Illumination Compensation Effect of Least Squares Migration on Seismic Imaging

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Extended abstract

Introduction

Seismic migration plays an increasingly crucial role in imaging complex regions, such as utilizing one-way wave equation migration (WEM) and reverse time migration (RTM)[1-2]. In comparison to velocity stack seismic sections, migration outcomes usually offer a more accurate positioning of subsurface structures. This leads to more reliable data for geological structure interpretation and well placement. However, as exploration depths increase and lithologic oil and gas reservoirs take precedence, conventional migration methods focused on structural imaging exhibit limitations that no longer meet the high-precision exploration requirements. As a result, seismic inversion technology has gained attention among scholars. It allows for the estimation of elastic parameters within reservoirs and provides quantitative descriptions with the support of petrophysics. In the 1980s, TARANTOLA[3] established a theoretical framework for seismic wave field inversion in the time domain, based on Bayes estimation theory. This approach enables direct inversion of elastic parameters in the underground medium using observed seismic data. The wave equation acts as a bridge between the observed data and the subsurface model, minimizing the total error between predicted and observed data. By doing so, an optimal estimation of subsurface medium parameters can be obtained.

Least squares migration refers to the migration outcome obtained through an inversion methodology framework. In one aspect, least squares migration corrects traditional migration results by using the inversion of the Hessian matrix[4]. This paper begins by presenting the fundamental theory of least squares migration and demonstrates that conventional migration results are essentially equivalent to applying the Hessian matrix to the least squares migration outcomes. The pre-stack migration result obtained conventionally is essentially a blurred and amplitude distorted least square imaging result[5]. Least squares migration imaging employs the inverse of the Hessian matrix to perform illumination compensation on conventional migration outcomes, thereby enhancing the resolution and fidelity of seismic imaging[6]. To begin with, this paper verify the illumination compensation effect of least squares migration by comparing the results of conventional one-way wave equation migration and least squares migration when dealing with missing shots in the Marmousi model. Subsequently, further experiments are conducted using Sigsbee model data. The salt body in the Sigsbee model obstructs the propagation of seismic waves, resulting in insufficient illumination beneath it. By comparing the outcomes of conventional one-way wave equation migration and least squares migration on the Sigsbee model data, we also validate the illumination compensation effect of least squares migration in the presence of salt bodies. Finally, the application of least squares migration to actual production data demonstrates its advantage in providing illumination compensation for deep layers and survey margins.

Method and Theory

Based on linear Born approximation of seismic wave field, seismic signal recorded by geophone is equivalent to the convolution of Green's function and seismic wavelet, which can be expressed as:

$$D(r, t \mid s, 0) = \int m(x)W(t) * G(r, t \mid x, 0) * G(x, t \mid s, 0)dx \qquad (1)$$

$D(r, t \mid s, 0)$ is seismic data recorded by observation system. $m(x)$ is the earth reflectivity model. $W(t)$ is source wavelet. $G(r, t \mid x, 0)$ and $G(x, t \mid s, 0)$ are Green's function from source s to scatter point x and from scatter point x to receiving point r respectively. The coordinate system is shown in Figure 1, and the matrix expression of formula (1) is:

$$d = Lm \qquad (2)$$

L is linear forward matrix operator, which is related to acquisition system, source wavelet, velocity model and density model. It represents the system of seismic wave field propagation.

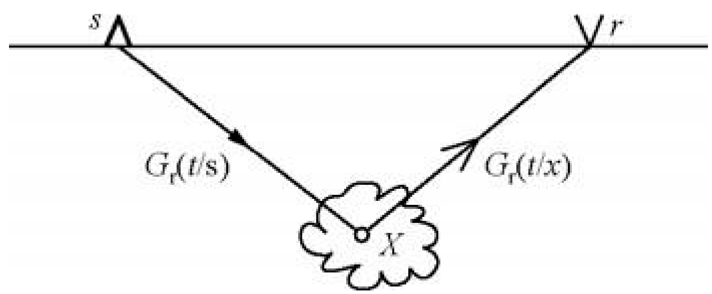


Figure 1. Schematic diagram of scatter model coordinate system

At present, the inversion operator is replaced by the transpose of the forward operator L in conventional migration, which can be expressed as:

