A Tour of High Resolution Transforms

Mauricio D. Sacchi*

Signal Analysis and Imaging Group, Department of Physics

University of Alberta, Edmonton, AB, Canada

msacchi@ualberta.ca

Summary

Transform methods have provided effective tools to remove undesired events and for seismic data reconstruction. The purpose of this talk is to review transform-based seismic data processing methods and their application to noise reduction and data reconstruction. In particular, I will discuss the implementation of high resolution transforms via regularization techniques. I will also provide an example where the Gabor transform is turned into a high resolution transform and used for seismic data reconstruction.

Introduction

Many problems in seismic data processing can be posed as inverse problems of the form

$$Lm = d, \qquad (1)$$

where d indicates the data, m the data in the transform domain and L the operator containing a family of basis functions used to fit the data. In particular, Radon transforms in time-space and in frequency-space domain are examples of transformations that can be posed via equation (1) (Hampson, 1986; Thorson and Claerbout, 1985). The seismic data reconstruction problem has also been tackled with the aforementioned framework. In this case L indicates the inverse Fourier transform, and m the vector of unknown Fourier coefficients needed to fit the data (Sacchi and Ulrych, 1996). It is interested to note that; in general, the solution m (the data representation in the transform space) is non-unique. A way of solving the problem (finding a stable solution) is via regularization methods. The latter requires solving for the minimum of the following cost function

$$J = \| Lm - d \|^2 + \lambda R(m). \qquad (2)$$

The first term on the right hand side of equation (2) is the misfit; the second term is the regularization term. It is clear that the misfit must be minimized in order to find a signal representation capable of recovering the data. The regularization term is used to stabilize the solution, and in addition, to increase focusing in the transform domain. This simple idea gave rise to various methods for signal enhancement and signal reconstruction. We will cite a few of them:

FX parabolic Radon transform: In this particular application the basis functions are complex chirps in the *FX* domain (parabolic travel-times in *TX*). Stability is often achieved by including a quadratic regularization term (Hampson, 1986).

FX high resolution parabolic Radon transform: In this formulation the regularization term is a non-quadratic form. In general, *R(m)* is a functional that provokes sparsity (Sacchi and Ulrych, 1995). Strategies exist to optimally control alias (Ribeiro et al., 2001) and to improve computational efficiency (Sacchi and Porsani, 1999).

TX high resolution parabolic Radon transform: A robust high-resolution parabolic Radon transform can be obtained by posing the problem in the *TX* domain (Cary, 1998). The computational cost of the transform is quite high when compared to *FX* algorithms (Trad et. al, 2003).

TX high resolution hyperbolic Radon transform: The basis functions are given in the *TX* domain and correspond to hyperbolic events (Thorson and Claerbout, 1985). *TX* Radon algorithms require the inversion of large linear operators as opposed to *FX* algorithms where a small operator is inverted for each temporal frequency.

Local Radon Transform: Local wave field operators with variable dip synthesize the operator. Each operator locally mimics spatio-temporal signals in a small aperture (Sacchi et. al, 2004).

Minimum Weighted norm interpolation (MNWI): The operator *L* is the inverse Fourier Transform. The regularization term uses a spectral norm similar to a sparsity norm. It is important to mention, however, that sparsity can lead to non-optimal solutions. MWNI adopts a regularization term that de-emphasizes sparsity via a smoothing strategy.

Interpolation and SNR enhancement with Curvelet Frames: The operator *L* is the adjoint of the Curvelet Transform (Herrmann et. al, 2008). The Curvelet Transform can decompose a dip-varying signal in a superposition of local and scale-dependent dipping events.

High Resolution Gabor Transform: An example

The machinery outlined above can be used to design other high-resolution transforms. I will show how one can utilize a *FX* high resolution Gabor transform to reconstruct seismic records. The Gabor transform is a Short Window Fourier Transform that utilizes a Gaussian window (Feichtinger and Strohmer, 1998) Following the notation introduced above, *L* indicates the adjoint of the Gabor transform and *m* the coefficients of the Gabor transform. We now minimize the following cost

$$J = \| T(Lm - d) \|^2 + \lambda R(m), \tag{3}$$

where we have incorporated a sampling operator *T*. The sampling operator is used to de-emphasize the influence of missing or bad quality data points (or traces in a seismic reconstruction problem). The strategy entails using the available data to compute *m* (the coefficients of the Gabor transform). The coefficients of the Gabor transform are then used to synthesize new data including missing observations.

Figure 1 portrays the application of the high resolution Gabor transform to a 1D reconstruction problem. In this case, the data represent a temporal chirp with erasures. Figure 1a shows the chirp prior to reconstruction. Figure 1b shows the chirp after reconstruction using the high resolution Gabor transform. To complete the analysis we portray the classical Gabor transform obtained from the incomplete data (Figure 1c) and the high-resolution Gabor transform obtained by minimizing equation (3). The regularization term in this example is a Cauchy norm (Sacchi and Ulrych, 1995).

The algorithm that minimizes equation (3) was also implemented in the *FX* domain. In this case, the name time-frequency analysis should be replaced by space-wavenumber analysis to reflect that the high-resolution Gabor transform is used to reconstruct spatial signals. Figure 2a shows a common offset section where 30 % of the traces are missing. Figure 2b shows the *FX* domain reconstruction using the high resolution Gabor transform.

