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Estimation of group velocity using slant stack generalized S transform based method

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Summary

Group velocity is an important factor that determines shear wave velocity in Multi-channel Analysis of Surface Wave (MASW) surveys. In this study, we present the slant stack generalized S transform based method for the estimation of the group velocity. The slant stack generalized S transform based method uses the slant stack idea in the time-frequency domain based on the generalized S transform. The method is robust to estimate group velocity where ground roll is multi-modal. We anticipate that, through inversion of the group velocities of multi-modal ground rolls, a better estimation of near surface shear wave velocity will be obtainable.

Introduction

The analysis of dispersed surface waves (Rayleigh and Love waves) is a well-known procedure to estimate shear wave velocity (e.g. Xia et al., 1999). The method is a based on the inversion of a surface wave dispersion curve (either phase or group velocity curves) calculated from a real data to a vertical shear wave velocity profile.

Since surface wave methods are based on the inversion of phase or group velocities to a shear wave velocity profile, the processing of data in order to truly estimate a dispersion curve is very crucial. In this study, we present a slant stack model based on the generalized S transform to estimate the group velocity of multi-channel ground roll.

The Generalized S-Transform

The S transform is a time-frequency transform which provides a time depended frequency distribution of a signal. The idea presented in the S transform is very similar to that in the Gabor transform (Gabor, 1946) which utilizes a Gaussian window for spectral localization. But, in the S transform, the Gaussian window is scalable with the frequency which enhances a better time-frequency resolution. The S transform is given by Stockwell et al. (1996) as

$$S[h(\tau)](t,f) = \int_{-\infty}^{+\infty} h(\tau)\left[\frac{|f|}{\sqrt{2\pi}} e^{-\frac{f^2(\tau-t)^2}{2}}\right] e^{-j2\pi f\tau} d\tau. \qquad (1)$$

Where as an operator, S transforms h into a function of frequency f and time t. Time t controls the position of the Gaussian window on the output time axis.

In the generalized version of the S transform, a multi-parameter factor is inserted in the Gaussian window in order to manipulate the time-frequency resolution based on processing or interpretational purposes. A version of the generalized S transform (Pinnegar and Mansinha, 2003) that has particular usefulness in our analysis is defined

$$S_g[h(\tau)](t,f,\sigma) = \int_{-\infty}^{+\infty} h(\tau) \frac{|f|}{\sqrt{2\pi}\sigma} e^{-\frac{f^2(\tau-t)^2}{2\sigma^2}} e^{-j2\pi f\tau} d\tau, \qquad (2)$$

where the scaling factor σ controls time-frequency resolution by changing the number of oscillations within the window. If we choose a small values of the scaling factor ($\sigma \cong 1$), the generalized S transform presents a better time resolution and a low frequency resolution. On the other hand, choosing a larger value of the scaling factor ($\sigma \gg 1$) results a better frequency resolution in the cost of reduce time resolution. This properly of the generalized S transform can be applied in the slant stack generalized S transform method to improve the estimation of the group velocity at different frequencies.

The Wave Propagation Operator

Assuming geometrical spreading correction has been applied on surface wave data, if $h_1(\tau)$ is the wavelet at station 1, the wavelet $h_2(\tau)$ recorded at station 2 can be expressed

$$H_2(f) = e^{-\lambda(f)d} e^{-j2\pi k(f)d} H_1(f), \quad (3)$$

where λ(f) is an attenuation function, and k(f) is a spatial wavenumber that controls wave propagation from station 1 to station 2. This wavenumber characterizes horizontal propagation of surface wave and is a function of elastic properties of the medium, and d is the distance between, the two stations.

Askari and Ferguson (2011) assume that attenuation function λ(f) and phase function k(f) vary slowly with respect to the effective size of the spectrum of the generalized S transform. Therefore, λ(f+α) and k(f+α) can be expressed

$$\lambda(f + \alpha) = \lambda(f) + O(\alpha), \quad (4)$$

and

$$k(f + \alpha) = k(f) + \alpha k'(f) + O(\alpha^2), \quad (5)$$

where k'(f) indicates a frequency derivative of k(f). Using the above approximations, the generalized S transform of the wavelet in station 2 can be linked to that in station 1 with the following equation (Askari and Ferguson, 2011)

$$S_g[h_2(\tau)] = e^{-j2\pi f d/v_p(f)} e^{-\lambda(f)d} S_g[h_1(\tau)]\big(t - d/v_g(f), f\big) \quad (6)$$

where $S_g[h_1(\tau)]\big(t - d/v_g(f), f\big)$ is the generalized S transform of h_1 shifted by - $d/v_g(f)$ and v_g is the group velocity which is the velocity of a wave pocket (envelope) of surface wave around frequency f.

