

Numerical Simulation of Fluid Pressure and Fracturing in CO₂ Sequestration*

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Abstract

CO₂ sequestration in underground aquifers has the potential to reduce atmospheric CO₂ emissions on the order of hundreds of gigatons. However, rock fractures, formed during injection, may release toxic species into the water table and release CO₂ into the atmosphere. A model to compute the internal rock stresses induced by the injection of CO₂-rich water is developed to simulate the initiation and propagation of rock fractures. A mixed finite element-finite volume model is used to calculate the fluid pressure, stresses, and strains induced by injection of CO₂ into a geologic saline aquifer-caprock formation. The Terzaghi effective stress is determined from the calculated overburden pressure and fluid pressure. When the Terzaghi effective stress exceeds the rock tensile stress at a given depth, simulated fractures are induced. The effect of the simulated fractures is upscaled to the reservoir scale by estimating an incremental permeability by means of Oda's permeability tensor. The effect of fractures on the mineralization of CO₂ is simulated by estimating the impact of fractures on the free surface area of the solid, rock phase. The free surface area impacts the dissolution and precipitation of carbonate minerals. We examine the optimal rate of fracturing to maximize the amount of CO₂ sequestered while minimizing the risk of damaging caprock integrity, and minimizing transport of CO₂ through the caprock layer. This poroelastic pressure and fracture module is used to approximate formation pressure during injection of CO₂ rich water into the Oligocene Frio Formation along the Texas Gulf Coast, with simulation parameters derived from the Frio Test Pilot Experiment. Simulation results are compared to bottom-hole pressure data obtained from an observation well 30 meters away from the injection well, during a 35-day monitoring phase beginning with four days of CO₂ injection.

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Anthropogenic CO₂ Sources



Chemical manufacturing
(cement, ethanol, metal
production, waste combustion)



Petroleum and natural gas
production



Coal-fired electric power



Agriculture and fertilizer
production

Hydraulic Fracturing in Carbon Sequestration

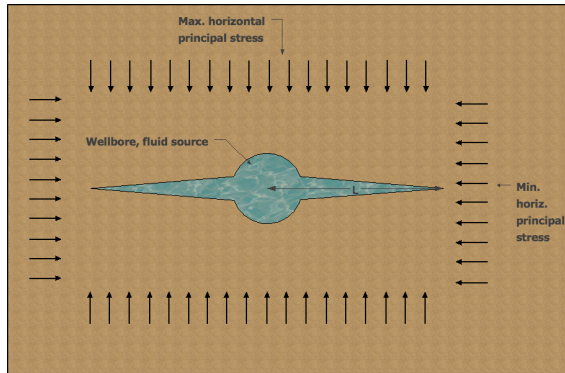
- Reservoir stimulation, also known as hydraulic fracturing, can be used to increase permeability in underground reservoirs
- Depleted oil and natural gas reservoirs, many of which have already been stimulated, show promise as potential CO₂ sequestration sites

Effects of Fracturing on CO₂ Sequestration

- Mechanical trapping of CO₂ by mineralization of carbonate species can keep CO₂ securely stored in an immobile solid phase
- Minerals hold the strongest promise for long-term CO₂ sequestration
- Cohen and Rothman (2015) have shown that this mineralization may be limited by the formation of a “carbonate-encrusted bubble”
- This simulator aims to assess the effects of hydraulic fracturing on mineralization, by combining reactive transport and fracture modeling

Mechanism of Hydraulic Fracturing

Fractures expected parallel to the axis of the injection well extending radially from the wellbore. The plane of the fracture is perpendicular to the maximum horizontal principal stress (minimum compressive stress).



