Empirical Mode Decomposition Based Instantaneous Spectral Analysis and its Applications to Heterogeneous Petrophysical Model Construction

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Summary

Spectral analysis is an important step in seismic data processing and interpretation. The frequency contents of seismic traces vary with time due to the fact that the earth is non-stationary medium. Methods were available to improve the temporal and spectral resolution, such as windowed Fourier transform, wavelet transform, S-transform and Matching Pursuit Decomposition, etc. This paper described the Hilbert-Huang Transform (HHT) based on Empirical Mode Decomposition (EMD) that was initially developed to analyze nonlinear and non-stationary water waves. The advantage of HHT-EMD is that it does not require presumed a set of functions as previous methods and allows projection of a non-stationary and non-linear signal onto a time-frequency plane using a set of adaptive Intrinsic Mode Functions (IMF) only determined from the signal itself. The comparisons with wavelet transform and S-transform were made before HHT-EMD was applied to decompose well logs into the wave number-depth (k-z) domain. The depth varying spectrum function was obtained and then used to simulate locally stationary heterogeneous petrophysical models.

Introduction

According to Castagna et al. (2006), spectral decomposition in seismic exploration is a method which produces a continuous time-frequency expression of a seismic trace. Short time Fourier transform was first used (e.g., Cohen, 1995) but due to its weak temporal resolution in a fixed window, was replaced by wavelet transforms (e.g., Daubechies, 1992). However, wavelet transform projects the signal onto a scale-time plane and hence a transform from scale to frequency must be done. The S-transform (Stockwell, 1996) combines the elements of the short time Fourier transform and the wavelet transform by changing the shape of the S-transform 'wavelet' (Pinnegar and Mansinha, 2003). Although the S-transform can do multi-resolution analysis and retain the frequency information, one cannot expect the predetermined Gaussian window to fit all signals. Matching Pursuit Decomposition (Mallat and Zhang, 1993) can provide a better resolution, but a redundant dictionary of functions must be predefined and makes the process computationally intensive. On the other hand, HHT-EMD generates necessary adaptive bases only from the data. After its debut (Huang et al., 1998), HHT-EMD increasingly gained wide application in Atmospheric science (e.g., Duffy, 2004), engineering (Huang and Attoh-Okine, 2005), hydrology (Rao, 2008), finance (e.g., Drakakis, 2008), and earthquake seismology (e.g., Huang et al., 2001). The comprehensive review can be found in Huang and Wu (2008). In seismic exploration, although spectral decomposition was widely used to derive time-frequency seismic data attributes (Sinha et al., 2003); to detect low-frequency shadows (Castagna et al., 2003); to determine layer-thickness (Puryear and Castagna, 2008); and to analyze reservoir fluids (Chen et al, 2008), HHT-EMD as a competitive spectral decomposition technique has not been widely seen in exploration literatures. To our knowledge, Magrin-Chagnolleau and Baraniuk (1999) was among the first introducing HHT-EMD as a time-frequency analysis tool. Hassan (2005, 2006) processed airborne gravity data using EMD as an adaptive dyadic filter bank. Battista et al. (2007) showed HHT-EMD application in processing reflection seismic data and Jeng et al. (2007) applied it to reduce noise observed in EM data. Bekara and van der Baan (2008a, 2008b) used EMD to reduce the random noise by eliminating the fastest oscillation. Considering that significant improvement has been made since 1998 (Wu and Huang, 2004 and 2009), in this study we applied the up-to-date HHT-EMD to time-frequency analysis and showed that superior results can be obtained. We also applied the HHT-EMD to well log analysis and demonstrated the idea to simulate locally stationary heterogeneous models.

The Empirical Mode Decomposition and Hilbert-Huang Transform

Two steps are required to analyze a real signal X(t): (1) use EMD adaptively to decompose X(t) in a limited number (N) of zero-mean, narrowband IMF; and (2) calculate the instantaneous frequency of each IMF by using the normalized Hilbert transform (Huang and Wu, 2008 and Huang et al., 2009) instead of a direct Hilbert transform on IMF due to the Bedrosian theorem (Bedrosian, 1963). The combination of the Hilbert spectrum yields the entire time-varying instantaneous spectrum. The recipes of two steps are outlined as follows:

Step 1. EMD

i. The extrema of the signal X(t) were identified together with two extra points at each edge to avoid the big swing of cubic spline (Wu and Huang, 2004 and 2009).

