Stochastic Modeling of Inclined Heterolithic Stratification With the Bank Retreat Model

Michael J. Pyrcz* and Clayton V. Deutsch
Department of Civil & Environmental Engineering
University of Alberta, 114 St - 89 Ave. Edmonton AB T6G 2E1
mpyrcz@ualberta.ca

ABSTRACT
Inclined heterolithic stratification (IHS) sets are commonly encountered in the host rock of the Alberta oil sands. Important features in petrophysical properties have been identified with respect to these sets. Remnant shales constrained by the IHS geometries are of primary interest. Trends in the shale fraction and grain size trends (e.g. fining upward, lateral and distal) are also present. These geologic features may have a significant impact on the response to exploitation through steam assisted gravity drainage (SAGD).

The IHS sets are derived from lateral accretion architectural elements within a fluvial depositional setting. The bank retreat fluvial model is a simplified process based model of stream meander migration. This model is parameterized by hydraulic parameters and initialized with realistic channel stream lines from the dampened harmonic model. A realistic channel profile with correctly positioned thalweg may be fit to the migrating streamlines. The migration of this channel results in the generation of realistic lateral accretion geometries.

Multiple realizations of the IHS sets may be calculated with the bank retreat model. The result is unconditional surface based IHS set geometry models. Petrophysical properties may be fit to these models. The resulting models may be applied as training images for multiple-point simulation and to assess the impact of IHS geometries and heterogeneities on reservoir response.

Introduction
Inclined heterolithic strata (IHS) are characterized as siliciclastic sequences that are parallel to subparallel with original dips (Thomas et. al., 1987). These strata generally result from the lateral growth of large-scale bedforms such as point bars (lateral accretion elements). The internal geometry is characterized by growth units with fining outward trends. These units are separated by accretionary surfaces (Thomas et. al., 1987). The external geometry is described by Miall (1996) as point bar lenses flanked by shale filled abandoned channels.

IHS sets demonstrate a variety of characteristic trends and forms. For example Thomas et. al. (1987) identified seven possible grain size fining trends associated with IHS deposits. These grain size trends include fining upwards, fining distal and fining perpendicular to the IHS sets. These trends may have a
significant control of the porosity and permeability distribution of IHS set dominant sand bodies.

Improved modeling of these deposits will be of economic consequence. IHS sets are common in the MacMurray formation in Northern Alberta. This formation illustrates the interaction of fluvial and estuary processes over a cycles of various time scales. The estuary influence amplifies the fining trends and mud drapes along the accretionary surfaces with the addition of brackish to marine sediments in the IHS sets.

These accretionary surfaces may provide valuable control for the construction of IHS property trend models and in the assessment of recovery factors. Yet, the geometry of accretionary surfaces in IHS sets is complicated. The geometry is the result of channel sinuosity, channel geometry and meander development and avulsion frequency. These IHS sets geometries have been discussed by authors such as Diaz-Molina (1993), Thomas and others (1987) and Willis (1993).

An unconditional stochastic IHS sets geometry model is proposed to reproduce the complicated geometries of accretionary surfaces. This model is relies on (1) the disturbed periodic meander models for the seeding of realistic channel streamlines, (2) the FLUVSIM channel geometry for a realistic meandering channel profile and (3) the bank retreat model for the realistic channel meander migration.

**Disturbed Periodic Meanders Models**

The disturbed periodic meanders model has been shown to produce realistic channel streamlines (Ferguson, 1976). This model is based on a dampened harmonic model that is continuously disturbed.

\[
\theta + \frac{2h}{k} \frac{d\theta}{ds} + \frac{1}{k^2} \frac{d^2\theta}{ds^2} = \varepsilon(s)
\]  

(1)

where \( k \) is related to wavelength of the largest amplitude \( (k = 2\pi / \lambda) \), \( 0 < h < 1 \) is the dampening factor and \( \varepsilon(s) \) is the disturbance value. The physical analogy for this model is a pendulum dampened by air resistance and continuously hit by rocks. This model may be applied as a discrete approximation and results in streamlines that are based on clear parameters and with statistical properties similar to natural streamlines. Figure 1 shows example streamlines calculated with the disturbed periodic model with sinuosity of 1.1 to 1.8.
Figure 1 example streamlines calculated with the disturbed periodic model. The sinuosity is labeled on the left side.

Channel Geometry

The channel geometry is based on FLUVSIM, a publicly available fluvial object based model (Deutsch and Tran, 2002). This model is consistent with the channel profile expected in meandering streams (Easterbrook, 1969). Channels are parameterized by a streamline, relative thalweg location, stochastic depth and a width to depth ratio. The relative thalweg is calculated as a function of channel curvature. The relative thalweg location is based on the following equation.

\[
  a(y) = \begin{cases} 
  \frac{W(y)}{2} \left( 1 - \frac{C_i(y)}{C_i^l} \right) & C_i(y) < 0 \\
  \frac{W(y)}{2} \left( 1 - \frac{C_i(y)}{C_i^c} \right) & C_i(y) > 0 \\
  \frac{W(y)}{2} & C_i(y) = 0 
  \end{cases}
\]

Equation 2

where \(a(y)\) is the relative thalweg, \(C_i^l\) and \(C_i^c\) are the maximum channel curvature in the clockwise and counter clockwise directions, and \(C_i(y)\) is the local curvature. The channel cross section geometry is defined by Equation 3 for a thalweg closer to the left bank and Equation 4 for a thalweg closer to the right bank.
\[
d(w, y) = 4 \cdot t(y) \cdot \left( \frac{w}{W(y)} \right)^{b(y)} \cdot \left[ 1 - \left( \frac{w}{W(y)} \right)^{b(y)} \right]
\]

(3)

where \( b(y) = -\ln(2)/\ln(a(y)) \).

\[
d(w, y) = 4 \cdot t(y) \cdot \left( 1 - \frac{w}{W(y)} \right)^{c(y)} \cdot \left[ 1 - \left( 1 - \frac{w}{W(y)} \right)^{c(y)} \right]
\]

(4)

where \( c(y) = -\ln(2)/\ln(1 - a(y)) \).

