# The Technology of the Joint Inversion of Conventional Well Logs for Evaluation of Double Porosity Carbonate Formations*

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## Abstract

This work presents a new technology of well log interpretation for determining lithology, pore system characteristics, and saturations of double-porosity carbonate formations. This technology includes a unified petrophysical model for calculating the effective physical properties of carbonates and joint inversion of the following conventional logs: P- and S-wave transit times, resistivity, density, neutron porosity, gamma ray, and photoelectric absorption property. The inversion procedure consists in minimizing the cost function, which contains the sum of weighted square differences between the measured and calculated logs, and the regularization functional with additional information, initial model parameters, and ranges of their variations. The following petrophysical parameters are obtained by the inversion: a) volumes of limestone, dolomite, and shale; b) porosities of matrix, fractures, and vugs; c) connectivity of fractures and vugs; and d) initial and residual oil saturations in different pore-systems.

To calculate the theoretical logs, we introduce the formation model that corresponds to a transversely isotropic medium composed of intercalated layers of carbonates and shales. The carbonate rocks are treated as a composite material that consists of a homogeneous isotropic matrix (solid skeleton and matrix pore system) where the secondary pores, represented by spheroids, are embedded. The saturation model includes different distributions of fluids (connate water, oil, gas, mud filtrate) in the matrix and secondary-pore systems for the invaded and virgin zones. For computing the effective physical properties of such composite media, we apply the symmetrical, self-consistent, effective medium method.

The results of petrophysical inversions obtained for various boreholes from vuggy and fractured carbonate reservoirs, show a good correspondence with geological information, core data, and image logs. Based on the numerous formation evaluations, we demonstrate that the technology developed improves the classification of carbonate lithotypes, determination of initial and residual saturations, and permeability prediction.

## Introduction

The determination of the pore microstructure in double-porosity formations is one of the most important problems in carbonate reservoir characterization. The integrated analysis of all borehole data is required in order to differentiate the matrix porosity from the secondary porosity and define the dominant secondary pore type at the borehole scale. This integrated interpretation can be performed based on a unified pore microstructure model of double-porosity carbonates and a joint simultaneous inversion of several well logs representing different physical properties.

The approach to evaluate secondary porosity in vuggy carbonate formations using acoustic velocities and electric conductivity was proposed by Brie et al. (1985). The authors of this paper represented secondary pores as spherical inclusions in a homogeneous matrix and described the acoustic and electric parameters by using the models of Kuster – Toksöz and Maxwell – Garnet models.

Rasmus and Kenyon (1985) carried out a joint interpretation of measured resistivity and dielectric permittivity to evaluate separately the amount of water in the oomoldic and intergranular pores. They calculated the electromagnetic properties of vuggy-porosity formations using the Maxwell – Garnett expressions for spherical shapes of secondary pores. To estimate dual porosity in carbonates Dutta et al. (1999) applied the analogous approach based on the models of Kuster – Toksöz and Maxwell – Garnet.

The technique of joint inversion of acoustic-wave velocities and electrical conductivities to determine the pore microstructure model in the carbonate double-porosity formations was proposed by Kazatchenko et al. (2007). This technique is based on the unified pore-space model where the secondary pores, approximated by ellipsoidal inclusions, are embedded in the solid and conductive homogeneous isotropic matrix with the primary porosity. The self-consistent effective media approximation method is applied to calculate the effective elastic moduli and electrical conductivity. This approach uses the simultaneous joint inversion of the following conventional logs: P- and S-wave transit times, resistivity, density, neutron porosity, and gamma ray. The petrophysical parameters determined by the inversion are limestone, dolomite and clay volumes, matrix, and fracture and vuggy porosities.

In this paper, we extend the technique proposed in (Kazatchenko et al., 2007) by including the log of photoelectric absorption property in the input data and by determining additionally to the mentioned petrophysical properties the connectivity of the secondary pores, and initial and residual oil saturations in different pore-systems.

The results of this joint inversion obtained for various boreholes from vuggy and fractured carbonate reservoirs show a good correspondence with core data, image logs, and geological descriptions. We have demonstrated with experimental data that the output parameters of the inversion improve the classification of carbonate lithotypes and the assessment of secondary pore interconnections required for permeability prediction.