$$m = L^T d \qquad (3)$$

From the formula provided above, it is evident that when approximating and simplifying the inversion operator as the transpose of the forward operator, the resolution and amplitude preservation of migration imaging may be reduced. To mitigate migration artifacts and achieve a more accurate geophysical model m, the concept of inversion is introduced. This involves constructing an error functional based on the principle of least square, which can be expressed as follows:

$$F(m) = \|d - Lm\|^2 \qquad (4)$$

By solving the objective function $F(m)$, the least square solution is obtained as follows:

$$m = (L^T L)^{-1} L^T d = H^{-1} L^T d \qquad (5)$$

In the given equation, m represents the outcome of least squares migration. H = $L^T L$ is defined as Hessian matrix . The inverse of Hessian matrix, H^{-1}, is also referred to as the illumination operator or deblurring operator, etc. $L^T d$ represents the result obtained through conventional migration. In the conventional migration algorithm, the inverse of the forward operator L^{-1} is always replaced by its transpose L^T, This substitution avoids the need for inverting the forward operator and enhances the stability of the solution process. However, without including the term $(L^T L)^{-1}$, conventional migration inevitably suffers from shortcomings such as low imaging resolution and poor amplitude preservation. The provided formula demonstrates that the outcome of least squares migration is obtained by applying the inverse of the Hessian matrix on conventional migration results. Conversely, conventional migration is the outcome of least squares migration filtered by the Hessian matrix. In other words, conventional migration represents the result of a blurred and amplitude-distorted least squares migration. In theory, least squares migration has the capability to compensate for illumination effects and improve amplitude preservation.

Least squares migration can be categorized into two methods: data domain and imaging domain. The imaging domain method involves solving the inverse operator of the Hessian matrix approximately and applying it to the conventional migration image. However, the accuracy of this method is currently relatively low. On the other hand, the data domain method avoids directly calculating the inverse operator of the Hessian matrix. Instead, it employs an iterative approach to gradually approximate the least squares migration result. The main calculation steps in data domain are as follows:

Step1: Calculate the residual of forward data and real observed data in step n, known that the migration result in step n is mn, L is forward operator, d is observed data, Lm_n is forward data. The residual d_{miss} is calculated by formula (6).

$$d_{miss} = Lm_n - d \qquad (6)$$

Step2: Use the migration operator to migrate the residual error to obtain the quantity of migration imaging correction δm by formula (7).

$$\delta m = L^T d_{miss} \qquad (7)$$

Step3: Calculate the iteration step length by formula (8).

$$k_n = \delta m^T \delta m / (L\delta m)^T (L\delta m) \qquad (8)$$

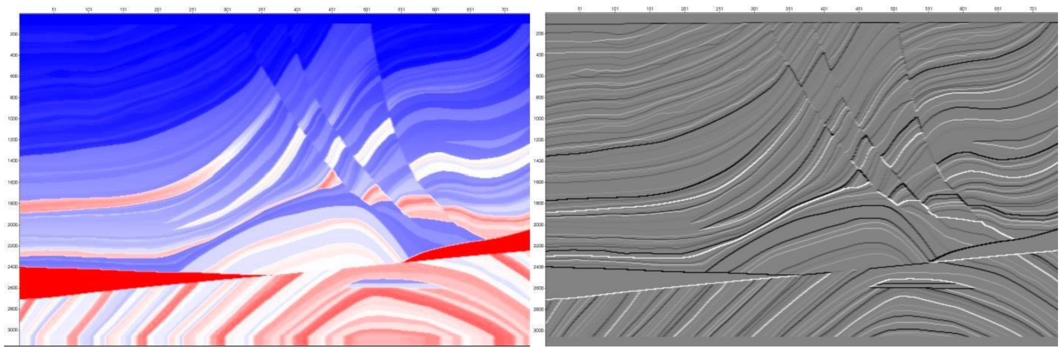
Step4: Calculate the updated migration result of least squares migration of step n+1 by formula (9)

$$m_{n+1} = m_n - k_n \delta m \qquad (9)$$

During the iterative process described above, the imaging results are continuously updated and corrected. The iteration continues until either the residual error decreases to the expected value or the iteration reaches the predefined number. At this point, the iteration process can be terminated, and an optimized imaging result can be obtained.

