P-Wave Impedance, S-Wave Impedance and Density from Linear AVO Inversion: Application to a VSP Data from Alberta

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Summary

In presented AVO (amplitude variation with offset) inversion the amplitudes of compressional and converted shear seismic data are inverted both separately and jointly to provide three parameters; compressional impedance, shear impedance, and density. The approximated Zoeppritz equation is least-squares fitted to the amplitude of all traces of a common-mid point gather (PP data) and a common-converted point gather (PS data) at each depth sample to obtain the band-limited reflectivity of the three parameter traces. Then, the reflectivity traces are integrated to obtain the three parameter traces with the missing low-frequency components provided from well log information.

The three parameters, especially the density, cannot accurately be resolved from AVO data due to the ill-posed nature of the inverse problem. The damped SVD (singular value decomposition) method has been utilized to stabilize the AVO inversion. The examination of the resolution matrix, after adding a damping factor, demonstrates that the shear velocity contributes more than the compressional velocity to improving the density estimate for the study area data namely, VSP data from a Red Deer coal bed methane site.

In the joint inversion, the converted shear wave data dominates in estimating the shear impedance and density and appears promising in providing shear impedance and density estimates from the PS inversion alone. In addition, in the joint inversion the compressional data dominates in estimating the compressional impedance and provides a good estimate for the compressional impedance in the PP inversion alone.

Methodology

The Zoeppritz equations describe the elastic, plane-wave reflection and transmission coefficients. The Aki-Richards (1980) linear approximations for PP and PS reflection coefficients, R_{PP} and R_{PS}, can be formulated as a function of density, ρ, P-impedance, I, and S-impedance, J, the resulting expressions are

$$R_{PP} = A\frac{\Delta I}{I} + B\frac{\Delta J}{J} + C\frac{\Delta\rho}{\rho},$$

(1)

$$R_{PS} = E\frac{\Delta J}{J} + D\frac{\Delta\rho}{\rho},$$

(2)

where the coefficients A, B, C, D and E are functions of the average P-wave incident angle, the average S-wave reflected angle, and the ratio of S-velocity to P-velocity across the interface. Assuming that the PP and PS reflection data provide estimates of R_{PP} and R_{PS} for a range of source-receiver offsets, the Aki-Richards approximations for different offsets, at a particular depth under consideration, can be used to express a linear system of $2n$ linear equations (n being the number of offsets) with three unknowns as

$$\begin{bmatrix} A_1 & B_1 & C_1 \\ & \vdots & \\ A_n & B_n & C_n \\ 0 & E_1 & D_1 \\ & \vdots & \\ 0 & E_n & D_n \end{bmatrix}_{2n*3} \begin{bmatrix} \dfrac{\Delta I}{I} \\ \dfrac{\Delta J}{J} \\ \dfrac{\Delta\rho}{\rho} \end{bmatrix} = \begin{bmatrix} R_{PP1} \\ \vdots \\ R_{PPn} \\ R_{PS1} \\ \vdots \\ R_{PSn} \end{bmatrix}.$$

(3)

The above is also a matrix equation which can be written symbolically as $Gm = d$, where G is the matrix of known coefficients, m the unknown parameter vector containing $[\Delta I / I \quad \Delta J / J \quad \Delta\rho / \rho]$, and d the reflection data vector. This system of equations is solved by SVD method, to obtain band-limited impedances and density reflectivity. Then, the estimated band-limited reflectivity traces are integrated to I, J and ρ, with the low frequency components provided from well logs, using a MATLAB routine called BLIMP (Mahmoudian and Margrave, 2003).

3-parameter linear AVO inversion is an ill-posed problem. In an ill-posed problem a small change in data will cause a large change in solutions. This is mainly due to the nonlinearity of the problem and limited data acquisition aperture. To overcome this problem the SVD method is used.