ii. Interpolation of the above points via cubic spline yields the top and bottom envelops (I_{top} and I_{bot}). The local mean m_{11}=(I_{top}+ I_{bot})/2 was then subtracted from X(t). The residual h_{11}=X(t)-m_{11} is subject to the criterion of IMF, which is that the difference of its zero crossings and the extrema point is no more than one. If it is not satisfied, h_{11} will be treated as the new signal and is returned to step i. This procedure is designated as ‘sifting’ by Huang et al., (1998). If it is satisfied consecutively up to k times (k=5 was suggested in Wu and Huang, 2009), the k^{th} residual h_{1k} is accepted as the first IMF, IMF_1.

iii. The residual r_1=X(t)-IMF_1 is treated as the new signal and is returned to step i.

iv. Repeat step i to iii, until the residual becomes the monotonic function from which no IMF can be extracted. The signal finally is expressed as: $X(t)=\sum_{i=1}^{N} IMF_i(t)+r_N(t)$, where N is the total number of intrinsic mode functions and $r_N(t)$ is the trend.

Step 2. HHT

i. The envelop $e_1(t)$ is obtained via the interpolation of the extrema points of the absolute value of the i^{th} IMF_i.

ii. The normalization is given by an iterative process: $f_1(t)=IMF_i(t)/e_1(t)$, $f_2(t)=f_1(t)/e_2(t)$,till $f_n(t)=f_{n-1}(t)/e_n(t)$, when $e_n(t)$ is unity, and $e_j(t)$ is obtained from $f_{j-1}(t)$ as in step i.

iii. The $IMF_i(t)$ is then expressed as $IMF_i(t)=A_i(t)\cos(\phi_i(t)),\phi_i(t)=\int_{\tau=0}^{t}\omega_i(\tau)\,d\tau$,where ω_i is the instantaneous angular frequency and thus the original signal is expressed as amplitude and frequency modulated signal: $X(t)=\sum_{i=1}^{N} A_i(t)\cos(\phi_i(t))+r_N(t)$.

In this study we have used the Matlab programs from http://rcada.ncu.edu.tw/research1_clip_program.htm.

Applications

1. Synthetic Time Series

The HHT-EMD outlined above was applied to three time series similar to the ones used in Castagna et al., (2006), Charkaraborty et al., (1995), and Miao et al. (2005). Figure 1 shows that the HHT-EMD provides superior results to complex wavelet transform (Charkaroborty et al., 1995) and S-transform in terms of temporal and spectral resolution. For display purpose, a Gaussian weighted filter was applied to the spectrum skeleton directly obtained from HHT. In the complex wavelet transform, the scale is converted to pseudo-frequency by a matlab internal function 'scal2frq' based on $F_a=F_c/(a\cdot\Delta)$, where a is a scale, Δ is the sampling period, F_c is the center frequency of a wavelet in Hz, and F_a is the pseudo-frequency corresponding to the scale a, in Hz. Due to the choice of time series, the S-transform did not show significant improvement over wavelet transform.