Example 1: Illumination compensation effect with shot missing

To validate the illumination compensation effect of least squares migration, we conducted tests using the Marmousi model data. The experiments were performed with a total of 369 shots, spaced 25 meters apart, effectively covering the entire Marmousi model from left to right. Similarly, we utilized 737 detector points, spaced at 12.5 meters intervals, to ensure comprehensive coverage of the Marmousi model. Each shot emitted a signal that was recorded by all detectors. In Figure 2, it can be observed the velocity field model and reflectivity model of the Marmousi model. These components provide crucial information for our testing and analysis.



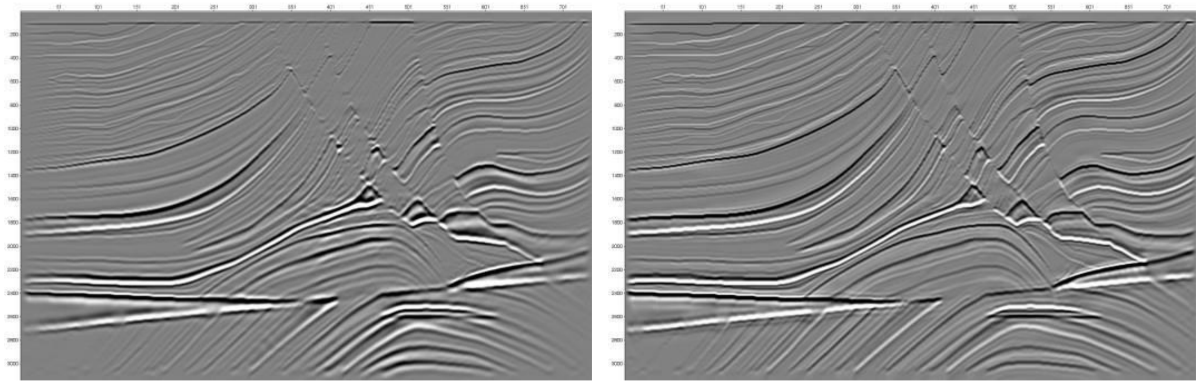
(a) (b)

Figure 2. (a) Marmousi velocity field model. (b) Marmousi reflectivity model.

Perform both least squares migration and conventional one-way wave equation migration in four different scenarios:

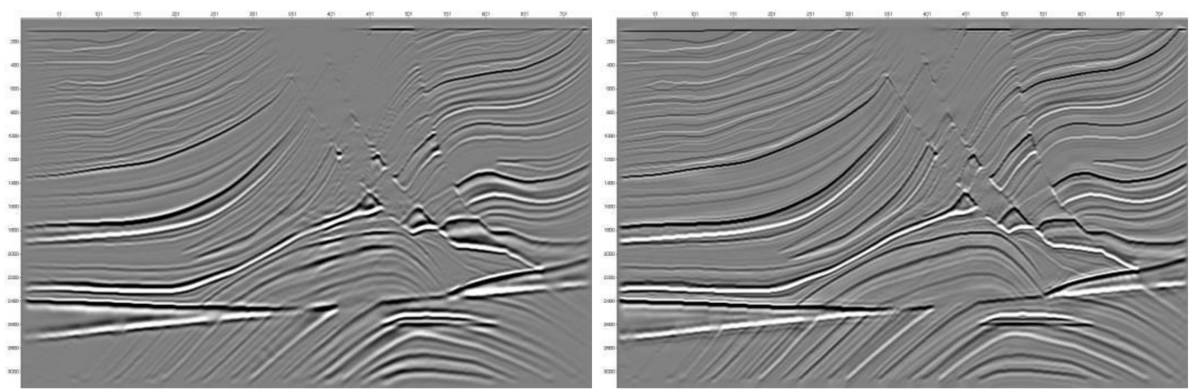
(a) No shots are missing.
(b) 60 shots are missing in the middle of section.
(c) 140 shots are missing in the middle of section.
(d) 120 shots are missing in the middle of section.

The corresponding results for each situation can be observed in Figures 3 to 6.