SVD Analysis

For a general matrix G, there is always a matrix decomposition called the SVD of matrix G. SVD allows the matrix G to be expressed as the product of three matrices $G = U\Lambda V^T$ (Menke, 1989), where U is the matrix of eigenvectors of GG^T, and V is the matrix of eigenvectors of the G^TG, both related to the non-zero singular values. The singular values of the matrix G are the positive square roots of the eigenvalues of the matrix G^TG. Λ is a matrix with the non-zero singular values of the matrix G in its main diagonal elements in a decreasing order.

The generalized inverse of matrix G is defined as $G_g^{-1} = V\Lambda^{-1}U^T$. The only potential difficulty in using SVD is when inverting a matrix that possesses some very small singular values. One solution to this problem is to include all the singular values while damping the smaller ones. This change has little effect on the larger eigenvalues but prevents the smaller ones from leading to large variances. The damped generalized inverse is defined as $G_g^{-1} = V\Lambda(\Lambda^2 + \varepsilon^2 I)^{-1}U^T$. The precise value of damping factor must be chosen by a trial-and error process which weighs the relative merits of having a solution with small variance against one that is well resolved (Menke, 1989). The matrix $G_g^{-1}G$ is

called the model resolution matrix. The model resolution matrix defines how well the estimated solution resolves the true solution.

In SVD analysis of the AVO inversion, the condition number has been examined as an indicator of the singularity of the matrix. The condition number is the ratio of the largest to the smallest singular values of a matrix. A matrix is well-posed when its condition number is not far from 1 (Jin at al., 2002), and an ill-posed matrix is a matrix with very large condition number.

AVO Inversion Testing: Synthetic Surface Seismic Data

The AVO inversion results for synthetic data from a well log, from Blackfoot field are presented. The high condition number from all three inversions indicates that the AVO inversions are ill-posed and unable to provide good estimates for all three parameters especially density. Figure 1 shows the joint inversion results, with 10% of the largest singular value added to the singular values as a damping factor. By adding a damping factor the I and J estimates are not changed noticeably; however the density estimate is considerably improved.
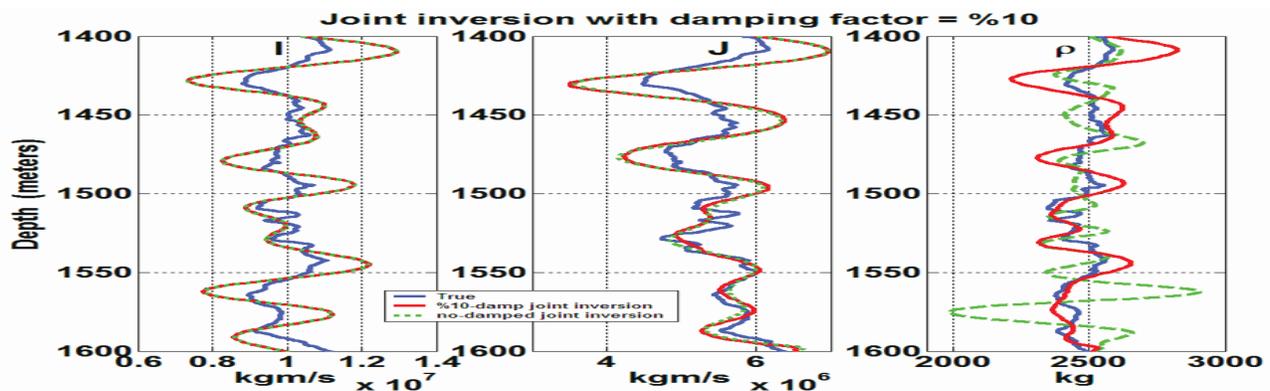
Figure 1. The three parameter estimate from joint inversion, with no damped SVD and %10 damped SVD, of Blackfoot synthetic data.

AVO Inversion of VSP Data: Case Study

The AVO inversions have been applied to a walkaway VSP data acquired at Red Deer, Alberta. At this location, Suncor Energy Inc., with industry partners, and the Alberta Research Council were evaluating the Ardley coal zone for its coal bed methane potential. The Ardley coal zone is 11.7 m thick, located at a depth of 284 m below surface. Coal has a low seismic velocity and low density with respect to its boundary strata. The deconvoleved upgoing wavefield in two way time is the input to the AVO inversion. The four walkaway shot points east of the borehole were at the offsets: 100 m, 150 m, 191 m, and 244 m from the borehole.