## Unified Model of the Formation Microstructure and Methods for Synthetic Log Simulations

The petrophysical characterization of real rocks requires the application of a microstructure model that can be used for a simultaneous simulation of different physical parameters. Such a unified model for double-porosity carbonates that contains small-scale primary and large-scale secondary pores was proposed in (Kazatchenko et al., 2004). Primary pores are approximated by ellipsoids in this model, and solid grains and secondary pores of two different types are represented by spheroids with different aspect ratios. Variation of the aspect ratio allows the modeling of different types of secondary pores such as vugs (close to sphere inclusions), connected vugs (prolate spheroids), and cracks (oblate spheroids).

The physical properties of the medium are calculated by applying a four-step hierarchical homogenization. The first step includes the determination of the effective properties of the saturating mixture that contains connate water and oil. Fluids saturating secondary and matrix pores may contain the fluid components in different proportions. The conducting constituent (water) is considered as a host where non-conducting inclusions (oil or gas) are embedded (Markov et al., 2012). Non-conducting inclusions have variable spheroidal shapes with aspect ratio $\alpha_{fl}$. The volume of these inclusions determines the oil saturation and the shapes simulate different wettabilities of the rock (Kazatchenko et al., 2006). We calculate the effective electrical conductivity of the saturating mixture using the differential effective medium method (DEM) (Sen et al. 1981; Norris et al., 1985),

$$\frac{d\sigma^*}{dC_o} = \frac{1}{1-C_o}(\sigma_o - \sigma^*)R, \quad (1)$$

where $C_o$ is the concentration of non-conducting inclusions (oil), $\sigma_o$ and $\sigma^*$ are the electrical conductivities of oil inclusions and the conductivity of the fluid mixture, respectively, and $R = D_{kk}/3$. Tensor $D^{(i)}$ is an analogue of Wu's tensor (equation 5) for the electromagnetic field. In this case, the electrical conductivity is a function of the inclusion aspect ratio. The initial condition for the equation (1) is $\sigma^*(C_o = 0) = \sigma_w$, where $\sigma_w$ is the electrical conductivity of the water.

Elastic moduli are calculated as the volumetric average between water and oil moduli

$$\frac{1}{K^*} = \frac{C_w}{K_w} + \frac{C_o}{K_o}, \quad (2)$$

where $K^*$, $K_w$, and $K_o$ are effective elastic modulus and elastic moduli of water and oil, respectively, and $C_w$ and $C_o$ are water and oil volumetric concentrations.

In the second step, the matrix is considered as a frame composed by rock grains and primary pores with effective physical parameters determined for the saturating fluid in the first step. In the third step, the effective properties of a medium with secondary pores are found. To calculate the effective properties in the second and third steps we use the symmetrical effective medium approximation (EMA) developed by Berryman (1980, 1992) and Norris (1985). The EMA self-consistent method treats all components of the composite material equally, with none considered as a host. The model guarantees a non–zero conductivity for low porosity and non-zero shear modulus for high porosity media. Taking into account that the frequency range of well-logging measurements is lower than Biot's critical frequency, fluid flows in pores are neglected.

The general EMA equations for the elastic properties of the medium of N components were obtained by Berryman (1980, 1992) and Norris (1985):

$$\sum_{i=1}^{N} C_i (L_i - L^*) T^{(i)} = 0, \qquad (3)$$

where $C_i$ is a volumetric concentration of the i-th component ($\sum C_i = 1$), $L^*$ and $L_i$ are the elastic tensors of the effective medium and the i-th component respectively, and $T^{(i)}$ is Wu's tensor (Wu, 1966) that relates the strain tensor inside individual element of the i-th component ($\varepsilon_i$) with the uniform field of strain ($\varepsilon_0$) far from it, $\varepsilon_i = T^{(i)} \varepsilon_0$.

The components of tensor $L_i$ for an isotropic medium are defined as:

$$L_{jklm} = K_i \delta_{jk} \delta_{lm} + \mu_i \left( \delta_{jl} \delta_{km} + \delta_{jm} \delta_{kl} - \frac{2}{3} \delta_{jk} \delta_{lm} \right), \qquad (4)$$

where $K_i$ and $\mu_i$ are the bulk and shear moduli of the i-th component, and $\delta_{ij}$ is the Kronecker delta.

