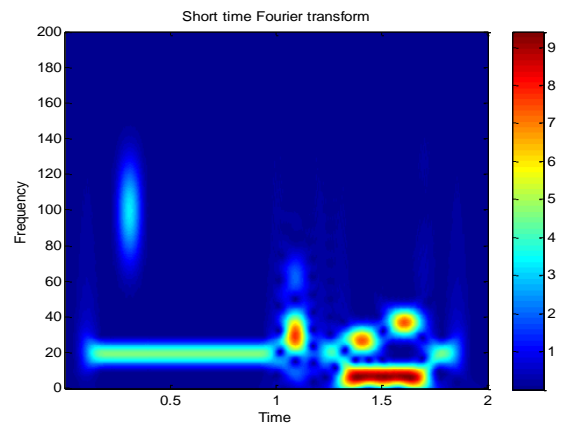

Figure 4: Short-time Fourier transform with a 170ms time window. It cannot distinguish the two Ricker wavelets at 1.1s due to its fixed time-frequency resolution.
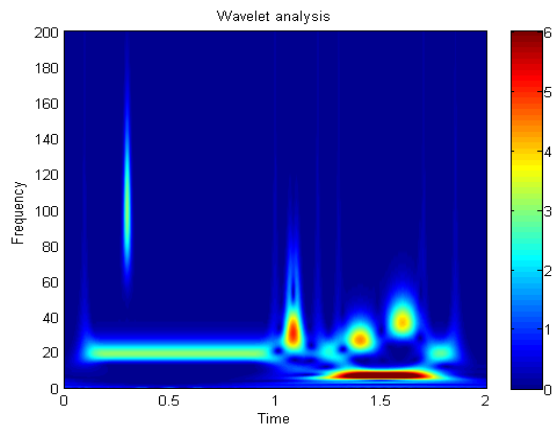
Figure 5: Wavelet transform analysis, which shows better time-frequency resolution than the short-time Fourier transform as it distinguishes both Ricker wavelets at 1.1s. Yet, the frequency resolution for the 100Hz Morlet wavelet at 0.3s is poor.
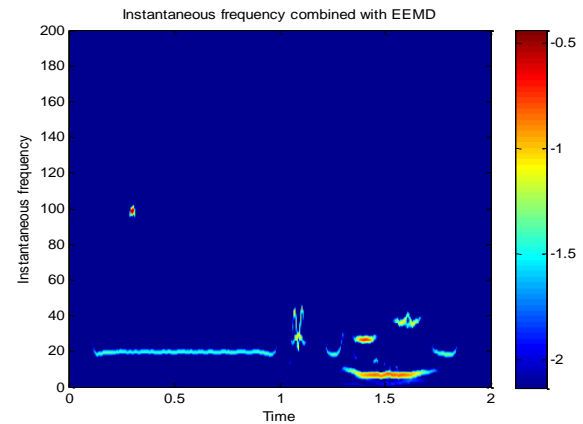

Figure 6: Instantaneous spectrum combined with EEMD which has the highest time-frequency resolution and identifies all individual components.

Conclusions

The synthetic data application indicates the suitability of shaping inversion approach to calculate instantaneous frequencies. Combined with empirical mode decomposition (EMD) or ensemble empirical mode decomposition (EEMD), instantaneous spectrum analysis shows higher temporal-spectral resolution than either the short-time Fourier transform or the wavelet transform.

Acknowledgements

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