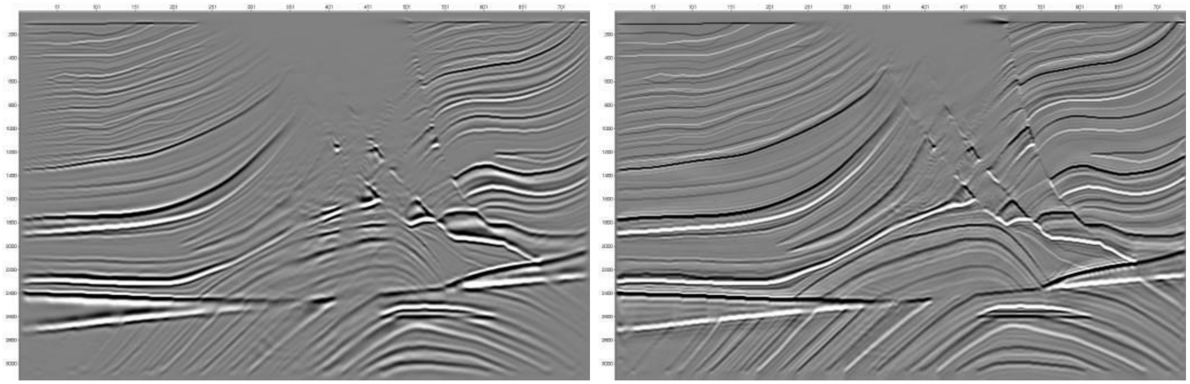
(a) (b)

Figure 3. Migration results of no shots are missing, (a) One-way wave equation migration, (b) Least squares migration.



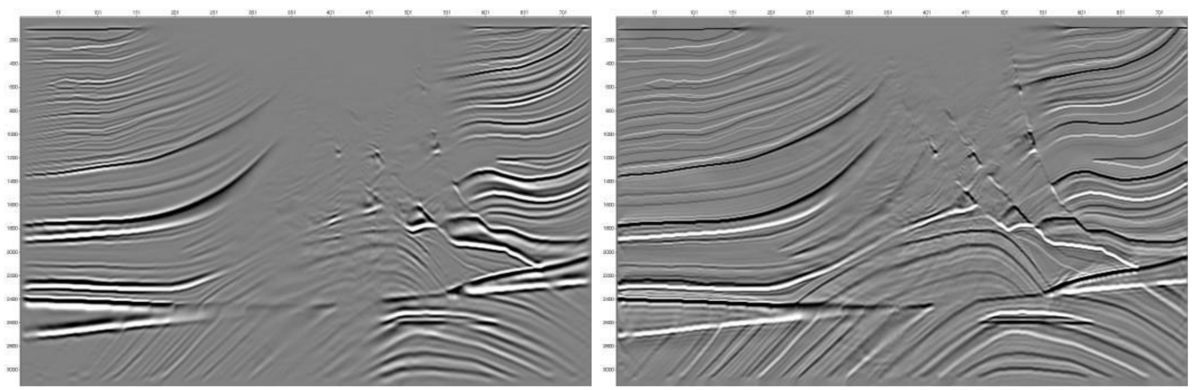
(a) (b)

Figure 4. Migration results of missing 60 shots in the middle of section, (a) One-way wave equation migration, (b) Least squares migration.



(a) (b)

Figure 5. Migration results of missing 140 shots in the middle of section, (a) One-way wave equation migration, (b) Least squares migration.



(a) (b)

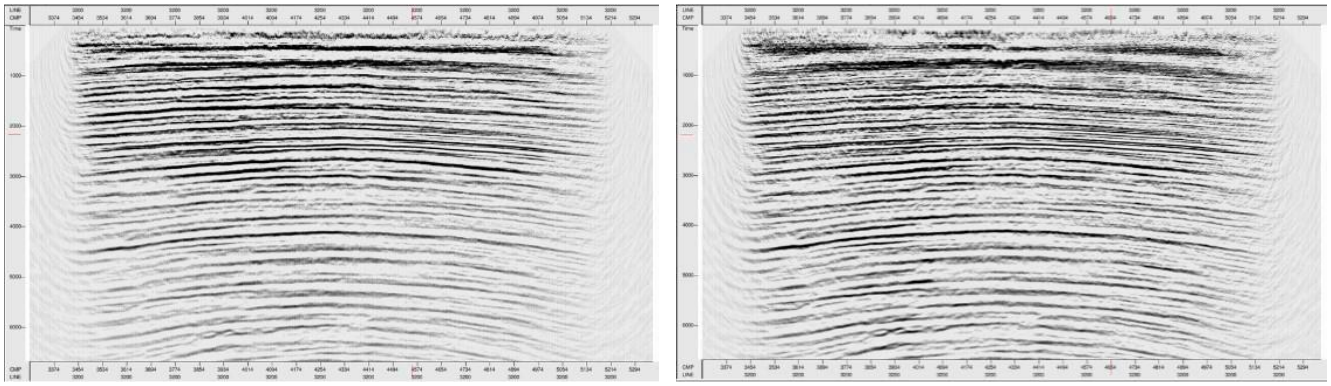
Figure 6. Migration results of missing 220 shots in the middle of section, (a) One-way wave equation migration, (b) Least squares migration.