The solution of a one-particle problem for the strain of an ellipsoid placed in an infinite effective medium provides the $T^{(i)}$ tensor as

$$T^{(i)} = \left[ I + S^{(i)} (L^*)^{-1} (L_i - L^*) \right]^{-1} \qquad (5)$$

where $I$ is the fourth-order isotropic identity tensor, and $S^{(i)}$ is Eshelby's tensor (Eshelby, 1957). The components of Eshelby's tensor depend on $L^*$ and the aspect ratio of the element of the i-th component.

Norris et al. (1985) obtained the EMA equations for the electrical conductivity of an isotropic medium with ellipsoidal components:

$$\sum_{i=1}^{N} C_i \left[ \sum_{k=1}^{3} \frac{(\sigma_i - \sigma^*)}{\sigma^* + n_k^{(i)} (\sigma_i - \sigma^*)} \right] = 0, \qquad (6)$$

where $\sigma_i$ and $\sigma^*$ are the electrical conductivities of the i-th component and an effective medium, respectively, and $n_k$ is the depolarization factor that can be determined by using the appropriate ellipsoidal coordinate system (Landau and Lifshitz, 1960)

$$n_k = \frac{a_1 a_2 a_3}{2} \int_0^\infty \frac{ds}{(s + a_k^2) \sqrt{(s + a_1^2)(s + a_2^2)(s + a_3^2)}}, \qquad (7)$$

where $a_1, a_2, a_3$ are the semi-axes of the inclusion ellipsoid.

We consider that the ellipsoid aspect ratios of saturated primary pores and the spheroid aspect ratios of solid grains are functions of the matrix porosity. The introduction of varied aspect ratios is based on the geological and geophysical data which show the difference in the microstructure of porous rocks (pore-size distribution and its network connection) for high and low porosities (Keynon et al., 1989; Song et al., 2002; Kazatchenko et al. 2004; Yang and Alpin, 2007). We use the relationships between the aspect ratios and the porosity obtained by adjusting the calculated acoustic velocities and the electrical conductivity with empirical petrophysical data (Kazatchenko et al. 2004).

In the fourth step, we introduced a model for carbonate formations containing clays. Based on the geological information that clay in carbonate is mostly presented as thin layers (Cook and Mullins, 1983, Tucker and Wright, 1990), we describe carbonate rocks as a transversally isotropic medium comprised by clay and carbonate layers (Kazatchenko et al., 2007). The carbonate constituent of the model is presented by a composite with effective physical properties determined in the third step. In this model, the axis of macroscopic anisotropy (appeared due to formation texture) is perpendicular to the stratification plan and the effective physical properties are described by tensors. The tensors of effective parameters have only two principal components for a vertical borehole and horizontal layers: vertical (perpendicular to layers) and horizontal (parallel to layers).

Taking into account that in the acoustic logs the vertical velocities of elastic wave are measured, we can apply the average time equation for the P- and S – waves' travel times

$$t_v^P = t_c^P(1 - C_{cl}) + t_{cl}^P(C_{cl}) \text{ and} \qquad (8)$$

$$t_v^S = t_c^S(1 - C_{cl}) + t_{cl}^S(C_{cl}), \qquad (9)$$

where $t_v^P, t_c^P, t_{cl}^P, t_v^S, t_c^S, t_{cl}^S$ are the travel times of the vertical effective-velocity component of P - and S – waves in carbonates and clays, and $C_{cl}$ is the clay concentration.

The resistivity well logs in thick layers measure, due to the anisotropy paradox, the horizontal component of the conductivity tensor,

$$r_h^{-1} = r_c^{-1}(1 - C_{cl}) + r_{cl}^{-1}(C_{cl}), \qquad (10)$$

where $r_h$ , $r_c$ , $r_{cl}$ are the horizontal component of the effective resistivity tensor, and the carbonate and clay resistivities, respectively.

The effective density $\rho_v$ and total porosity $\phi_v$ in this model is given by

$$\rho_v = \rho_c(1 - C_{cl}) + \rho_{cl}C_{cl} \quad \text{and} \qquad (11)$$

$$\emptyset_v = \emptyset_c(1 - C_{cl}) + \emptyset_{cl}C_{cl} \qquad (12)$$

where $\rho_c, \rho_{cl}, \emptyset_c, \emptyset_{cl}$ are the densities and porosities of carbonates and clays, respectively.