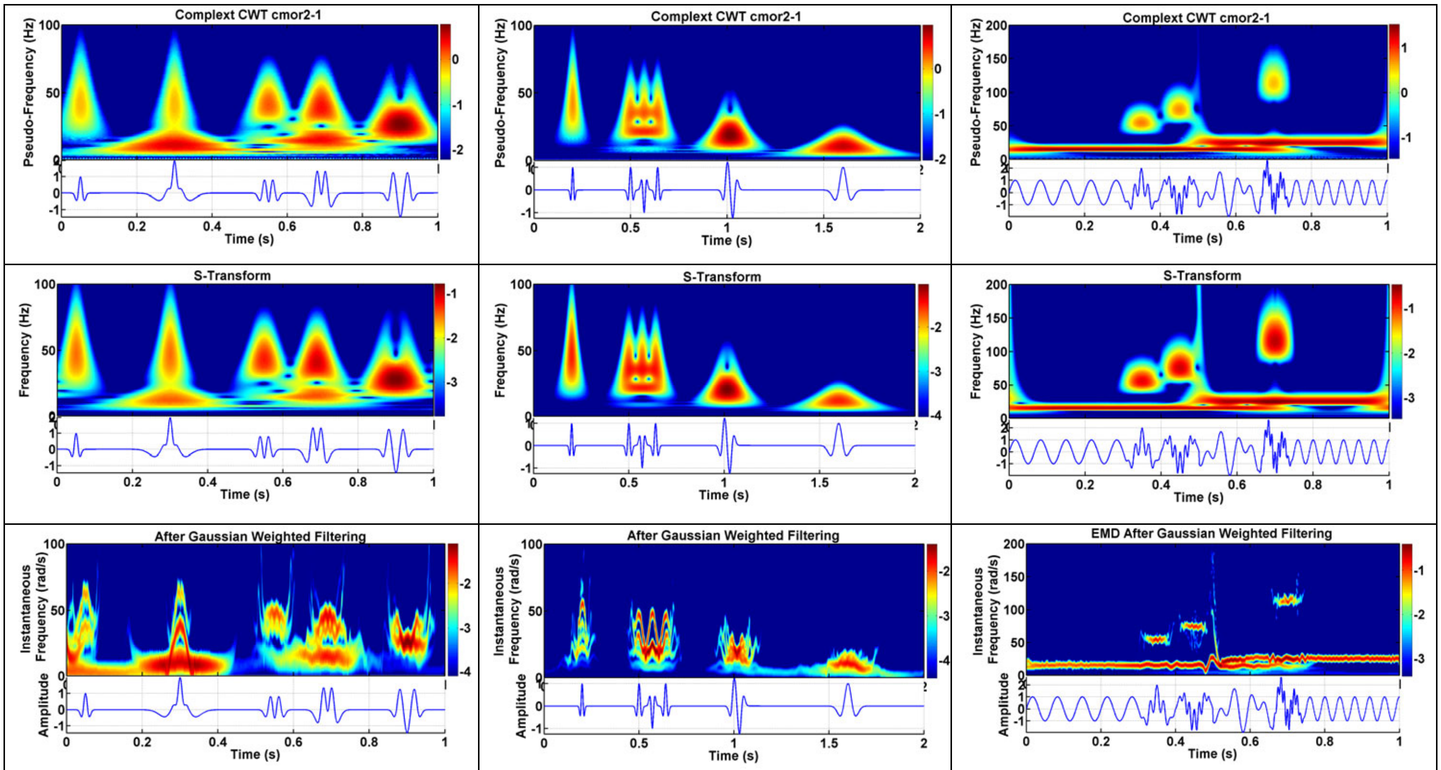

Figure 1. HHT-EMD was applied to three time series samples (third row) and compared with complex wavelet transform (first row), S-transform (second row). HHT-EMD demonstrated its high temporal and spectral resolution. In addition, for HHT-EMD without the average effect in a window, no significant edge effect can be observed and the energy amplitude has absolution meaning. The Gaussian weighted filter was applied to HHT-EMD spectrum for display purpose. The color code represents the logarithm of the power.

2. Well logs

Huang et al., (2008) proposed to simulate a heterogeneous earth medium assuming that it was piece-wise stationary. However, in reality the geology usually varies much more rapidly along the vertical direction than the lateral directions. The stationary assumption that implicitly applied the stochastic parameters estimated along the borehole direction to the other two dimensions may be too harsh. To include this reality, we propose to use an available instantaneous spectrum analysis technique to project 1D well logs to wave-number and depth in 2D domain, thus obtaining a depth varying Power Spectrum Density Function (PSDF). Then at each depth, a stationary 2D slice can be simulated and the lateral continuity can be reserved by using the same set of random series. As an example, figure 2 shows the expression of borehole sonic velocity logs from Mallik 5L-38 (Northwest Territory of Canada) in the plane of wave-number and depth using complex continuous wavelet transform (a), S-transform (b), and HHT-EMD (c). The decomposed intrinsic mode functions including the identified trend (IMF 10) are also shown in figure 2. The Mallik 5L-38 covers two strata: fluid saturated sediment (670m – 889m) and hydrated sediment (890m – 1115m). The occurrence of hydrates corresponds to increasing of the power spectrum. The usage of windows in complex wavelet transform and S-transform introduced the artificial increase of power spectrum at the edge. In figure 3(a), the instantaneous PSDF based on HHT-EMD at depth of 916m showed a von-Karman style and according to the modeling algorithm of Huang et al. (2008), the standard deviation (σ), vertical correlation length (a) and Hurst number (ν) can completely specify a stochastic model. If external information on lateral correlation length (a_x and a_y) is known, one time slice of heterogeneous model at that depth can be simulated. In our case, four difference geological situations were simulated with various lateral correlation lengths: 10m, 100m, 1000m, and 40.8m, respectively (Figure 3). Physical rock properties such as the porosity and the amount of hydrocarbon can be estimated. The bimodal probability distribution function for gas hydrates (Huang et al. 2008) was adopted and the background trend (the 10th IMF) was added. The combination of slices at every depth yields the full 3D heterogeneous model which is adaptive to the non-stationary along the borehole direction but stationary in the orthogonal plane.