Breakdown Pressure Modeling

- Hubbert and Willis pioneered the theoretical models for the pressure required to induce a hydraulic fracture. (Hubbert and Willis, 1972)
- This pressure required to initiate a fracture is known as the breakdown pressure.
- Haimson and Fairhurst expanded on the Hubbert-Willis model to include poroelastic effects in the calculation. (Haimson and Fairhurst, 1967)

Notation

Symbol	Property	SI Units
p	Fluid pressure	Pa
p_c	Hydraulic fracturing breakdown pressure	Pa
σ_{ii}	Principal stress in the i direction	Pa
σ	Mean stress	Pa
T	Tensile strength of Rock	Pa
k	Permeability	m ²
ϕ	Porosity	unitless
μ	Dynamic viscosity	Pa-sec
ν	Poisson's Ratio	unitless
α	Biot-Willis coefficient	unitless
η	Poroelastic stress coefficient	unitless
S	Specific storage coefficient	Pa ⁻¹

Breakdown Pressure Models

Hubbert-Willis Model

$$p_c - p_0 = T - (3\sigma_h - \sigma_H)$$

Haimson-Fairhurst Model

$$p_c - p_0 = \frac{T - (3\sigma_h - \sigma_H) - 2\eta p_0}{2(1 - \eta)}$$

Here, p_0 is the in situ fluid pressure and σ_h and σ_H are the minimum and maximum principal horizontal stresses respectively.

Note: The Hubbert-Willis breakdown pressure is the special case of the Haimson-Fairhurst model where $\eta = 0.5$.

Calculation of Initial and Boundary Conditions

- Vertical confining stress on the reservoir

$$\sigma_{zz} = - \int_z^0 z' g \rho_s dz'$$

where z is the depth of the upper boundary of the reservoir

- Hydrostatic fluid pressure

$$p_{\text{hstat}} = - \int_z^0 z' g \rho_w dz'$$

- Initial pore pressure initialized to match monitoring well measurements from Frio pilot test data.

Initiation of Fractures

- At the breakdown pressure, the effective tangential stress in the rock bordering the wellbore reaches the tensile strength of the rock.
- Fractures occur when the pore pressure at the well bore exceeds the breakdown pressure extending radially from the wellbore wall.

Reactive Transport in Porous Media

$$\phi \frac{\partial c_{\alpha}}{\partial t} = \phi D_{\alpha} \nabla^2 c_{\alpha} - \phi \vec{\nabla} \cdot (c_{\alpha} \vec{u}) - \sum_{\gamma=1}^M \nu_{\beta\gamma} \rho_{\gamma} A_{\gamma} G_{\gamma}$$

Where,

ϕ - porosity

c_{α} - chemical species activity

D_{α} - diffusion coefficient

\vec{u} - water flow velocity

$\nu_{\beta\gamma}$ - stoichiometric coefficient

ρ_{γ} - density

A_{γ} - surface area

G_{γ} - reaction rate

Effect of Fracture on Reactive Transport

The occurrence of fractures within a reference elementary volume, or REV, expose more of the solid phase to the pore fluid.

The reactive transport model uses growing and shrinking spherical grains to model the solid phase and their surface area exposed to reaction.

A fracture coefficient C_{frac} is applied to the surface area of the grains of each solid phase species, sp .

$$A_{\text{eff}}^{sp} = C_{\text{frac}} A^{sp}$$

C_{frac} formula

$$C_{\text{frac}}^{sp} = \frac{A_{\text{frac}} VF^{sp}}{A^{sp}}$$

Here A_{frac} is the area of the fracture within the REV, VF^{sp} is the volume fraction of the solid species sp within the REV, and A^{sp} is the reactive surface area of the species within the REV before fracturing.

Effects on Permeability

The incremental permeability due to fractures is determined using the permeability tensor from Oda.