\( d(w, y) \) is a function describing the channel cross section, \( t(y) \) is the local channel thickness, \( W(y) \) is the local channel width and \( w \) and \( y \) are the transverse and longitudinal coordinates. An illustration of the channel profile is shown in Figure 2.

Figure 2 channel profile based on FLUVSIM.

The channel parameters, location, thalweg and depth, are calculated at discrete locations along the streamline. Cubic splines are fit to these properties to allow for a smooth transition along the channel length and interpolation at any channel position. These cubic splines act as the backbone for the channel geometry (Wietzerbin and Mallet, 1993).

**The Bank Retreat Fluvial Model**

The bank retreat model is applied to predict the migration of meandering stream channels based on hydraulic and host material parameters. This model has been proposed by Howard (1992) and applied to construct fluvial facies models by Sun et al. (1996). Also, Lopez et al. (2001) applied the bank retreat model to construct fluvial reservoir models with proposed methods for honor areal and vertical trends.

The algorithm proceeds in the following order (1) seed a channel, (2) discretize the channel with control nodes, (3) calculate the near bank velocity at the control nodes, (4) calculate the node migration as a function of the near bank velocity and host material erosion coefficient and (5) migrate the nodes in a direction normal to the channel.
The equation for the near bank velocity is (Sun et al., 1996):

\[
\tilde{u}_{sb} = -bu_{s0}\tilde{C} + \frac{bC_f}{u_{s0}}\left[ \frac{u_{sb}^2}{gh_0} \right] \cdot \int_{s_0}^{s} \exp\left( \frac{-2C_f}{h_0} \right) \cdot \tilde{C}(s-s')ds'
\]

(5)

where \( \tilde{u}_{sb} \) is the near bank velocity, \( b \) is the channel half width, \( u_{s0} \) is the stream mean velocity, \( \tilde{C} \) is the local channel curvature, \( C_f \) is the friction coefficient, \( g \) is the gravitational constant, \( h_0 \) is the average depth of channel, \( A' \) is a positive factor describing the scour factor and \( s \) is the coordinate along the channel. The integration component accounts for the inertial effects on the near bank velocity.

The channel migration is calculated with (Sun et al., 1996):

\[
\xi = E\tilde{u}_{sb}
\]

(6)

where \( \xi \) is the channel migration distance, \( E \) is the local erosion coefficient and \( \tilde{u}_{sb} \) is the near bank velocity calculated in Equation 5.

Time steps are applied to model the channel migration. As the channel migrates an underlying facies model is modified with the formation of channel, point bar and abandoned channel architectural elements. The erosion coefficient grid is modified as a function of facies.

**Methodology**

The proposed methodology for constructing stochastic IHS set geometries is (1) apply the disturbed periodic model to generate a realistic seed channel, (2) fit the FLUVSIM channel geometry to the sinuous channel streamline, (3) apply a modified bank retreat model to model streamline meander migration and (4) continue migrating and avulsing channel streamlines until the target NTG is reached.

The bank retreat model is modified with the addition of a uniform erodability coefficient. This uniform erodability coefficient is set during each time step such that the maximum streamline displacement is equivalent to a user supplied maximum thickness of the IHS couplet. This provides the user with explicit control over the IHS thickness, but removes the feedback of facies on subsequent meander migration.

The user supplies a probability of avulsion proximal of the model. Prior to each meander step, there is a random draw to determine if avulsion occurs. Avulsion is integrated into the model by coding the current channel as abandoned channel and generating a new channel streamline independent of the current streamline.
The avulsion probability controls the extent of the IHS sets and the frequency of abandoned channel fills.

The model is constructed from the bottom up. A simplified aggradation schedule is applied to improve control over the degree of amalgamation of the IHS complexes and the preserved thickness of the IHS sets. The number of levels and the elevation of each level are set by the user. The algorithm migrates and avulses channels until the NTG is reached for the current level and then advances up to the next level. The channels within a level are all constant elevation and the elevation is constant within each channel. It is assumed that the model is corrected for stratigraphic correlation.

For each time step the bottom surface of the channel is stored in an array. A post processing step is applied to apply erosion rules. The model output is a surface based model of IHS geometry.

An example IHS sets geometric model is shown in Figure 3. This model is based on low sinuosity channels (mean sinuosity of 1.2) with an average channel width of 400 meters, accretionary surfaces with a maximum spacing of 20 meters and infrequent avulsions (0.02 probability of avulsion for each time step).
Conclusions
These IHS sets geometric models may be applied to construct property models. There is a wide variety of anticipated applications for the IHS set training images. (1) These training images may be applied to aid in the inference to input statistics for conventional semivariogram and multiple-point based geostatistical models (Strebelle, 2002). (2) They may be utilized in comparative flow studies and for the calculation of recovery factors for reserves and to assess connectivity. (3) Multiple models with a variety of geometries may be calculated for scenario based uncertainty analysis. In general these models will aid in quantifying the impact of IHS trends and mud drapes along accretionary surfaces on reservoir response.

References
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