Implications from Transfer Functions when Comparing Seismic Data from MEMS Accelerometers and Geophones

Michael S. Hons*
University of Calgary, Calgary, AB, Canada
msjhons@ucalgary.ca

and

Robert R. Stewart
University of Calgary, Calgary, AB, Canada

Summary

Digital sensors based on micro electro mechanical systems (MEMS) accelerometers are one of the newest technologies being used in seismic acquisition. As such, there remains some confusion surrounding the similarities and differences relative to the coil-over-magnet geophone. By modeling the transfer functions of these devices and convolving displacement domain wavelets of varying frequency content with them, it will be demonstrated that raw electrical signals output by the sensor elements are expected to be similar in appearance. Also, the dominant frequency of the wavelet relative to the geophone's resonant frequency determines whether the MEMS accelerometer result is an apparent phase lead or phase lag relative to the geophone result.

Introduction and Theory

A transfer function is given by the form

$$\frac{B}{A} = H ,$$

where B is the output, A is the input and H is the transfer function. When the transfer function operates on the input, the output is obtained. Both seismic sensors are based on an inertial mass held by a spring. They are both single-degree-of-freedom damped harmonic oscillators and are thus governed by the equation

$$\frac{\partial^2 x}{\partial t^2} + 2\lambda\omega_0 \frac{\partial x}{\partial t} + \omega_0^2 x = -\frac{\partial^2 u}{\partial t^2}$$

in the presence of a driving signal, where u is the ground displacement (i.e., driving signal), x is the displacement of the inertial (i.e., proof) mass, λ is the damping ratio of the sensor and ω_0 is the resonant frequency of the spring-mass system.

Any spring-mass system responds differently depending on what frequency is used to drive it. When driving frequencies are very low relative to the resonant frequency, the spring appears to be very tight and the proof mass only displaces away from its undriven hanging position under acceleration. Here we can write $x \propto (d^2u)/dt^2$, when $\omega << \omega_0$. When driving frequencies are near the resonant frequency, the proof mass displacement is related to the ground velocity, though in a complicated manner fully described by the damped simple harmonic oscillator equation. This is the reason a geophone is referred to as a 'velocimeter', and a MEMS is an 'accelerometer'.

The ground motion is transformed into an electrical signal at the transducer. By stating the input to the transducer in terms of ground motion (as above), and the electrical output of the transducer in terms of the proof mass displacement, we can fashion a transfer function directly from the damped simple harmonic oscillator equation.

Geophones use magnetic induction to transform the proof mass motion into an electrical signal. For this reason, the proof mass velocity will be referred to as the 'unscaled' electrical output from a geophone transducer. MEMS accelerometers use capacitors, so in this case the proof mass displacement is the 'unscaled' electrical output from a MEMS transducer. Some MEMS sensors use force feedback to the capacitor to prevent the proof mass from moving any substantial amount. This does not change the fact that capacitors sense displacement change.

By taking the Fourier transform and rearranging the damped simple harmonic oscillator equation accordingly, we arrive at transfer functions for a geophone

$$H_G = \frac{\frac{\partial X}{\partial t}}{\frac{\partial U}{\partial t}} = \frac{\frac{\omega^2}{\omega_0^2}}{-\frac{\omega^2}{\omega_0^2} + \frac{2j\lambda\omega}{\omega_0} + 1}$$

and for a MEMS accelerometer

$$H_A = \frac{X}{\frac{\partial^2 U}{\partial t^2}} = \frac{-\frac{1}{\omega_0^2}}{-\frac{\omega^2}{\omega_0^2} + \frac{2j\lambda\omega}{\omega_0} + 1}.$$

These equations are valid over the frequency band for which they were derived, which means that the MEMS accelerometer transfer function reduces to

$$H_A = \frac{X}{\frac{\partial^2 U}{\partial t^2}} \cong -\frac{1}{\omega_0^2},$$

since $\omega << \omega_0$. Example transfer functions for a 10 Hz geophone and 1000 Hz MEMS accelerometer, each with 0.7 damping ratio, are shown in Figures 1 and 2.

Modeling

Running a wavelet through a sensor is the same as convolving the wavelet with the sensor transfer function. In the following examples, the input wavelet represents ground displacement, and so the time derivative (velocity wavelet) is convolved with the geophone transfer function, and the double time derivative (acceleration wavelet) is convolved with the MEMS accelerometer transfer function.

Figure 3 shows the result of time differentiating an input wavelet and convolving it with a geophone transfer function. A simple phase rotation of this result produces a symmetrical 'zero-phase' appearance (Figure 4), which very closely resembles the result of double time differentiation and convolution with a MEMS accelerometer transfer function (Figure 5). Further modeling has shown that wavelets with low dominant frequencies must be phase rotated the opposite direction to resemble 'zero-phase', and at some intermediate dominant frequency, determined by the resonant frequency of the geophone, the geophone and MEMS outputs will be nearly identical.