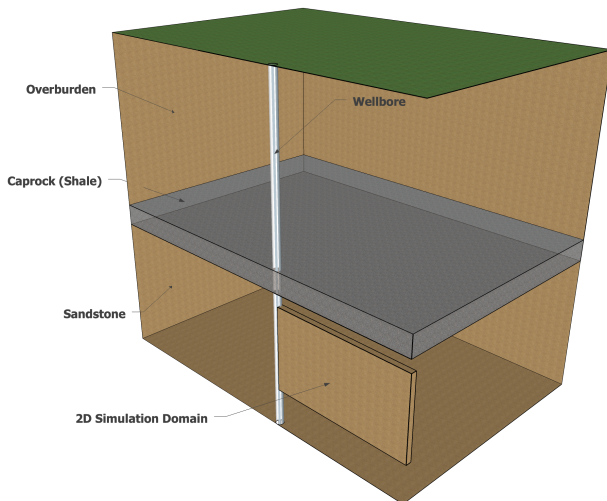
$$k_{ij} = \lambda(P_{kk}\delta_{ij} - P_{ij}), \quad (1)$$

where P_{ij} is the “crack tensor”:

$$P_{ij} = \frac{\pi\rho}{4} \int_0^{r_m} \int_0^{t_m} \int_{\Omega} r^2 t^3 n_i n_j E(\vec{n}, r, t) d\Omega dt dr \quad (2)$$

Simulation Configuration

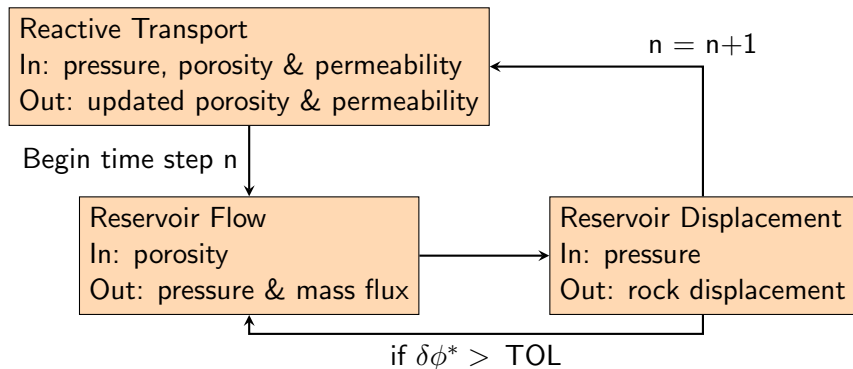
2-D finite volume simulation domain:



Pressure, flux and stress calculations

Following an approach outlined by Ganis, et al. (2014) for the calculation of the fluid flux, pressure profile and stresses in the reservoir between reactive transport time steps. The iterative scheme calculates the pressure, fluid flux and displacements, using constitutive relations for fluid density, porosity and stress iterating until the system is in quasi-equilibrium for each time step.

Iterative scheme



Reservoir flow equations

Given a reservoir domain, Ω , reservoir flow in the porous medium is governed by the following equations.

Darcy Flow (mass flux):

$$v_{\text{res}} = -\frac{K}{\mu} \rho_0 (\nabla p_{\text{res}} - \rho g \nabla z)$$

Fluid Conservation:

$$\partial_t(\rho_0 \phi) + \nabla \cdot v_{\text{res}} = q_{\text{res}}$$

Strain and stress in a homogeneous isotropic material

Displacement vector is denoted u .

Strain:

$$\epsilon = \frac{1}{2} (\nabla u + \nabla u^T)$$

Stress-strain relationship in the material:

$$\sigma(u) = \lambda(\nabla \cdot u)I + 2G\epsilon(u)$$

Poroelastic Stress:

$$\sigma_{\text{por}}(u, p) = \sigma(u) - \alpha p I$$

Reservoir displacements

Under a quasi-static assumption, the body force due to gravity, f , is balanced by the divergence of the reservoir stress tensor.

$$-\nabla \cdot \sigma_{\text{res}}(u_{\text{res}}) = f.$$

Here, the reservoir stress is computed using the poroelastic stress relationship

$$\sigma_{\text{res}} = \sigma(u_{\text{res}}) - \alpha p I$$

Reservoir boundary conditions

We can impose boundary conditions on the reservoir by decomposing the boundary $\partial\Omega$ into sections with Dirichlet boundary conditions Γ_D and Neumann conditions Γ_N .

$$\partial\Omega = \Gamma_D \cup \Gamma_N$$

The Dirichlet conditions may represent an immovable boundary

$$u_{\text{res}} = 0,$$

where the Neumann conditions may represent the pressure of the overburden on the reservoir as a traction vector

$$\sigma_{\text{res}} n = t_N.$$

In this simulation, we assume a closed system with no mass flux across the boundary ($\partial\Omega$),

$$v_{\text{res}} \cdot n = 0.$$

Mixed Finite element formulation: Reservoir flow

The domain is discretized by decomposition into quadrilateral elements.