Conclusions

The Hilbert-Huang Transform based on Empirical Mode Decomposition was applied to three time-series. The comparison with complex wavelet transform and S-transform demonstrated that the HHT-EMD can produce superior results. HHT-EMD was then applied to decompose well logs and the instantaneous PSDF provided the depth varying stochastic properties which can be used to simulate a time slice of heterogeneous medium at every depth. The combination of the 2D slices yields a heterogeneous 3D earth model adaptive to the non-stationary along the borehole.

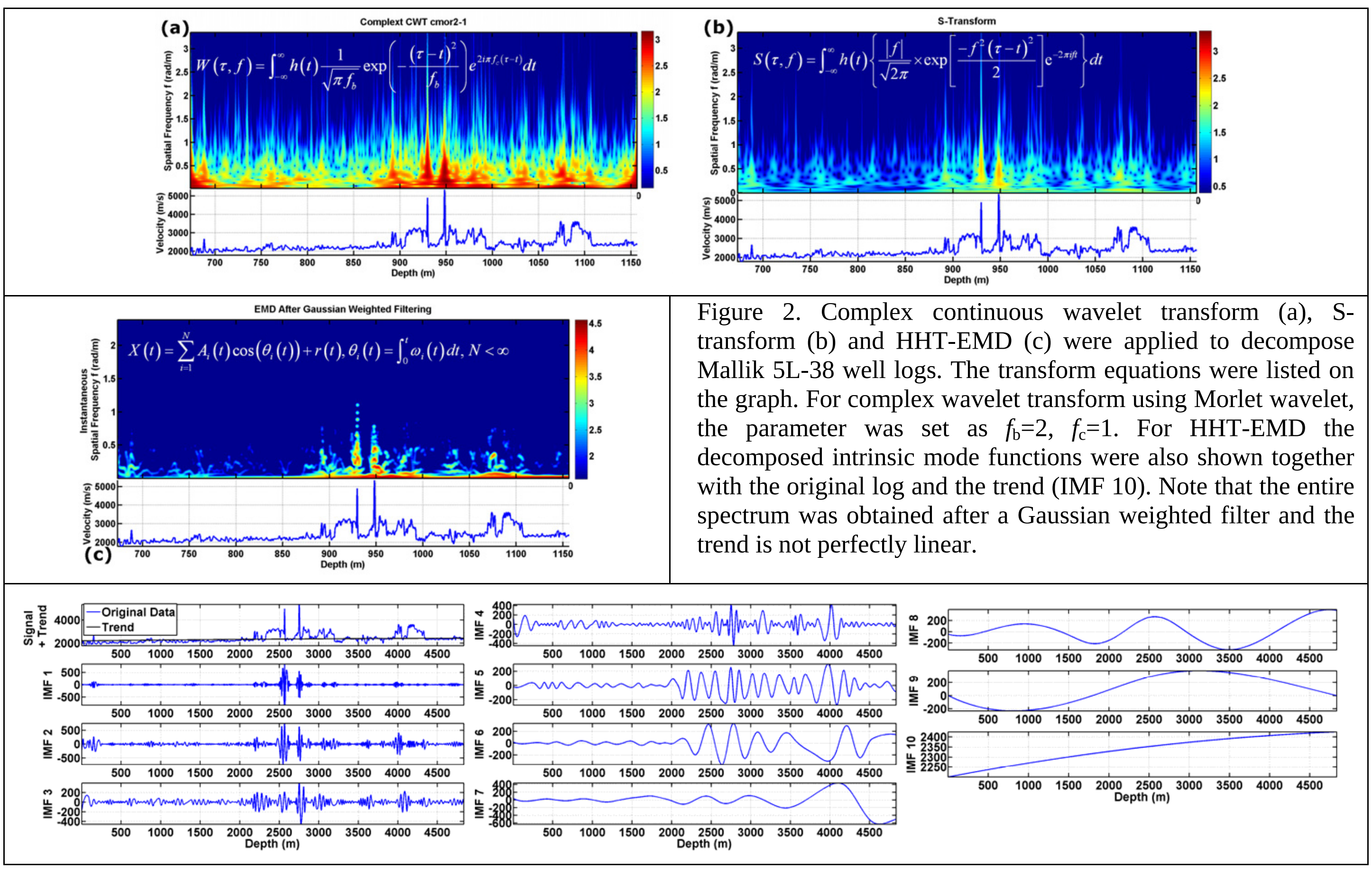

Figure 2. Complex continuous wavelet transform (a), S-transform (b) and HHT-EMD (c) were applied to decompose Mallik 5L-38 well logs. The transform equations were listed on the graph. For complex wavelet transform using Morlet wavelet, the parameter was set as f_b=2, f_c=1. For