Conclusions

Over a common seismic exploration bandwidth (~10-100 Hz), the raw electrical output from the sensor elements of a geophone and a MEMS accelerometer is expected to be similar. The results should be comparable apart from a small phase rotation, less than 90 degrees. The dominant frequency of the signal and the geophone resonant frequency have a bearing on whether the phase rotation is positive or negative. A dominant frequency exists where the sensor element output over this bandwidth from a geophone and a MEMS accelerometer will be nearly identical.

References

Keller, F.; 2006; Seismometer Documentation; TU Clausthal;
http://www.ifg.tu-clausthal.de/java/seis/seis_doc-e.html

Meirovitch, L; 1975; Elements of Vibration Analysis; McGraw-Hill; p. 52-55

Havskov, J and Alguacil, G; 2006; Instrumentation in Earthquake Seismology, 1st edition; Springer; p. 25-27

Longoria, R. F.; 2000; Dynamic Systems and Controls Lab Notes; Department of Mechanical Engineering, University of Texas (Austin)
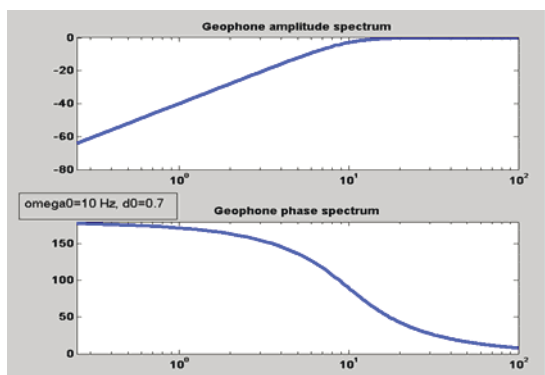
Figure 1. Amplitude and phase spectra of the geophone transfer function. Resonant frequency is 10 Hz and damping ratio is 0.7.
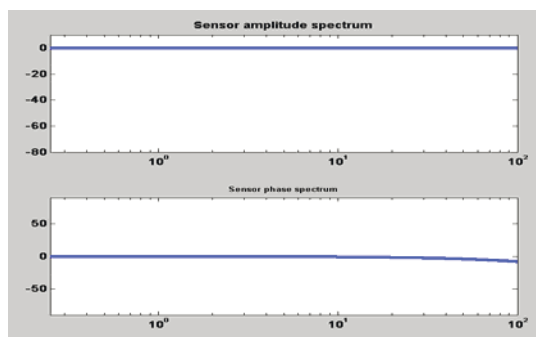

Figure 2. Amplitude and phase spectra of the accelerometer transfer function. Resonant frequency is 1000 Hz, and damping ratio is 0.7.
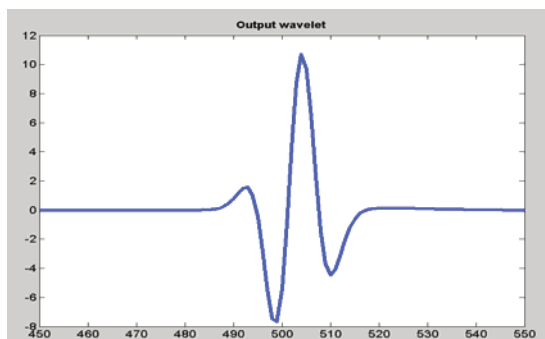

Figure 3. Ricker wavelet, 30 Hz dominant frequency, after differentiation and convolution with a 10 Hz geophone, 0.7 damping ratio
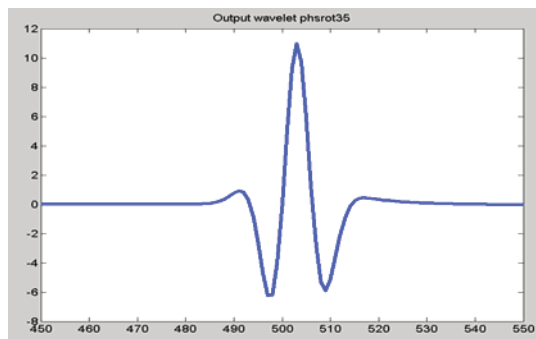

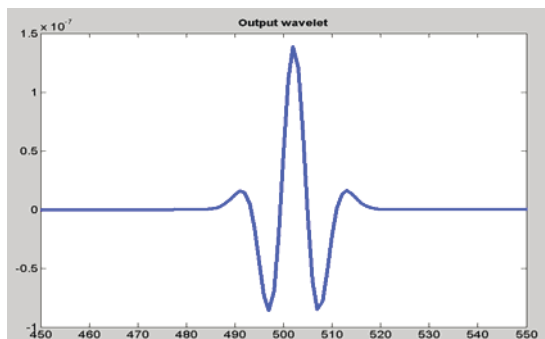**Figure 4.** Result from Figure 3 phase rotated by 35 degrees

Figure 5. Ricker wavelet, 30 Hz dominant frequency, double differentiated and convolved with a 1000 Hz MEMS, 0.7 damping ratio.