Let V_h and W_h be the mixed finite element space over the quadrilateral elements with vector and scalar elements respectively. Find the flux and pressure functions $(v_{\text{res}}, p_{\text{res}}) \in V_h \times W_h$, such that

$$\begin{aligned} \int_{T_h} \mu \rho_0^{-1} K^{-1} v_{\text{res}} \cdot \psi dV - \int_{T_h} p_{\text{res}} \nabla \cdot \psi dV - \int_{T_h} \rho g \nabla z \cdot \psi dV \\ = 0, \forall \psi \in V_h, \\ - \int_{T_h} \phi^* \rho \chi dV - \delta t \int_{T_h} \nabla \cdot v_{\text{res}} \chi dV - \int_{T_h} (\phi^* \rho^{n-1} - \delta t q_{\text{res}}) \chi dV \\ = 0, \forall \chi \in W_h. \end{aligned}$$

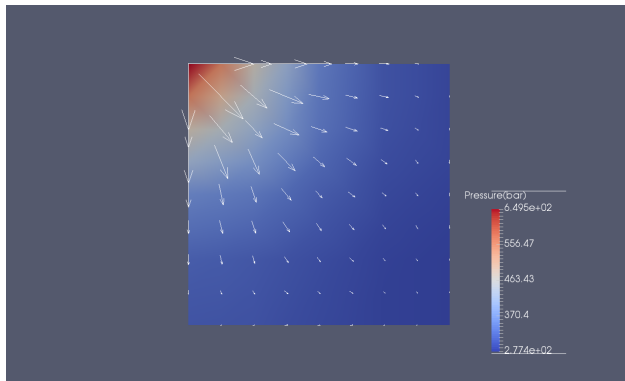
Finite element formulation: Reservoir displacement

Let X_h be the Galerkin finite element space with two-dimensional vector elements that are piecewise linear over the quadrilateral elements. Find the reservoir displacement vector u_{res} , such that

$$\int_{\Omega} \sigma(u_{\text{res}}) : \epsilon(\psi) dV = \int_{\Omega} (f - \alpha \nabla p) \psi dV + \int_{\Gamma_N} t_N \psi dS, \quad \forall \psi \in X_h.$$

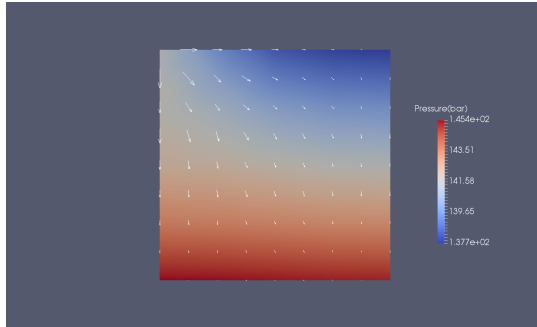
Here, f is the body force due to gravity.

Preliminary Results: Pressure profile and mass flux field in a reservoir with no-flux boundary



Injection of fluid occurs at top left.

Preliminary Results: Pressure profile and mass flux field in a reservoir with no-flux boundary



This is a lower rate of injection, so the hydrostatic pressure gradient dominates.

Note: the fluid flux is determined by the pressure gradient net of the hydrostatic gradient.

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Assessment

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Future Work

- Model fluid flow in fracture separate from reservoir flow
- Allow for textured, i.e. non-planar fractures
- Incorporate into the Subflow subsurface transport simulator utilizing the mimetic finite difference library MTK for the advection/diffusion equations