The relationship between the gamma ray log and the clay concentration was approximated by a linear relation (Bassiouni, 1994)

$$\gamma_v = (\gamma_{cl} - \gamma_c)C_{cl} + \gamma_c, \quad (13)$$

where $\gamma_v, \gamma_{cl}, \gamma_c$ are the effective gamma and gamma values for clays and clean carbonates, respectively.

### Model of a Saturating Fluid Distribution in Invaded and Virgin Zones

We consider that the oil trapped by lifting of the oil-water contact during the field exploitation can be predicted by the evaluation of oil saturation in the invaded zone. We assume that fluid mixtures saturating primary and secondary pores contain water and oil in different proportions (different electrical resistivities). In a virgin zone, primary and secondary pores are saturated with mixtures of connate water and oil with the effective electrical resistivities $R_{f1}$ and $R_{f2}$ respectively (Figure 1). In the general case, $R_{f1} \neq R_{f2}$. Pores related to clay are saturated with a connate water. Resistivities $R_{f1}$ and $R_{f2}$ characterize the initial oil saturations ($S_{oi}$) in the pore systems. In the invaded zone, mud filtrate completely fills the connected primary and secondary pores that have higher permeability. Non-connected pores are filled with the same saturating fluids as in the virgin zone ($R_{f1}$ and $R_{f2}$). Pores related to clay keep their saturation of connate water. The residual oil saturation ($S_{or}$) depends on the number of pores (primary and secondary) non-washed with the mud filtrate (Figure 1).

For low-porosity matrix (<5%) the model can be simplified by representing the matrix as impermeable for oil (during initial saturation) and mud filtrate. The matrix is saturated in this case with connate water in both virgin and invaded zones.

### Joint Inversion of Well Logs

The joint inversion of well log data includes a solution of an optimization problem where a quadratic cost function is minimized for each measured point

$$F(z_i) = \|\boldsymbol{W}\_d\ (d(M) - d\_obs)\|^2 + \lambda_m \|\boldsymbol{W}_m (M - M_0)\|^2 + \lambda_s \|\boldsymbol{W}_s (M - \overline{M})\|^2, \quad (14)$$

where the vectors $d_{obs}$ and $d(M)$ are the measured and theoretically calculated well-log data, respectively. The variable $M$ is the vector of unknown pore microstructure parameters that have to be found. The vectors $M_0$ and $\overline{M}$ represent a reference model that can be given using *a priori* information and the model obtained on the previous point of inversion, respectively. $\boldsymbol{W}_d, \boldsymbol{W}_m, \boldsymbol{W}_s$ are the diagonal matrixes of weight coefficients. For each log, the selection of the weight coefficient $\boldsymbol{W}_d$ is based on the log dispersion and information about the log quality. The diagonal matrices $\boldsymbol{W}_m$, and $\boldsymbol{W}_s$ assign the weights for the model parameters based on *a priori* information. The regularization scalar parameters λ introduce relative weights between the misfit term (first term) and Tikhonov's stabilizers (second and third term) of the cost function. The minimal deviation from the reference model $M_0$ and the model obtained at the previous point of inversion $\overline{M}$ are provided by the optimal values of these parameters that keep the misfit between the simulated and measured logs within a prescribed error. We have applied the simplex-simulated annealing algorithm (Cardoso et al., 1996) to solve the optimization problem for the cost function in equation 14. This algorithm provides the stable global minimum in a large search space. The details of the inversion procedure are described in (Kazatchenko et al., 2007).

In this work, we formulate the inversion procedure that includes estimations of the original and residual oil saturations in addition to the pore system parameters. We have modified the cost function (14) adding to the vectors $d_{obs}$ and $d(m)$ the resistivity log with high investigation depth (LLD) and the log of the photoelectric absorption properties (PEF) to stabilize the lithology determination. We increase the vector $M$ of unknown pore microstructure parameters by adding the initial and residual oil saturations $S_{oi}, S_{or}$,

$$M = \left[\phi_{PP,}\phi_{v,}\phi_{fr,}\alpha_{v,}\alpha_{fr,}V_{Cl,}V_{D,}S_{oi}, S_{or}\ \right]^T, \qquad (15)$$

where $\phi_{PP}$ is the primary porosity, $\phi_v$ is vug porosity, $\phi_{fr}$ is microfractures porosity, $\alpha_v$ and $\alpha_{fr}$ are the aspect ratios of spheroids representing vugs and microfractures, respectively, and $V_{Cl}$and $V_D$ are the clay and dolomite volumes, respectively.