We can draw the following conclusions based on the aforementioned outcomes:
(1) In scenarios where no shots are missing, least squares migration exhibits higher resolution and a more balanced amplitude section compared to conventional one-way wave equation migration. The results of least squares migration also align more closely with the true underground reflectivity.
(2) When shots are missing, regardless of the number of missing shots, least squares migration outperforms conventional one-way wave equation migration in terms of illumination effect.

(3) However, there are limitations to the illumination compensation effect of least squares migration. When a significant number of shots are missing, least squares migration may not achieve the ideal level of illumination compensation.

Example 2: Illumination compensation effect at survey margin

Least squares migration is applied to actual marine production data at Bock A. At the margin of the survey, the CMP fold is lower than in the middle. A comparison is made between the result derived from the conventional one-way wave equation method and that obtained through least squares migration. These comparative results are visually represented in Figure 7. It is evident that least squares migration effectively compensates the illumination effects at survey margin. The energy of least squares migration at survey margin is stronger than that of conventional one-way wave equation migration.



(a) (b)

Figure 7. (a) Conventional one-way wave equation migration result of actual marine data; (b) Least squares migration result of actual marine data

Example 3: Illumination compensation effect at subsalt layers

Sigsbee model data is used for further tests. Figure 8 displays the velocity field of Sigsbee model data. Due to the presence of a high-speed salt body in the Sigsbee model, the underlying layers suffer from a significant lack of illumination, as the salt body acts as a strong barrier. To address this issue, conventional one-way wave equation migration and least squares migration techniques were employed and their results are depicted in Figure 9. It is evident that conventional one-way wave equation migration fails to effectively address the problem of insufficient illumination in the subsalt layers. On the other hand, least squares migration provides a more suitable solution by compensating for the illumination of subsalt layers.

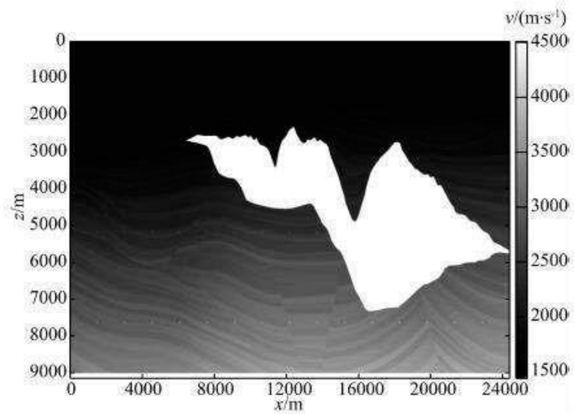
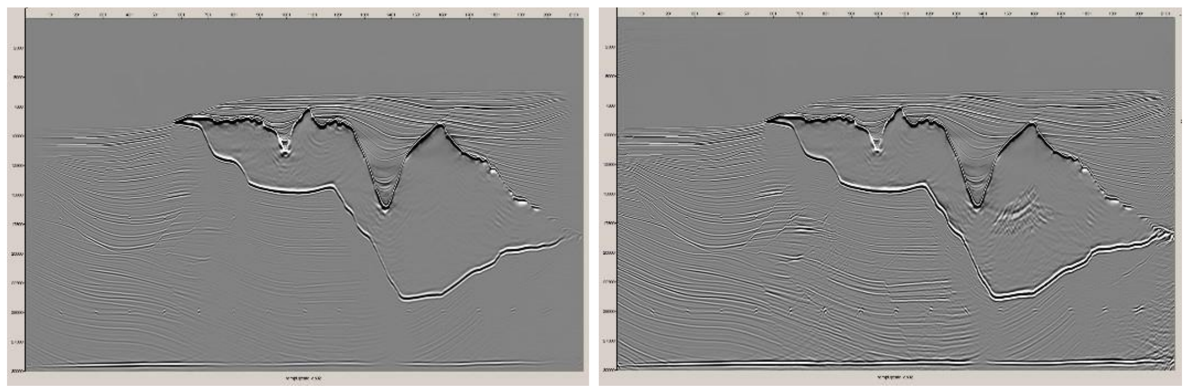