Based on equation (6), any point at time-frequency plane (t, f) of station 1, is equivalent to the time shifted-frequency plane (t-$d/v_g(f)$, f) of station 2 whose phase difference is $-j2\pi f d/v_p(f)$, and whose amplitude is proportional to $e^{-\lambda(f)d}$ that of station 1. Thus, the group velocity can be obtained from the time difference of the ridge of the transforms for any frequency

$$v_g = d/\Delta t, \quad (7)$$

where $\Delta t = t_2 - t_1$ is the time difference between two ridges. In the real world, a seismic record is composed of multi-modal ground roll therefore the estimation of the group velocity is not as straightforward as equation (7). Therefore in order to adjust the method for a real case, the slant stack generalized S transform based method is introduced.

Slant Stack Generalized S Transform Based Method

The plane wave of a wavefield such as a common shot gather can be transform from the offset-time domain to the intercept time-slowness domain. This transformation is achieved by applying linear moveout and summing amplitudes over the offset axis. This procedure is called slant stack (Yilmaz, 1987). The first step which is a linear moveout is expressed as

$$U(x, T) = U(x, T = t - px), \quad (8)$$

where T is intercept time and p is slowness. The second step which is the staking of all the amplitude over the offset with respect to their slownesses and intercept times is expressed as

$$P(p, T) = \textstyle\sum_x U(x, T = t - px), \quad (9)$$

where P is the slant stack function. Therefore it is possible to find the slownesses of linear phenomena on a seismic record using slant stack idea. Combining the idea represented by equation (7) with that in equation (9), we can provide the group velocity of multi-modal ground roll. In this study we call this approach the slant stack generalized S transform based method.

Figure 1 shows synthetic data which is composed of bi-modal ground roll. Figure 2a shows the generalized S transform for the first trace. For any specific frequency such as 40Hz in the generalized S transform domain, there is a pseudo-seismic trace which is a function of time (Figure 2b). Putting together all of the pseudo-traces for different offsets, we make a pseudo-seismic record for that frequency (Figure 3). According to the linear propagation of ground roll on surface, the apparent velocity of the ground roll can be estimated by applying the slant stack transform. Figure 4a shows the slant stack of the pseudo-seismic record. We show the slant stack in the intercept time-velocity domain instead of the intercept time-slowness domain for giving a better understanding of the velocity ranges of the group velocity. As seen, at T=0 (Figure4b), we can estimate the group velocity for that frequency. Therefore, we can make a 2D image of the group velocity of the ground roll by putting together all the velocities estimated at intercept time T=0, for all frequencies. Figure 5 shows the group velocity of the record based on the scaling factor σ=1. As seen, the group velocity is well estimated for frequencies smaller than 90Hz. However the estimated group velocity for frequencies larger than 90Hz is overestimated. This can be explained due to the frequency uncertainty of the generalized S transform at higher frequencies. In order to better estimate group velocity for higher frequencies, a larger scaling factor can be chosen. Based on our imperial observations on synthetic data, scaling factors 1 to 5 give reasonable results for the ranges of frequencies (5-100Hz) in the MASW survey. We have applied the slant stack generalized S transform on the real data in Figure 6a to estimate the group velocity. Figure 6b shows the group velocity. The ranges of the group velocity vary from 100m/s at frequency 18Hz to 400m/s at frequency 3Hz. These rages of the group velocity are also consistent with the group velocity of ground roll for near surface.

Conclusions

We introduce the slant stack generalized S transform based for the estimation of the group velocity. The resolution of the group velocity is manipulated by a scaling factor. Smaller scaling factor should be chosen for the low frequency ground roll. However, for higher frequencies, a larger scaling factor should be chosen. Therefore, there is a trade of between the resolution of low and high frequency ground roll. Based on our imperial observations, for the ranges of the frequencies that we deal with in the MASW survey (3-100Hz), scaling factors from 1 to 5 are recommended.

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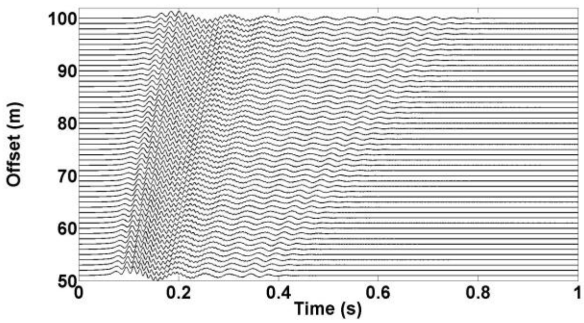

Figure 1: A synthetic data.