## Example of the Method Application on Experimental Data

The petrophysical inversion technique was fulfilled for the water-oil zone (Figure 2 and Figure 3). We apply the simplified model of fluid distribution with impermeable matrix due to the low matrix porosity. The formations are presented by dolomite with low concentration of clay. The processed intervals of the boreholes are characterized by total porosity of 10-20%. Secondary porosity is represented by vugs and fractures.

Comparison of the obtained residual saturation with laboratory measurements on cores shows a good agreement of these data (Figure 3).

## Conclusions

We have presented a new technique for petrophysical characterization of double porosity carbonate formations. The method uses the simultaneous joint inversion of conventional well logs. The technique permits quantitative estimation of matrix, vuggy and fracture porosities, determination of connected porosity that defined rock permeability, and obtaining the initial and residual oil saturation.

Application of the proposed technique in various boreholes and different oil fields demonstrates that the new approach in the well-log interpretation provides estimation of additional parameters of the pore microstructure of double-porosity formations. Based on the adequate petrophysical model we can improve significantly assessment of the initial oil saturation and predict residual saturation with high degree of probability.

Given the obtained results we consider the technique proposed as a complete and integrated system for the petrophysical characterization of double-porosity carbonate formations.

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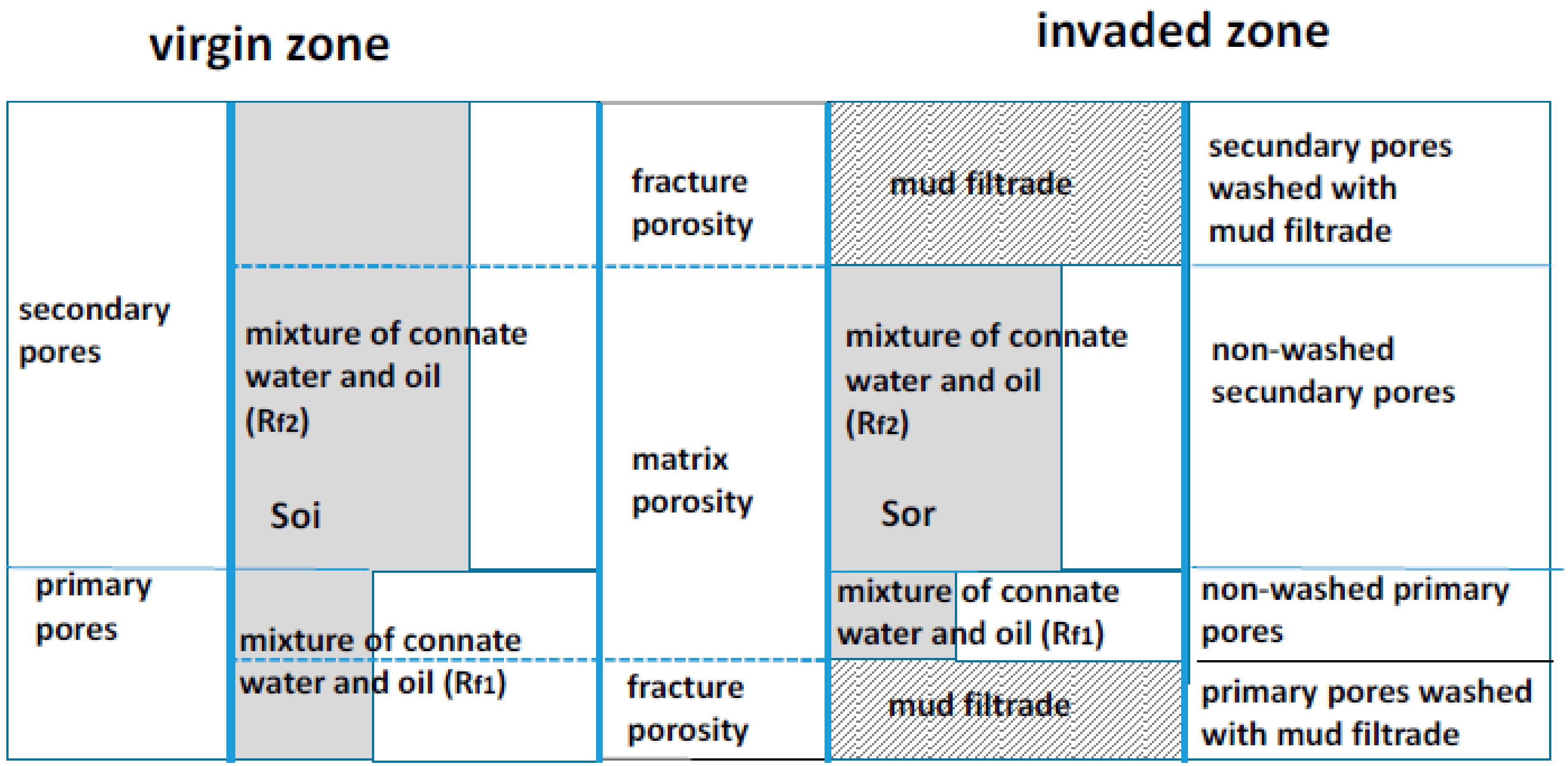
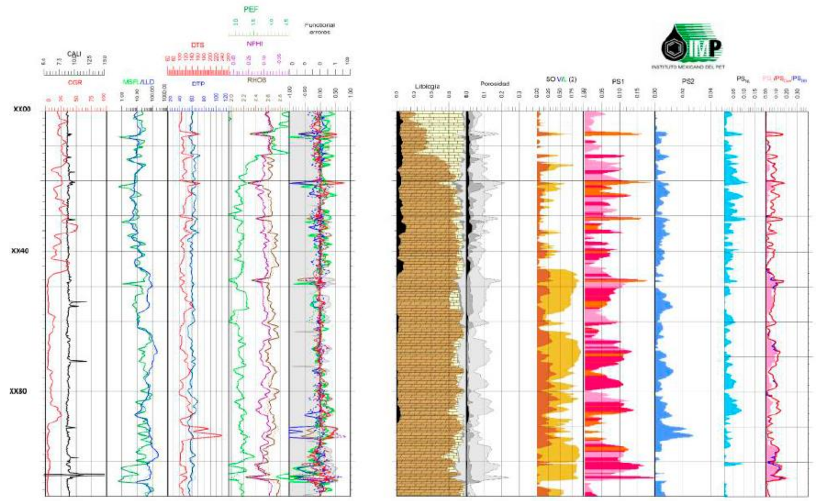