Figure 8. Sigsbee velocity model

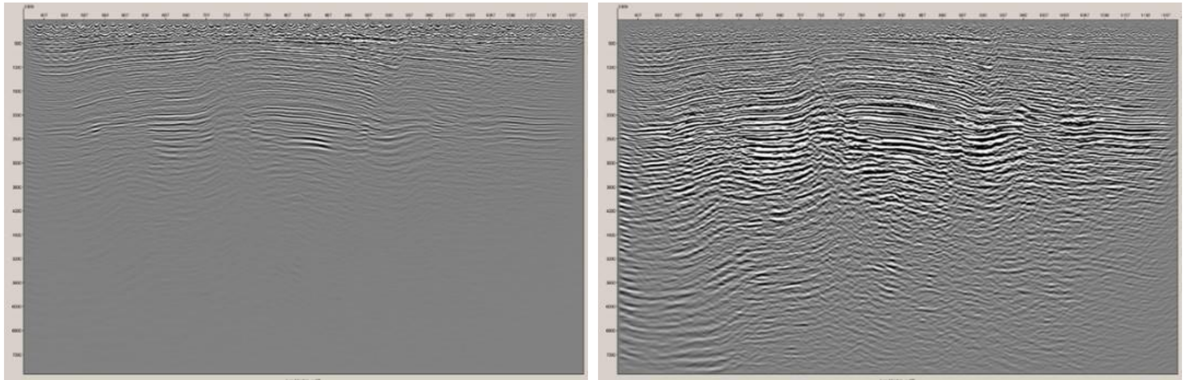


(a) (b)

Figure 9. (a) Conventional one-way wave equation migration result. (b) Least squares migration result.

Example 4: Illumination compensation effect on deep layers

Least squares migration is applied to actual production land data at Block B. A comparison is made between the results derived from the conventional one-way wave equation method and those obtained through least squares migration. These comparative results are visually represented in Figure10. It is evident that least squares migration effectively compensates the illumination of deep layers, resulting in stronger energy of the deep part.



(a) (b)

Figure 10. (a) Conventional one-way wave equation migration result of actual land data; (b) Least squares migration result of actual land data

Conclusions

Compared to traditional one-way wave equation migration methods, least squares migration offers a wider range of frequency bandwidth, improved imaging resolution, and increased amplitude consistency in seismic results. The effectiveness of illumination compensation through the use of least squares migration is demonstrated through the Marmousi model, particularly in cases involving missing shots, as well as in the imaging of subsalt layers in the Sigsbee model. Through practical implementation of least squares migration, it has been observed that this technique enhances the imaging of deep targets and survey margins by compensating for illumination issues.

Reference

1. Yang Qinyong, Duan Xinbiao. Development status and trend of reverse time migration technologyh [J]. Petroleum Geophysical Prospecting, 2010,49(1):92-98.

2. BAYSAL E, KOSLOFF D D, SHERWOOD J W C. Reverse time migration[J]. Geophysics, 1983, 48(11): 1514- 1524.

3. TARANTOLA A. Inversion of seismic reflection data in the acoustic approximation[J]. Geophysics, 1984, 49(8): 1259-1266.

4. KUEHL H, SACCHI M D.Split-step WKBJ least-squares migration/inversion of incomplete data[R]. Tokyo: The 5th SEG International Symposium-Imaging Technology.

5. REN H, WU R S, WANG H. Wave equation least square imaging using the local angular Hessian for amplitude correction[J]. Geophysical Prospecting, 2011, 59(4): 651-661.

6. Wang Huazhong, Hu Jiangtao, Guo song. Theory and method of least square prestack depth migration and imaging[J]. Petroleum Geophysical Prospecting,2017, 56(2). 159-170.