The high condition number indicates that the AVO inversions are ill-posed for most of the depths, especially for the Ardley coal zone at a depth of 284-300 m (Figure 2). Therefore, none of the three inversions could result in favorable estimates, at least for the third parameter; also the estimates plots confirm this. To examine the damping factor effect on the AVO inversion estimates, a damping factor, ε, varying from 0 to 9 percent is applied to the joint inversion of the walkaway offset 3 data (Figure 3); the error of the ρ estimate has been lowered by the SVD damping, especially at the Ardley coal zone.
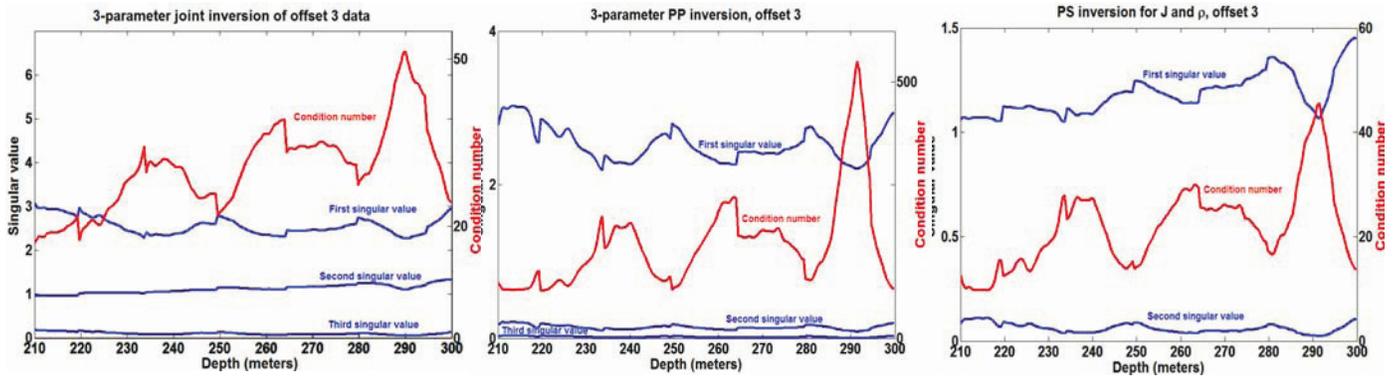
Figure 2. The condition number (in red) versus depth from the PP, PS and the joint inversion of walkaway offset3 data.

However, there is a corresponding decrease in resolution with a decrease in error of the estimates. For a perfect resolution, the resolution matrix should be an identity matrix. By increasing the damping factor the resolution matrix will deviate more from the identity matrix and the less resolution is achieved. The third row of the resolution matrix suggests that for the study area the S-velocity contributes to improving the density estimate more than the P-wave velocity. To decide about the damping factor, the maximum correlation between the estimates and the true model parameters (calculated from well logs) is examined. The maximum correlation investigation suggests that a damping factor equal to 3% has the best correlation between the model parameter estimates and the true values.
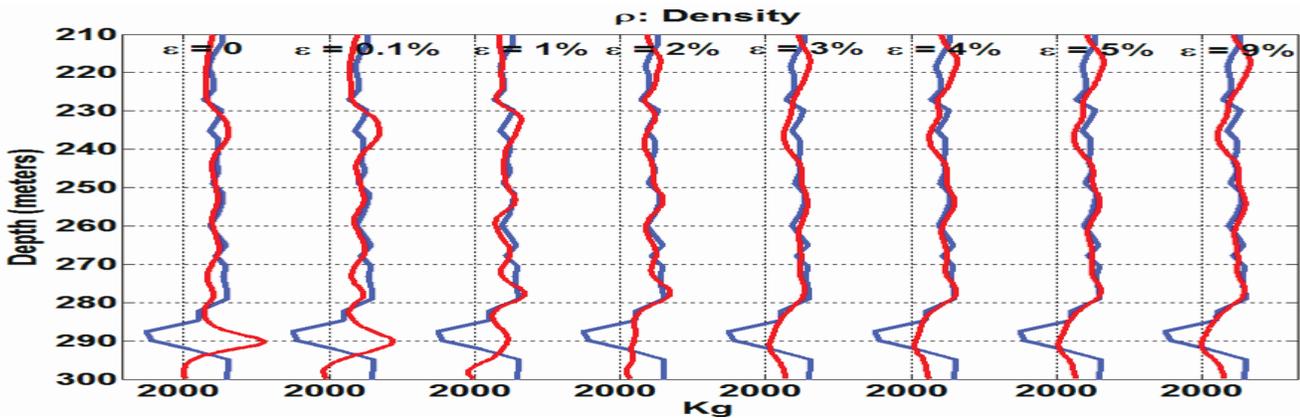
Figure 3. The density estimate from the joint inversion of walkaway offset 3, with various SVD damping factors. The blue curves are values from the well logs, and the red curves are estimates from the joint inversion. ε varies from 0 to 9 percent from left to right.