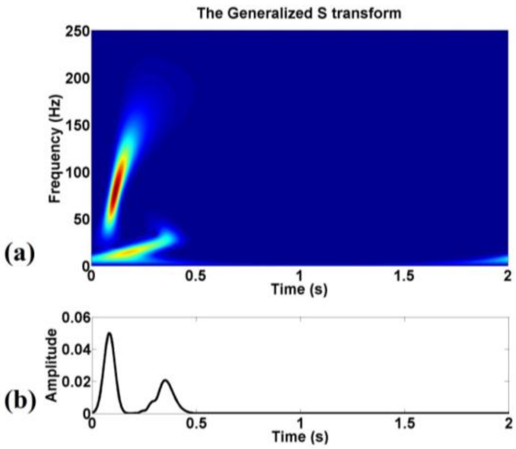

Figure 2: (a) The generalized S transform of the first trace in Figure 1. (b) Time representation of the generalized S transform at the single frequency 40Hz.

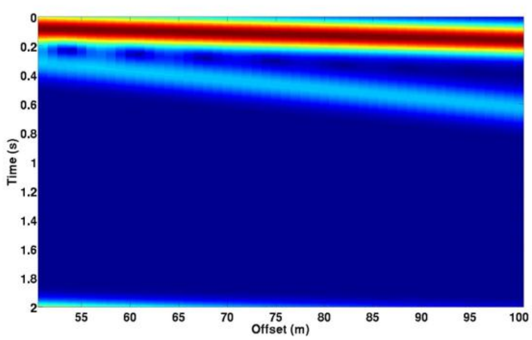

Figure 3: A pseudo-seismic record based of the generalized S transform of the traces of the record at Frequency=40Hz.

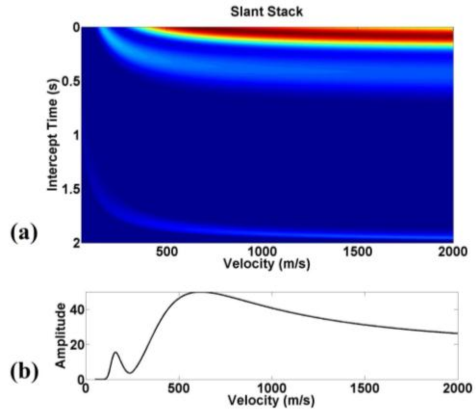

Figure 4: (a) The slant stack of the pseudo-record in Figure 3. (b) Single representation of the slant stack at intercept time T=0.

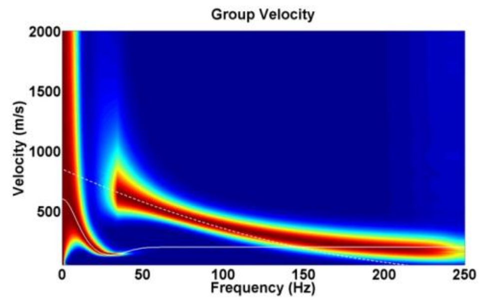

Figure 5: The estimated group velocity for the record. The solid and dashed lines correspond to the theoretical values.

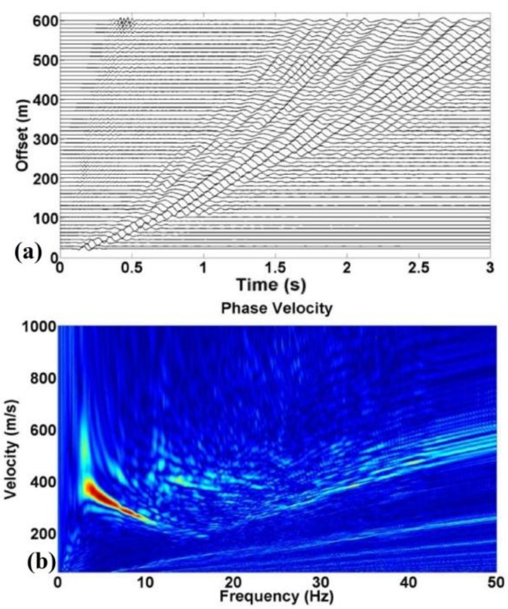

Figure 6: (a) A real data. (b) Its estimated group velocity.