Figure 1. General model of a fluid distribution for a double-porosity medium in virgin and invaded zones.

Borehole A




Borehole B

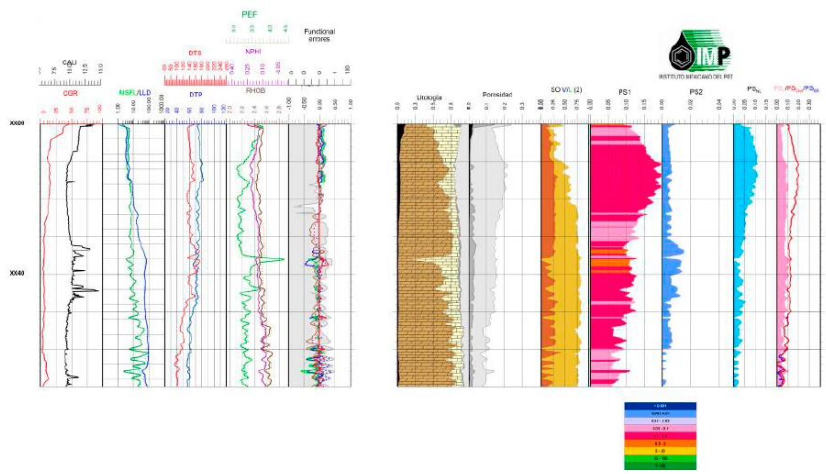



Figure 2. Results of the petrophysical inversion of experimental logs. Input data: track 1 – gamma log and caliper; track 2 – electrical resistivity measured by MSFL and LLD tools; track 3 – arrival times of P- (DTP) and S-waves (DTS); track 4 – density, neutron, and PEF logs. Solid lines represent measured logs and dotted lines show synthetic logs. Output data: track 5 – adjusting errors for the each log; track 6 – volumes of clay (black), dolomite (coffee), and limestone (yellow); track 7 – porosity types: secondary (bright grey), primary (dark grey), and pore associated with clay (black); track 8 – original oil saturation (yellow) and residual oil saturation (coffee); track 9 – vuggy porosity; track 10 – fracture porosity; track 11 - no connected secondary porosity; track 12 – connected secondary porosity. Rose area – pore flash with mud filtrate, rose line – connected pores determined by using percolation theory. The color legend corresponds to the secondary pore shapes (from microfractures to channels). The numbers are the aspect ratios of spheroids representing secondary pores.