Conclusions

A general framework for building high resolution transforms was described. I have reviewed different methods that use the concept of sparsity to enhance resolution and to design data reconstruction strategies. Finally, I have provided an example where the Gabor transform was utilized to design an algorithm for data reconstruction. The algorithm operates in the *FX* domain and unlike most *FX* reconstruction methods can tolerate spatial variations of dip.

Acknowledgments

This is a good opportunity to acknowledge the important contribution of Dan Hampson to the field of transform-based methods for seismic data enhancement and reconstruction.

References

P. W. Cary,1998, The simplest discrete Radon transform, SEG, Expanded Abstracts,17, 1999.

H.G. Feichtinger, Th. Strohmer, "Gabor analysis and algorithms" , Birkhäuser (1998)

D. Hampson, 1986, Inverse velocity stacking for multiple elimination Canadian Journal of Exploration Geophysicists, 22, 44.

F. J. Herrmann, D. Wang, G. Hennenfent, and P. P. Moghaddam, 2008, Curvelet-based seismic data processing: A multiscale and nonlinear approach, Geophysics, 73, A1.

Bin Liu and M. D. Sacchi, 2004, Minimum weighted norm interpolation of seismic records Geophysics, 69, 1560.

C. Ribeiro, P. Herrmann, and P. Hugonnet, 2001, High Resolution Radon - a Review, EAGE,Expanded Abstracts,,Session: IM

M. D. Sacchi and T. J. Ulrych, 1995, High resolution velocity gathers and offset space reconstruction, Geophysics, 60, 1169.

M. D. Sacchi and T. J. Ulrych, 1996, Estimation of the discrete Fourier transform, a linear inversion approach Geophysics, 61,1122.

M. D. Sacchi and M. Porsani, 1999, Fast high resolution parabolic Radon transform, SEG, Expanded Abstracts,18,1477.

M. D. Sacchi, D. J. Verschuur, and P. M. Zwartjes, 2004, Data reconstruction by generalized deconvolution , SEG, Expanded Abstracts,23,1989.

J. R. Thorson and J. F. Claerbout, 1985, Velocity stack and slant stack stochastic inversion

Geophysics, 50, 2727.

D. Trad, T. J. Ulrych, and M.D. Sacchi, 2002, Latest views of the sparse Radon transform Geophysics, 68, 386.
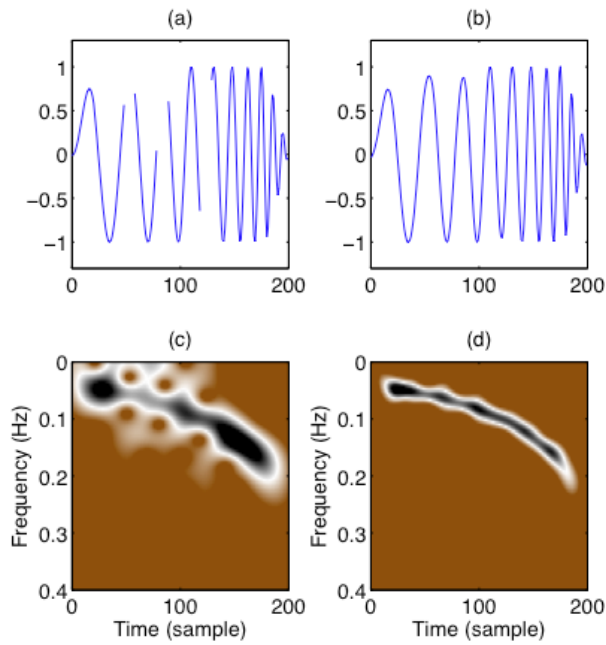
Figure 1. (a) A hyperbolic chirp with missing samples (the incomplete data). (b) Recovery of the hyperbolic chip using a high resolution Gabor transform. (c) Classical Gabor transform of the incomplete data. (d) Inverted high resolution Gabor transform from the incomplete data.
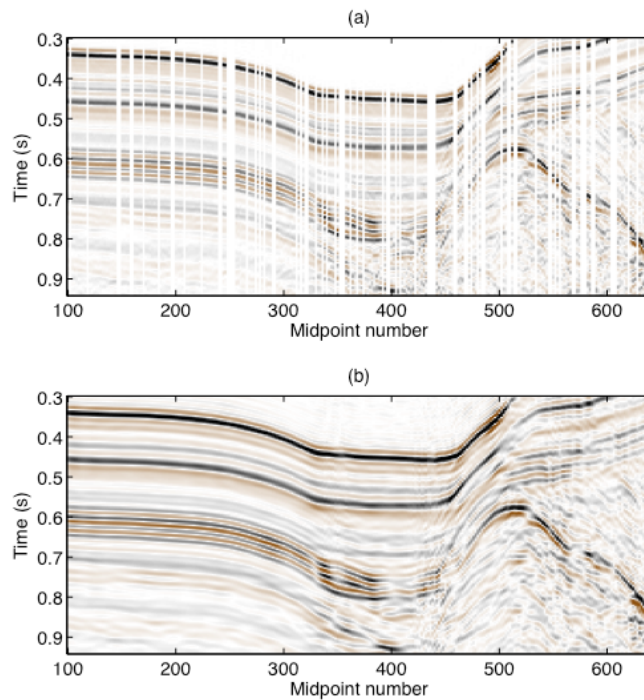

Figure 2. (a) Common offset section with missing traces. (b) Data reconstructed using the *FX* high resolution Gabor transform.