General Anisotropic Travel Times – A Purpose Built Solution

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Summary

Seismic migration and data processing are dependent on travel time functions in the image region. This paper presents a novel method of using purpose built computer hardware for general anisotropic media. The eikonal solver for isotropic media is extended to any general twenty-one elastic constant anisotropic media. The examples used illustrate VTI media, but the methodology is applicable to any general anisotropy. In this work using hardware for geophysical algorithms such as finite difference methods and ray tracing is extended to travel times.

Introduction

The eikonal equation is a non-linear partial differential equation, which has many applications in geophysical modeling and inversion [1, 2 and 3]. To simulate earthquake travel times computer methods were pioneered by Vidale [4] for an expanding grid rather than tracing rays in the isotropic inhomogeneous media However, the earth is not always isotropic and structures and media can be shown to be anisotropic. Media in which the velocity of propagation is dependent on direction is anisotropic.

Recently hardware for isotropic travel time engines have been designed [5] for seismic applications, ray tracing in hardware [6], and least time fast marching methods [7]. These innovations were compared to multi-core and GPU computing models in [8] and found to be very competitive. The primary hardware models used hardware description languages VHDL and field programmable gate arrays, FPGA's. These transformations into hardware allow for intrinsic parallel and pipeline data movements. These hardware devices for geophysical applications [9,11,12, and 13] are causing disruptions in the use of computer technology [10], so much so that orders of magnitude in performance improvement are possible if a suitable algorithm mapping can be found.

Thus hardware methods can be applied to resolve ray paths, parameters, and travel times [14, 15, and 16]. The above work was directed to the solution of travel times in isotropic media. In this paper, we examine the requirements for algorithms, and successful implementations for a general two dimensional anisotropic media.

Theory and Method

The partial differential form of the eikonal equation is shown in equation (1).

$$(\delta T/\delta x)^2 + (\delta T/\delta y)^2 + (\delta T/\delta z)^2 = S^2 \qquad (1)$$

Here the solution methods in two and three dimensions for isotropic in-homogeneous media are extended to anisotropic media. Using finite difference grid based isotropic algorithms; the slowness function is expanded to be angle dependent.

A seed region is created with a small area of pre-computed solution, for the remaining region is expanded around this area. The edge of the region, which has known, and unknown solution grid points is called the active region. In this active region some points have current solution times, which are smaller than all the rest. If these points are used for the next computation a meaningful physical solution is obtained, and further the model parameters can have significant jumps in value. The grid spacing is *h*.

The Triangle

For isotropic media, the use of a grid allows for a simple geometric argument for computing new times. This is shown in Figure 1. The computation requires two known points to compute a third. The *Ta* and *Tb* points are known, to compute *Tc* we allow a linear interpolation of time between points *(a,b)*. The interpolation parameter is *mu* on the interval [0,1]. This interpolation value is added to the distance slowness time from the point (*c*) to the line *x=L(a,b,mu)* that computes the new time *Tc*.

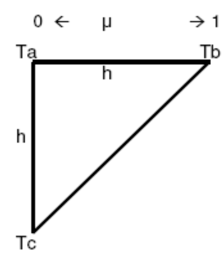

Figure 1 – Grid cell triangle geometry

For isotropic media we assume this slowness is a constant with the cell.

$$Tc = Ta + \mu(Tb - Ta) + \sqrt{(1+\mu^2)}\ (S*h) \qquad (2)$$

The least time to *Tc* is computed by taking the derivative of (2) with respect to *mu* and setting it to zero. Then we can solve for *mu*. In practice, this method has limitations but can be modified to work; the important consideration is to bound the solution for *mu* to the interval [0,1].

For general anisotropic media, the slowness function is defined as angle dependent. In the model results shown here we use a vertical transverse isotropic model, where the velocity (3) is a function of the angle theta. The angle theta Θ is measured from the vertical axis of symmetry. The equation for velocity is used and for each angle theta computed. Once the velocity V is known the slowness is determined, with a vertical velocity *Vv* of 2200 m/s and a horizontal velocity *Vh* of 2900 m/s.

$$V^2(\Theta) = Vh^2(\Theta)\sin^2(\Theta) + Vv^2(\Theta)\cos^2(\Theta) \qquad (3)$$

The active set of points have adjacent points which are in the initial state, because the nature of wave propagation, energy could be coming from any direction and to properly evaluate this least time point we must consider all points attached to the least time point as in the initial state. By examination if we find a point attached to the least time point as un-computed then we examine all adjacent possible solutions. In doing this we might have one or more solutions, it is the minimum of all possible solutions, which is needed to replace an initial state. Once this new time is computed it is placed the time array, making it part of the active set and placing it on the sort list for further evaluation. The data are on a 5 by 5 grid that supports the evaluation process in a structured way. An FPGA can be organized to support these calculations in parallel.