Figure 4 shows the *I*, *J* and ρ estimates obtained from the PP, PS and joint inversion of the walkaway VSP data, with 3% damping factor; the joint inversion provides *I* estimate similar to those of the PP inversion; also the joint inversion produces the *J* and ρ estimates almost identical to those obtained from the PS inversion.
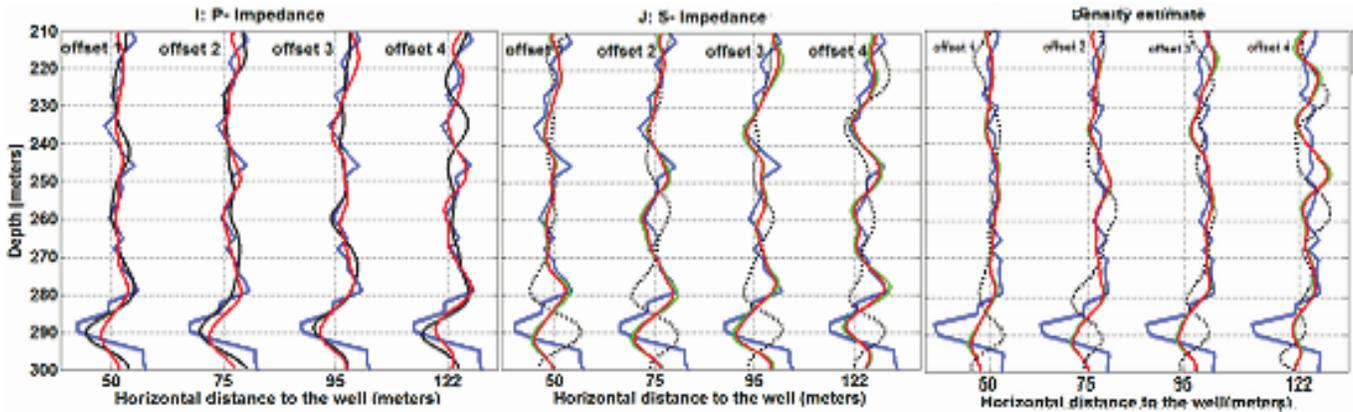
Figure 4. The I,J and ρ estimate from the AVO inversions of Red Deer VSP data. The blue curves are the real values calculated from the well logs, and the red, black and the green curves are estimates by the joint, PP and PS inversions respectively.

Conclusion

The 3-parameter PP, joint inversion and the PS inversion (for the *J* and *ρ*) of Red Deer VSP data are ill-posed problems especially for the Ardley coal zone. Therefore, the SVD damping method is utilized. The damping factor contributes to the stability of the inversion by suppressing the effect of small singular values. The examination on the resolution matrix demonstrates that the S-velocity contributes to improving the density estimate more than the P-velocity for the study area. The joint inversion provides the *I* estimate similar to those of the PP inversion; also the joint inversion produces the *J* and *ρ* estimates similar to those obtained from the PS inversion.

References

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