HHT-EMD the decomposed intrinsic mode functions were also shown together with the original log and the trend (IMF 10). Note that the entire spectrum was obtained after a Gaussian weighted filter and the trend is not perfectly linear.

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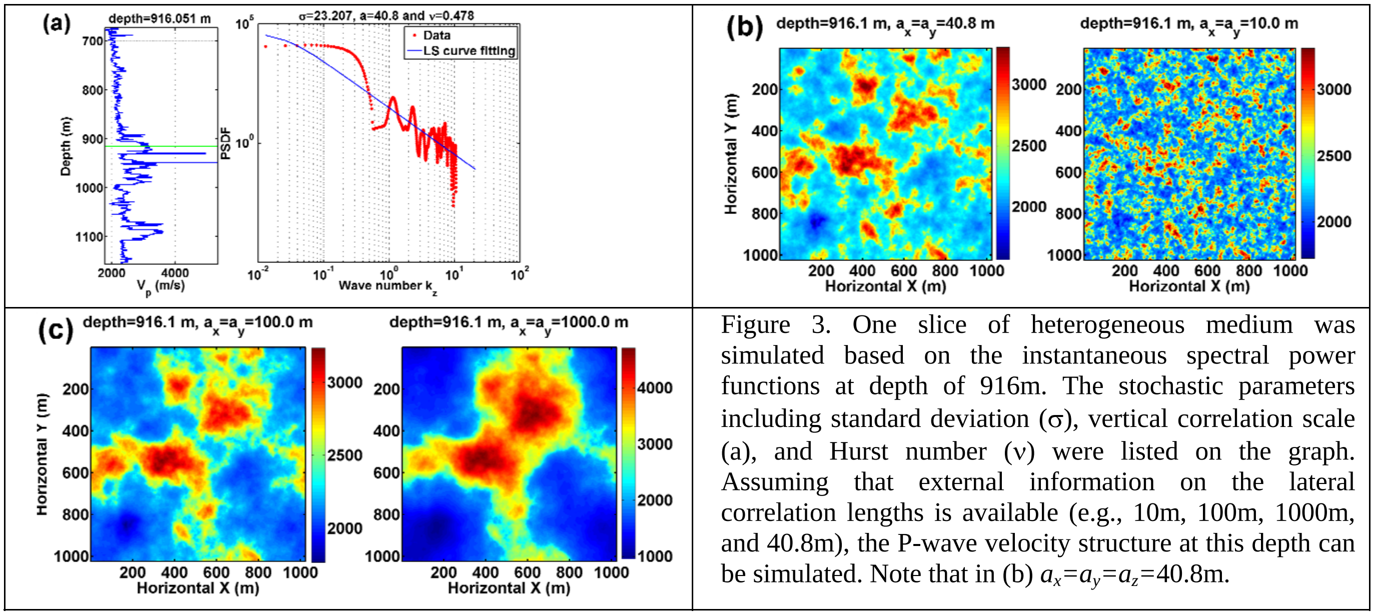

Figure 3. One slice of heterogeneous medium was simulated based on the instantaneous spectral power functions at depth of 916m. The stochastic parameters including standard deviation (σ), vertical correlation scale (a), and Hurst number (ν) were listed on the graph. Assuming that external information on the lateral correlation lengths is available (e.g., 10m, 100m, 1000m, and 40.8m), the P-wave velocity structure at this depth can be simulated. Note that in (b) $a_x=a_y=a_z=40.8$m.

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