Data Examples

To illustrate the need for these eikonal solutions examples travel time contours of a simple geological structure of uniform material are shown in Figure 2a, 2b, and 2c.

To model this geological material with a simulation we assume a horizontal velocity of 2900m/s and a vertical velocity of 2200m/s. In Figure 2a the constant velocity first arrival travel time curves are shown, the velocity used here is isotropic and 2550m/s. As indicated all the plot contours are circles around the source point located at (400, 0)m. In Figure 2b we see the anisotropic velocity model results, and now see time spreading faster in the horizontal direction as expected. To complete the examples of this solution technique the velocity values are reversed the vertical is now 2900m/s, the results are shown in Figure 2c. Notice the expansion of the travel time curves as they are perpendicular at the z=0 surface, this expansion of the lines of constant time is because the horizontal velocity is the slowest.

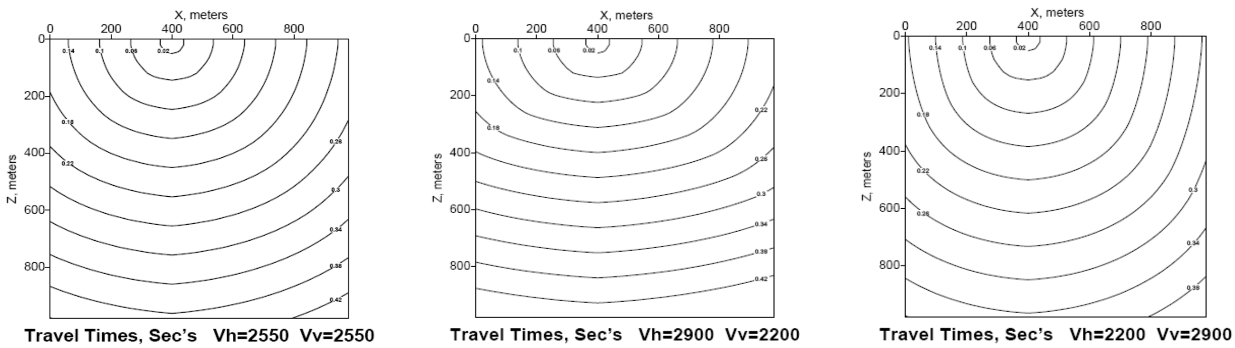

Figure 2(a=isotropic media with constant velocity, b-anisotropic horizontal, c-anisotropic vertical)

Hardware Description

The hardware platform utilized was the Berkeley Emulation Engine, a third generation commercial FPGA based computer system commonly known as BEE3 (BeeCube). Each BEE3 module contains four large Xilinx Virtex-5 LXT/SXT FPGA chips, up to 64GB DDR DRAM, and eight 10GigE interfaces for inter-module communication. BEE3 targets a wide range of application domains, including system emulation and simulation acceleration of multi-processor computer systems [17, 18, and 19].

The proposed system architecture of this paper comprised of one shared memory controller, and a calculate engine processor implementation on Xilinx FPGAs. Given hardware's ability to process in parallel, we can also explore the option of multiple instantiations of the calculating engine. Figure 6 illustrates a parallel processing architecture across four FPGAs. With this parallelization, it will provide a significant performance improvement.

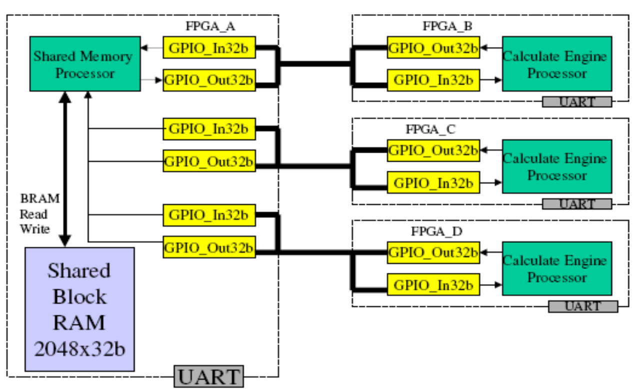

Figure 6 – Multi-FPGA Architecture

Future Work

The transformation of the eikonal algorithm into VHDL using an appropriate fixed point binary representation is next. The data movement for triangle evaluation has been tested and is working. Once a single triangle calculation has been demonstrated, a set of four triangle compute cells will be constructed for the FPGA. This will allow for a complete row of the 5 by 5 memory data structure to be mapped into the hardware.

Conclusions

The existing isotropic hardware methods have been extended to angle dependent anisotropic materials. These extensions are viable for use in geophysical algorithms. The new methods can be mapped into FPGA subsystems using available tools and hardware. Further work is needed to complete these mappings, but the basis for memory data movement has been demonstrated and that is the critical bottleneck. The triangle arithmetic processes have been reduced to the minimum by new matrix transformations.

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