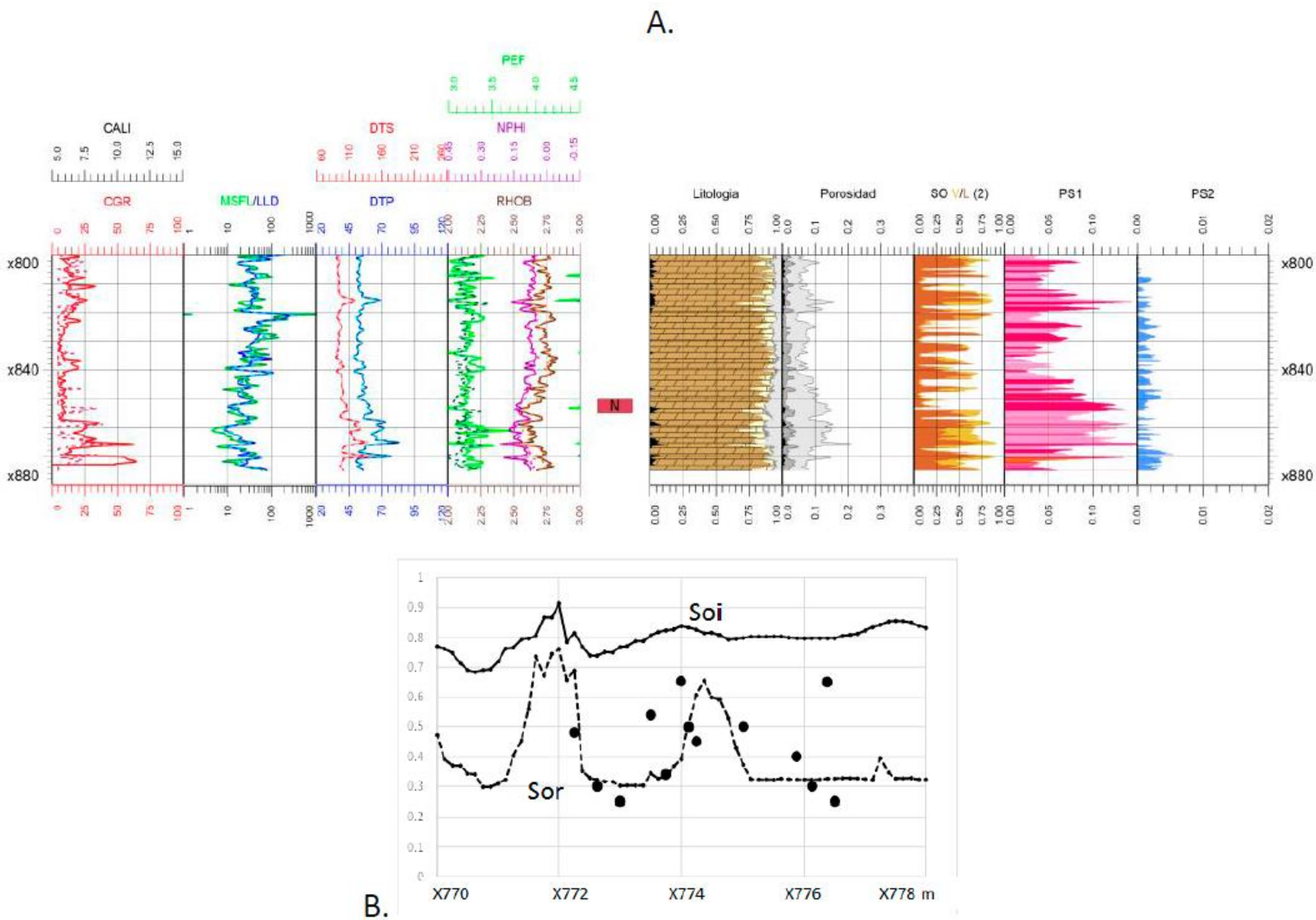



Figure 3. Borehole C: A. - Results of the petrophysical well-log inversion. B. - Comparison of the determined $S_{or}$ with laboratory measurements on the core (points).